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OF THE

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VOL. III.

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CONTENTS.

VOLUME III.

1845–1847.

APPENDICES.

INDEX

OF CONTRIBUTORS' NAMES TO PROCEEDINGS.

VOLUME III.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1844–45. No. 48.

November 11, 1844.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

HIS Grace the Duke of Leinster, Most Noble the Marquis of Downshire, Baron Farnham, and Baron Wallscourt, were elected Members of the Academy.

The Chair having been taken, *pro tempore*, by the Rev. J. H. TODD, D.D., V.P., the President gave an account of some additional researches in the theory of Quaternions, or of a new system of Imaginaries in Algebra.

In the theory which Sir William Hamilton submitted to the Academy in November, 1843, the name *quaternion* was employed to denote a certain quadrinomial expression, of which one term was called (by analogy to the language of ordinary algebra) the *real part*, while the three other terms made up together a trinomial, which (by the same analogy) was called the *imaginary part* of the quaternion: the square of the former part (or term) being always a positive, but the square of the latter part (or trinomial) being always a negative quantity. More particularly, this imaginary trinomial was of the form $ix + jy + kz$, in which x, y, z were three real and independent coefficients, or *constituents*, and were, in several applications of the theory, constructed or represented by three rectangular coordinates; while i, j, k were certain *imaginary units*, or symbols, subject to the following *laws of combination* as regards their *squares and products*,

$$i^2 = j^2 = k^2 = -1, \tag{A}$$

$$ij = k,\ jk = i,\ ki = j, \tag{B}$$

$$ji = -k,\ kj = -i,\ ik = -j, \tag{C}$$

but were entirely *free from any linear relation* among themselves; in such a manner, that to establish an equation between two such imaginary trinomials was to equate *each* of the three constituents, xyz, of the one to the corresponding constituent of the other; and to equate two quaternions was (in general) to establish FOUR separate and distinct equations between real quantities. *Operations* on such quaternions were performed, as far as possible, according to the analogies of ordinary algebra; the *distributive* property of multiplication, and another, which may be called the *associative* property of that operation, being, for example, retained: with one important departure, however, from the received rules of calculation, arising from the abandonment of the *commutative* property of multiplication, as *not* in general holding good for the *mixture* of the new imaginaries; since the product ji (for example) has, by its definition, a different sign from ij. And several constructions and conclusions, especially as respected the geometry of the sphere, were drawn from these principles, of which some have since been printed among the Proceedings of the Academy for the date already referred to.

The author has not seen cause, in his subsequent reflections on the subject, to abandon any of the principles which have been thus briefly recapitulated; but he conceives that he has been enabled to present some of them in a clearer view, as regards their bearings on geometrical questions; and also to improve the algebraical method of applying them, or what may be called the CALCULUS OF QUATERNIONS.

Thus he has found it useful, in many applications, to dismiss the *separate* consideration of the three real constituents,

x, y, z, of the imaginary trinomial $ix+jy+kz$, and to denote that trinomial by some *single* letter (taken often from the Greek alphabet). And on account of the facility with which this so called *imaginary* expression, or square root of a negative quantity, is constructed by a *right line having direction in space*, and having x, y, z for its three rectangular components, or projections on three rectangular axes, he has been induced to call the trinomial expression itself, as well as the line which it represents, a VECTOR. A *quaternion* may thus be said to consist generally of a *real* part and a *vector*. The fixing a special attention on this last part, or element, of a quaternion, by giving it a special name, and denoting it in many calculations by a single and special sign, appears to the author to have been an improvement in his method of dealing with the subject: although the general notion of treating the constituents of the imaginary part as coordinates had occurred to him in his first researches.

Regarded from a geometrical point of view, this algebraically imaginary part of a quaternion has thus so natural and simple a signification or representation in space, that the difficulty is transferred to the algebraically real part; and we are tempted to ask what this last can denote in geometry, or what in space might have suggested it.

By the fundamental equations of definition for the squares and products of the symbols i, j, k, it is easy to see that any (so-called) real and positive quantity is to any vector whatever, as that vector is to a certain real and negative quantity; this being indeed only another mode of saying that, in this theory, *every vector has a negative square*. Again, the product of any two rectangular vectors is a third vector at right angles to both the factors (but having one or other of two opposite directions, according to the order in which those factors are taken); a relation which may be expressed by saying, that the fourth proportional to the real unit and to any two rectangular vectors is a third vector rectangular to both; or, conversely, that the *fourth proportional to any three rectangular vectors* is a quantity *distinct from every vector*, and of the kind called *real* in this theory, as contrasted with the kind called *imaginary*.

Now, in fact, what originally led the author of the present communication to conceive (in 1843) his theory of quaternions (though he had, at a date earlier by several years, speculated on *triplets* and *sets** of numbers, as an extension of the theory of *couples*, or of the ordinary imaginaries of algebra, and also as an additional illustration of his views respecting the Science of Pure Time), was a desire to form to himself a distinct conception, and to find a manageable algebraical expression, of a *fourth proportional to three rectangular lines*, when the DIRECTIONS of those lines were taken into account; as Mr. Warren† and Mr. Peacock‡ had shewn how to conceive and express the fourth proportional to any three lines having direction, but situated *in one common plane*. And it has since appeared to Sir William Hamilton that the subject of quaternions may be illustrated by considering more closely, though briefly, this question of the determination of a fourth proportional to three rectangular directions in space, rather in a geometrical than in an algebraical point of view.

Adopting the known results above referred to, for proportions between lines having direction in a single plane (though varying a little the known manner of speaking on the subject), it may be said that, in the horizontal plane, "West is to South as South is to East," and generally as any direction is to one less advanced than itself in azimuth by ninety degrees. Let it be now assumed, as an extension of this view, that in *some analogous sense* there exists a fourth proportional

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* See Transactions of the Royal Irish Academy, vol. xvii. p. 422. Dublin, 1835.

† Treatise on the Geometrical Representations of the Square Roots of Negative Quantities, by the Rev. John Warren. Cambridge, 1828.

‡ Treatise on Algebra, by the Rev. George Peacock. Cambridge, 1830.

to the three rectangular directions, West, South, and Up; and let this be called, provisionally, *Forward*, by contrast to the opposite direction, *Backward*, which must be assumed to be (in the same general sense) a fourth proportional to the directions of West, South, and Down. We shall then have, inversely, Forward to Up as South to West; and therefore, as West to North: if we admit, as it seems natural and almost necessary to do, that (for directions, as for lengths) the inverses of equal ratios are equal; and that ratios equal to the same ratio are equal to each other. But again, Up is to South as South to Down, and also as North to Up: and we can scarcely avoid admitting, or defining, that (in the present comparison of directions) ratios similarly compounded of equal ratios are to be considered as being themselves equal ratios. Compounding, therefore, on the one hand, the ratios of Forward to Up, and of Up to South; and on the other hand the respectively equal (or similar) ratios of West to North, and of North to Up, we are conducted to admit that Forward is to South as West to Up. By a reasoning exactly similar, we find that Forward is to West as Up to South; and generally that if X, Y, Z denote any three rectangular directions such that A : X : : Y : Z, A here denoting what we have expressed by the word Forward, then also A : Y : : Z : X (and of course, for the same reason, A : Z : : X : Y); so that the *three* directions XYZ may be *all changed together* by advancing them in a *ternary cycle*, according to the formula just written, without disturbing the proportionality assumed. But also, by the principle respecting proportions of directions in one plane, we may cause *any two* of the three rectangular directions XYZ to revolve together round the third, as round an axis, without altering their ratio to each other. And by combining these two principles, it is not difficult to see that *because* Forward has been supposed to be to Up as South to West, *therefore* the same (as yet unknown) direction "Forward" must be supposed to be *to any direction* X *whatever*, as *any* direction Y, perpendicular to X, is to *that third* direction Z which is perpendicular to both X and Y, and which is obtained from Y by a right-handed (and not by a left-handed) rotation, through a right angle, round X; in the same manner as (and because) the direction West was so chosen as to be to the right of South, with reference to Up as an axis of rotation. Conversely we must suppose that if *any three rectangular directions*, XYZ, be arranged, as to order of rotation, in the manner just now stated, then Z : Y : : X : A; or in other words, we must admit, if we reason in this way at all, that the direction called already Forward, will be the fourth proportional to ZYX. And if we vary the order, so as to have Z to the left, and not to the right of Y, with reference to X, then will the fourth proportional to ZYX become the direction which we have lately called Backward, as being the opposite to that named Forward.

Again, since Forward is to Up as South to West, that is in a ratio compounded of the ratios of South to East and of East to West, or in one compounded of the ratios of West to South, and of any direction to its own opposite; or, finally, in a ratio compounded of the ratios of Up to Forward and of Forward to Backward, that is, in the ratio of Up to Backward, we see that the third proportional to the directions Forward and Up is the direction Backward: and by an exactly similar reasoning, with the help of the conclusions recently obtained, we see that if X be ANY *direction in tridimensional space*, then A : X : : X : B; B here denoting, for shortness, the direction which has been above called Backward.

The geometrical study of the relations between directions in space, combined with a few very simple and guiding principles respecting the composition of relations generally, might therefore have led to the conception or assumption of a certain *pair of contrasted directions*, namely, those which we have called Forward and Backward, and denoted by the letters A and B. And these are such that if we conceive a quantitative element to be combined with each, and give the name of POSITIVE UNITY to the unit of magnitude measured in the direction of *Forward*, but that of *Negative Unity* to the same magnitude measured backward; and if we extend to *this* positive unity and to lines having direction in space the received definitions of multiplication, that "Positive Unity is to Multiplier as Multiplicand is to Product," and that "the product of two equal factors is the square of either;" we may then consistently and naturally be led to assert the same results as those already enunciated from the theory of quaternions respecting the product of two vectors, in the two principal cases, first, where those two vectors are rectangular, and second, where they are coincident with each other. And thus may we justify, or at least interpret and explain, the fundamental definitions (A) (B) (C) of this theory, by regarding the symbols *ijk* as denoting three vector-units having three rectangular directions in space.

But farther, we derive from this view of the whole subject an illustration (if not a confirmation) of the remarkable conclusion that the so-called *real and positive unit* + 1 is not (in this theory) to be confounded with *any* vector unit whatever, but is to be regarded as of a kind essentially distinct from every vector. For this positive unit + 1 is in the direction above called Forward, and denoted by A. Now if this could coincide with a direction X in tridimensional space, then, whatever this latter direction might be supposed to be, we could always, by the general formula A : X : : Y : Z (where X is arbitrary), deduce the inadmissible proportion X: X : : Y : : Z, in which the two directions in one ratio are identical, but those in the other are rectangular to each other. If then we resolve to retain the assumption of the existence of a fourth proportional A to three rectangular directions in space, as subject to be reasoned on at all in the way already described, and as determined in direction by its contrast to its own opposite B (corresponding to an opposite order of rotation in the system XYZ), we must think of these two opposite directions A and B as merely *laid down upon a scale*, but must abstain from attributing to this SCALE any one direction rather than another in tridimensional space, as having such or such a zenith distance, or such or such an azimuth, rather than such or such another. And the progression *on this scale* from negative to positive infinity, obtained by combining a quantitative element with the contrast between two opposite directions, corresponds less to the conception of *space* itself (though we have seen that considerations of space might have suggested it) than to the conception of *time;* the variety which it admits is not *tri-* but *uni-* dimensional; and it would, in the language of some philosophical systems, be said to appertain rather to the notion of *intensive* than of *extensive* magnitude. Though answering precisely to the progression of the quantities called *real* in algebra, it has, when viewed from the geometrical side, somewhat the same sort of *imaginariness*, and yet (it is believed) of *utility*, as compared with lines in space, which the square root of an ordinary negative has, when compared with positive and negative quantities. This analogy becomes still more complete when we observe that (in this theory) the fourth proportional to any direction X in space, and either of the two directions A or B upon the scale, is the direction opposite to X; so that, *if a vector-unit* in any determined direction X *had been taken for positive unity*, then each of the two *scalar* units in the directions A and B (in common, it is true, with every vector-unit perpendicular to X) *might have been called*, by the general nomenclature of multiplication, *a square root of negative one.*

It is, however, a peculiarity of the calculus of quaternions, at least as lately modified by the author, and one which seems to him important, that it selects *no one direction in space as eminent* above another, but treats them as all equally related to that *extra-spatial,* or simply SCALAR direction, which has been recently called "Forward." In this respect it differs in its processes from the Cartesian method of coordinates, and seems often to admit of being more simply and directly applied to the treatment of geometrical problems, because it requires no previous selection of axes, rectangular or other. The author is, indeed, aware that the cooperation of other and better analysts will be necessary in order to bring the method of quaternions to anything approaching to perfection. But he hopes that an instance or two of the facility with which *some* questions at least allow themselves to be treated by this method, even in its present state, may not be without interest to the Academy. And he conceives that two examples in particular, one relating to the composition of translations, and the other to the composition of rotations in space, may usefully be selected for statement on the present occasion.

As preliminary illustrations of the operations employed, it may be remarked that for any system of lines having direction in space, it is required by many analogies (and is, for lines in one plane included among the definitions or results of the theories of Mr. Warren and Mr. Peacock), that the *sum* should be regarded as being equal to that one line which constructs or represents the *total effect* of all the different rectilinear motions which are expressed by the different summands. *Vectors* are therefore to be *added* to each other by a certain geometrical composition, exactly analogous to the composition of motions or of forces, and following the same known rules. *Scalars*, on the other hand (that is to say, the so-called real parts of any proposed quaternions), admitting only of a progression in quantity, and of a change of sign, without any other changes of direction, are to be added among themselves by the known rules of algebra, for the addition of positive and negative numbers. The addition of a scalar and a vector to each other can be no otherwise performed, or rather indicated, than by writing their symbols with the sign + interposed; each being, as we have seen, in some sense, imaginary with respect to the other. These operations of addition are all of the commutative, and also of the associative kind; that is to say, the order of all the summands may be changed, and any group of them may be collected or associated into one partial sum.

Scalars are *multiplied*, as well as added, by the known rules of ordinary algebra, for the multiplication of real numbers, positive or negative; because the positive unity of the system has been assumed to be itself a scalar, and not a vector unit.

For the same reason, *to multiply any vector by any scalar* a, is in general to change its length in a known ratio, and to preserve or reverse its direction, according as a is $>$ or < 0; the product is therefore a new vector, which may be denoted by $a\alpha$. The same new vector is obtained, under the form αa, when we multiply the scalar a by the vector α. If $a + \alpha$ and $b + \beta$ be two quaternion factors, of which a and b are the scalar parts, and α, β the vectors, then with a view to preserving the distributive character of multiplication, it is natural to define that the product may be distributed into the four following parts:

$$(a+\alpha)\ (b+\beta) = ab + a\beta + \alpha b + \alpha\beta.$$

And if the multiplicand vector β be decomposed into two parts, or summands, one $= \beta_1$ and in the direction of the multiplier α, or in a direction exactly opposite thereto, and the other $= \beta_2$, and in a direction perpendicular to the former (so that β_1 and β_2 are the projections of β on α itself, and on the plane perpendicular to α), then it may be farther defined that the *multiplication of any one vector* β *by another vector* α may be accomplished by the formula

$$\alpha\beta = \alpha(\beta_1 + \beta_2) = \alpha\beta_1 + \alpha\beta_2;$$

in which, by what has been shewn, the partial product $\alpha\beta_1$ is to be considered as equal to a scalar, namely, the product of the lengths of α and β_1, taken with the sign $-$ or $+$, according as the direction of β_1 coincides with, or is opposite to that of α; while the other partial product $\alpha\beta_2$ is a vector, of which the length is the product of the lengths of α and β_2, while its direction is perpendicular to both of their's, being obtained from that of β_2, by making it revolve right-handedly through a right angle round α as an axis. These definitions, which are compatible with the formulæ (A) (B) (C), and may serve to replace them, will be found sufficient to prove generally, and perhaps with somewhat greater geometrical clearness than those formulæ, the distributive and associative properties of quaternion multiplication, which have been already stated to exist. They give easily the following corollaries, which are of very frequent use in this calculus:

$$\alpha\beta + \beta\alpha = 2\alpha\beta_1 = -2\,\mathrm{AB}\cos(\mathrm{A}, \mathrm{B})\,; \tag{a}$$

$$\alpha\beta - \beta\alpha = 2\alpha\beta_2 = 2\gamma\mathrm{AB}\sin(\mathrm{A}, \mathrm{B})\,; \tag{b}$$

A and B denoting here the *lengths* of the lines α and β, and (A, B) the *angle* between them; while γ is a vector-unit perpendicular to their plane, and such that a right-handed rotation, equal to the angle (A, B), performed round γ, would bring the direction of α to coincide with that of β. For example, when $\beta = \alpha$, then $\mathrm{B} = \mathrm{A}$, $(\mathrm{A}, \mathrm{B}) = 0$, and

$$\alpha\beta = \beta\alpha = \alpha^2 = -\mathrm{A}^2,$$

so that the length A of any vector α, in this theory, may be expressed under the form

$$\mathrm{A} = \sqrt{-\alpha^2}. \tag{c}$$

More generally we have the equation

$$\alpha\beta - \beta\alpha = 0, \tag{d}$$

when the lines α and β are coincident or opposite in direction; while, on the contrary, the condition for their being at right angles to each other is expressed by the formula

$$\alpha\beta + \beta\alpha = 0. \tag{e}$$

These simple principles suffice to give, in a new way, algebraical solutions of many geometrical problems, of various degrees of difficulty and importance. Thus, if it be required, as an easy instance, to determine the length of the resultant of several successive rectilinear motions, or the magnitude of the statical sum of several forces acting together at one point, as a function of the amounts of those successive motions, or of those component forces, and of their inclinations to each other, we have only to denote the components by the vectors $a_1, a_2, \ldots a_n$, and their sum by a, the corresponding magnitudes being A_1, A_2, ... A_n, and A; and the equation

$$a = a_1 + a_2 + \ldots + a_n$$

will give, by being squared,

$$a^2 = a_1^2 + a_2^2 + \ldots + a_n^2$$
$$+ a_1 a_2 + a_2 a_1 + \ldots + a_1 a_n + a_n a_1 + \ldots;$$

that is, by the foregoing principles (after changing all the signs),

$$A^2 = A_1^2 + A_2^2 + \ldots A_n^2$$
$$+ 2 A_1 A_2 \cos(A_1, A_2) + \ldots + 2 A_1 A_n \cos(A_1 A_n) + \ldots;$$

a known result, it is true, but one which can scarcely be derived in any other way by so very short a process of *calculation*. For it is not *quite* so easy, on the *algebraical* side of the question, to see that

$$(\Sigma x)^2 + (\Sigma y)^2 + (\Sigma z)^2 = \Sigma(x^2 + y^2 + z^2) + 2\Sigma(xx' + yy' + zz'),$$

however easy this may be, as it is to see that

$$(\Sigma a)^2 = \Sigma(a^2) + \Sigma(aa' + a'a): \qquad \text{(f)}$$

although the *geometrical interpretation* of the first of these two formulæ is of course more obvious than that of the latter, to those who are familiar with the method of coordinates, and not with the method of quaternions.

Again, let us consider the more difficult problem of the composition of any number of successive rotations of a body, or, at first, of any one line thereof, round several successive axes, through any angles, small or large. Let the axis of the first of these rotations have the direction of the vector-unit a, ($a^2 = -1$), and let the amount of the positive rotation round this axis be denoted by a, which letter here represents still a scalar or real number. Let β be the revolving line, considered in its original position; β' the same line, after it has revolved through the angle a round the axis α. The part, or component, of β, which is in the direction of this axis, is that which was denoted lately by β_1; and the formula (a), when multiplied by $-\frac{1}{2}\alpha$, gives, as an expression for this part,

$$\beta_1 = \tfrac{1}{2}(\beta - \alpha\beta\alpha), \qquad \text{(g)}$$

because it has been supposed that $\alpha^2 = -1$. This part of β remains unaltered by the rotation. The other part, or component of β, is, in like manner, by (b),

$$\beta_2 = \tfrac{1}{2}(\beta + \alpha\beta\alpha); \qquad \text{(h)}$$

and this part is to be multiplied by $\cos a$, in order to find the part of β', which is perpendicular to α, but in the plane of α and β. Again, multiplying by α, we cause β_2 to turn through a right angle in the positive direction round α, and obtain, for the result of *this* rotation,

$$\alpha\beta_2 = \tfrac{1}{2}(\alpha\beta - \beta\alpha);$$

an expression which is the half of that marked (b), and which is to be multiplied by $\sin a$, in order to arrive at the remaining part of the sought line β', namely, the part which is perpendicular to the plane of α and β. Collecting, therefore, the three parts, or terms, which have been thus separately obtained, we find,

$$\begin{aligned}\beta' &= \beta_1 + (\cos a + \alpha \sin a)\beta_2 \\ &= \tfrac{1}{2}(\beta - \alpha\beta\alpha) + \tfrac{1}{2}\cos a\,(\beta + \alpha\beta\alpha) \\ &\qquad + \tfrac{1}{2}\sin a\,(\alpha\beta - \beta\alpha) \\ &= \left(\left(\cos\frac{a}{2}\right)^2 . \beta - \left(\sin\frac{a}{2}\right)^2 . \alpha\beta\alpha + \cos\frac{a}{2}\sin\frac{a}{2} . (\alpha\beta - \beta\alpha)\right.;\end{aligned}$$

that is,

$$\beta' = \left(\cos\frac{a}{2} + \alpha\sin\frac{a}{2}\right)\beta\left(\cos\frac{a}{2} - \alpha\sin\frac{a}{2}\right); \qquad \text{(i)*}$$

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* [*Note added during printing.*]—The printing of this abstract having been delayed, the Author desires to be permitted to append the following remarks:

the operations here indicated being thus sure to make no change in the part β_1, which is in the direction of the axis of rotation, but to cause the other part β_2 to revolve round that axis a through an angle $= a$. Again, let the same line β' revolve round a new axis of rotation denoted by a new vector unit a', through a new angle a', into a new position β''; we shall have, in like manner,

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If we should make, for abridgment

$$a \tan \frac{a}{2} = -\gamma,$$

the formula (i) for any single rotation might be thus written,

$$\beta' = (1 + \gamma)^{-1} \beta (1 + \gamma). \tag{i'}$$

And if we then made

$$\beta = ix + jy = kz, \quad \beta' = ix_, + jy_, + kz_,, \quad \gamma = i\lambda + j\mu + k\nu,$$

i, j, k, being the same three rectangular vectors, or imaginary units, as in the formulæ (A) (B) (C), but $x, y, z, x', y', z', \lambda, \mu, \nu$, being nine real or scalar quantities, we should obtain the same general formula for the transformation of rectangular coordinates (with the same geometrical meanings of the coefficients λ, μ, ν,) as that which Mr. Cayley has deduced, with a similar view, but by a different process, and has published, with other "Results respecting Quaternions," in the Philosophical Magazine for February, 1845.

The present writer desires to return his sincere acknowledgments to Mr. Cayley for the attention which he has given to the Papers on Quaternions, published in the above-mentioned Magazine: and gladly recognizes his priority, as respects the printing of the formula just now referred to. But while he conceives it to be very likely that Mr. Cayley, who had previously published in the Cambridge Mathematical Journal some elegant researches on the rotation of bodies, may have perceived, not only independently, but at an earlier date than he did himself, the manner of applying quaternions to represent such a rotation; he yet hopes that he may be allowed to mention, that a formula differing only slightly in its notation from the formula (i) of the present abstract, with the corollaries there drawn respecting the composition of successive finite rotations, had been exhibited to his friend and brother Professor, the Rev. Charles Graves, of Trinity College, Dublin, in an early part of the month (October, 1844), which preceded that communication to the Academy, of which an account is given above.

$$\beta'' = \left(\cos\frac{a'}{2} + \alpha' \sin\frac{a'}{2}\right) \beta' \left(\cos\frac{a'}{2} - \alpha' \sin\frac{a'}{2}\right); \qquad \text{(j)}$$

and so on, for any number n of rotations. Let the last position of β be denoted by β_n; and since it can easily be proved, by the theory of the multiplication of quaternions, that the continued products which present themselves admit of being thus transformed:

$$\left.\begin{array}{l} \left(\cos\frac{a^{(n-1)}}{2} + \alpha^{(n-1)} \sin\frac{a^{(n-1)}}{2}\right) \ldots \left(\cos\frac{a'}{2} + \alpha' \sin\frac{a'}{2}\right) \\ \qquad \left(\cos\frac{a}{2} + \alpha \sin\frac{a}{2}\right) = \cos\frac{a_n}{2} + \alpha_n \sin\frac{a_n}{2}; \\ \qquad \left(\cos\frac{a}{2} - \alpha \sin\frac{a}{2}\right)\left(\cos\frac{a'}{2} - \alpha' \sin\frac{a'}{2}\right) \ldots \\ \left(\cos\frac{a^{n-1}}{2} - \alpha^{(n-1)} \sin\frac{a^{(n-1)}}{2}\right) = \cos\frac{a_n}{2} - \alpha_n \sin\frac{a_n}{2}; \end{array}\right\} \text{(k)}$$

in which α_n is a new vector unit, and a_n a new real angle, we find that the result of all the n rotations is of the form

$$\beta_n = \left(\cos\frac{a_n}{2} + \alpha_n \sin\frac{a_n}{2}\right) \beta \left(\cos\frac{a_n}{2} - \alpha_n \sin\frac{a_n}{2}\right). \qquad \text{(l)}$$

It conducts, therefore, to the same final position which would have been attained from the initial position β, by a *single rotation* $= a_n$, round the *single axis* α_n; the amount and axis of this resultant rotation being determined by either of the two equations of transformation (k), and being independent of the direction of the line β which was operated on, so that they are the same for all lines of the body.

If the present results be combined with the theorem marked (R), in the account, printed in the Proceedings of the Academy, of the remarks made by the Author in November, 1843, it will at once be seen that if the several axes of rotation be considered as terminating in the points of a spherical polygon, and if the angles of rotation be equal respectively to the doubles of the angles of this polygon (and be taken with proper signs or directions, determined by those angles), then the total effects of all these rotations will vanish;

or, in other words, the body will at last be brought back to the position from which it set out.

Finally, it may be mentioned that the author is in possession of a general method for expressing by quaternions the tangent planes and normals to curved surfaces; and that in applying this method to find the cone of tangents enveloping a given sphere, and drawn from a given point, the geometrical impossibility of the problem, when the point is an internal one, is expressed by the square of a vector becoming in this case positive.

The special thanks of the Academy were voted to Doctor Simpson of Christiania, for his donation of a *fac simile* of an Irish MS. in the Library of Copenhagen.

DONATIONS.

Statutes of Trinity College (1844), *Dublin.*

Catalogue of the Egyptian Manuscripts in the Library of Trinity College, Dublin. By Edward Hincks, D. D.

A Catalogue of Roman Silver Coins in the Library of Trinity College. Presented by the Provost and Senior Fellows.

Recueil des Actes de la Séance Publique de l'Académie Imperiale des Sciences de St. Petersbourg, tenue le 29 *Decembre*, 1840, *et* 12 *Janvier*, 1843.

Memoires de l'Académie Imperiale des Sciences de St. Petersbourg; sixth series. *Sciences Mathématiques*, tom. 3, livraisons 4, 5, 6; tom. 4, livraison 1. *Sciences Politiques*, tom. 6, livraisons 4, 5, 6; tom. 7, livraisons 1, 2, 3.

November 30. (Stated Meeting.)

JAMES APJOHN, M. D., Vice-President, in the Chair.

Mons. Arago, of Paris, was unanimously elected an Honorary Member of the Academy.

Certain Returns ordered by the Council, on the 1st July, to be furnished to the Academy, were laid upon the table, viz.:

1. A List of all Papers or Essays read before the Academy, in the departments of Belles Lettres and Antiquities, which were referred to Council for publication, from the 17th March, 1828, to 17th March, 1844, containing the dates of such reading, the names of the authors, whether ordered by the Council for publication, and, if published in the Transactions, with the dates of the Council's order for publication.*
2. A statement of all payments made by the Academy for wood-cuts and engravings for Proceedings and Transactions, from 1816 to 1845, inclusive.
3. A statement of all payments made by the Academy for letter-press printing, from 1816 to 1845, inclusive.
4. Extract from Minutes of Council, June 29th, 1840, containing Mr. Petrie's contract to supply the Academy with 450 copies of his Essay on the Round Towers, at thirty shillings per copy.
5. A statement of premiums, in money and medals, given by the Academy from 15th March, 1828, to June 24th, 1844.
6. A statement of the available assets and liabilities of the Academy.

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* With the view of making this Return as full as possible, it also included the Papers on Science which were read, and it was continued to 24th June, 1844, when the session ended.

December 9, 1844.

REV. J. H. TODD, D.D., Vice-President, in the Chair.

Read, a letter from Rev. T. R. Robinson, D. D., on the periodical Meteors of the 10th August.

Rev. H. Lloyd gave an account of two remarkable halos and paraselenæ, observed in May and June last:

On the 27th of June, at $10^h\ 30^m$ P. M., a very remarkable phenomenon of paraselenæ was seen in Dublin. The moon was encompassed, as usual, by a halo, whose radius was about 22 degrees, but so faint, that its presence was unnoticed by some of the observers. A cross of light traversed the place of the moon, the arms of which were horizontal and vertical, the light fading off insensibly towards their extremities. The remaining parts of the space within the halo were darker than the surrounding sky. At the extremities of the horizontal diameter of the circle were two brilliant paraselenæ, having tails of light extending from the moon; of these the eastern was the most distinct. The whole phenomenon is represented in the lithograph sketch in the Appendix.

This beautiful phenomenon was witnessed by many observers. The appearances are briefly described in the records of the Magnetical Observatory, by the assistant whose duty it was to observe at 10 P. M.; and I have likewise received notes of them from Mr. O'Neill, formerly my assistant in the observatory (who has likewise furnished me with an interesting sketch), and from our Assistant Secretary, Mr. Clibborn.

The state of the sky at 10 P.M., shortly before the appearance of the phenomenon, is thus recorded in the day-book of the Observatory: "Sky all very lightly overcast; small dark masses of cumulo-stratus above the southern horizon, the moon shining weakly through them." The barometer had been rising uninterruptedly during the 27th, and the three days which preceded it, and continued to rise until the morning of the 29th; at the time of the phenomenon it stood at 30.052. The temperature of the air was 55.0, and that indicated by the wet-bulb thermometer 53.4. During the phenomenon the sky is described in Mr. Clibborn's notes as "covered with a white fog, apparently composed of a thin stratum of very opaque and fine vapour, sufficient to obscure the stars, which I do not recollect having noticed near the moon, nor within the limits of the imperfect circle." The halo was faint, excepting those parts of it which were at about the same altitude as the moon, and where the prismatic colours were distinctly visible. The intensity, however, was continually changing, apparently with the changes of brilliancy of the moon's disc. "The arch was all along quite imperfect at top, where we could at no time discover any trace of a false moon, or even any diffused light or appearance of arch."

"The true moon," writes Mr. Clibborn, "appeared to be elongated, and its outline was hazy and indistinct; that of the false moons was still more so. At times," he adds, "we were disposed to think there were three false moons on the eastern side, placed a little behind and above each other, with three distinct tails of light stretching towards the east. On the west, the moon was too diffuse to say that any appearance of the kind at any time presented itself."

"The whole phenomenon disappeared, by the clouds closing over the moon, at 7 minutes before 11 o'clock."

From some rough measurements made by Mr. Clibborn, it seemed that the two false moons were not exactly at the same altitude as the true moon, appearing to fall below it by about half its own diameter. "The false moon towards the east was the more perfect; and the tail which extended from it appeared to fade away into space, and must have been perceptible for 30 or 40 degrees."

The most remarkable, and at the same time the most uncommon feature of this phenomenon was the beautiful cross, which was sharply defined and distinct throughout, except at the ends of the arms, where its light gradually melted away. On looking through the numerous notices of paraselenæ, which are to be found in the early volumes of the Philosophical Transactions, I find but one, seen by Hevelius at Dantzic, in the year 1660, which was similar to the present phenomenon, or to that of the preceding month described by Dr. Robinson. The non-appearance, in any instance, of a complete vertical circle, seems to forbid the supposition that this cross can have been produced, like the horizontal white circle, by reflection from the *facets* of the prisms of ice. It is probably a phenomenon of *diffraction ;* and indeed it is described by one of the gentlemen who witnessed it, as resembling the cross of light which one sees, in looking at the sun or any bright object, through the silk of an umbrella. It would be important, with reference to the physical explanation of the phenomena, that the light, both of the horizontal circle and of the cross, should be analyzed with a tourmaline or double-refracting prism. The received explanation of the former may thus be easily tested ; for, it follows from the hypothesis upon which that explanation rests, that the light of the circle must be partially polarized in every part, the polarization increasing with the distance from the moon or sun on either side, up to a certain angle, at which it should be complete, and again diminishing from that point to 180° of distance, where it should disappear.

A lunar halo, with a pair of false moons, similar to that above described, but without the cross, was seen at Bandon, in the County of Cork, on the night of the 1st of May. The appearances are thus described by Mr. Richard Allman : " A faint halo surrounded the moon, at a distance which appeared equal to that of the pole star from the nearest point in the Plough. In this halo, at the extremities of the horizontal diameter, appeared two nebula-like, luminous masses, between which an intermittent stream of faint light seemed to play. I first perceived it about 11 o'clock. It continued unchanged for about an hour." The phenomenon was seen, under a much more complex form, by Mr. Lowe, at Lenton, Nottinghamshire (see Phil. Mag., Nov. 1844), and the fact indicates the very wide outspread of the high cirrus and cirro-stratus cloud, by the frozen particles of which it is produced. The existence of this cloud in the neighbourhood of the moon is also recorded in the Day-book of the Dublin Magnetical Observatory, at 10 P. M. of the same night. It would be interesting, in this point of view, to multiply the records of such phenomena, so as to be able to trace the extent and limits of the cloud in question. I find, in the Philosophical Transactions, that a remarkable halo surrounding the sun, accompanied with parhelia, was seen on the same day (Oct. 20, 1747), at Paris and Berlin; but the evidence derivable from such a fact is incomplete, in the absence of any account from intermediate stations.

Rev. Thomas Porter, D. D., presented an ancient wooden table and dish, and communicated the following notice:

The wooden table and dish to which this notice relates, were dug up in a peat moss, or turf-bog, near the road from Donaghey, in the townland of Killygarvan, parish of Desertcreight, or Dysertcreaght, County of Tyrone.—(Ordnance Survey, Sheet 38.) They were found four or five feet below the surface. With the dish there was a quantity of hazel nuts. Each article was cut out of a solid piece of wood, apparently fir. The table is of an oblong shape, with the ends curved inwards towards the centre.

The four short legs, about six inches high, are in the form of truncated cones, and about four inches thick. They are connected at their bases, except on one side, by a low rim, about one inch high, in the longest side of which are two holes, capable of admitting a cord or thong.

The dish was a long oval, four or five inches deep, clumsily hollowed out of a piece of a trunk of fir. Its length, when found, was exactly the same as the extreme length of the table, two feet. Its breadth was about ten inches, and just sufficient to *cover* the extremities of the four legs, but *not* to allow of its lying between them. It has since split lengthwise, in such a manner as to alter its proportions materially. In the edge of one side, where it has been somewhat injured, are the marks of two holes, exactly answering to the two in the rim under the table. From those particulars it may be inferred that the table was used by persons who sat on the ground at their meals; and that the dish, when not in use, was attached by a thong to the under surface of the table, which might be hung against the wall of the dwelling, or slung on the baggage when the owners migrated from place to place in the woods. It is not improbable, too, that the curved ends of the table may (as has been suggested by an observant person) have been of use in transferring meal, when ground in a quern or hand-mill, from the table to the dish. Possibly, also, the rim on the under surface may have been of use in kneading dough for cakes, the table being inverted for that purpose.

The workmanship and appearance of both articles are rude in the extreme, and indicate a very low state of civilization in the people who used them.

Nothing very remarkable has been found in the same bog, or in any of the many adjoining ones, except some stags' horns, whieh were dug up in the next little valley, in the townland of High Cross, and which are now in the possession of John Lindesay, Esq., of Loughry.

DONATIONS.

Annales des Sciences Physiques et Naturelles d'Agriculture et d'Industrie. Publièes par La Sociéte Royale d'Agriculture, &c. de Lyon. Tome V. Année 1842.

Flora Batava, door Jan. Koops. (132.) Presented by the King of Bavaria.

Synopsis of the Carboniferous Limestone Fossils of Ireland. Presented by the Author, Richard Griffith, Esq., M. R. I. A.

Greenwich Astronomical Observations, 1842. *Appendix to do.*, 1842. Presented by the Royal Astronomical Society.

Proceedings of the Zoological Society of London. Part 2. (243). *Reports of the Council and Auditors of do.* Presented by the Society.

Proceedings of do., with Plates in Illustration of the Papers abstracted, Session 1843, 1844. Vol. IV., No. 98. Presented by the Society.

Journal of the Statistical Society of London. Vol. VII. Part 3. Presented by the Society.

Statistical Returns of the Dublin Metropolitan Police, for the Year 1843. Presented by the Commissioners.

Journal of the Franklin Institute. Third Series. Vol. VI. Presented by the Institute.

Sixth Annual Report of the Commissioners of the Loan Fund Board of Ireland. Presented by the Commissioners.

Fourteenth Annual Report of the Belfast District Asylum for the insane Poor. March, 1844. Presented by the Governors.

Tenth Annual Report of the Poor Law Commissioners, 1844 (London). Presented by the Commissioners.

The First Part of the Ninth Volume of the Transactions, with the new Laws of the Batavian Society of Experimental Philosophy. Presented by the Society.

The Blade of an ancient Bronze Sword, mounted with a modern Iron Hilt, found in the County Kerry. Presented by Maurice O'Connell, Esq., M. P.

Several Cannon Balls found on both Sides of the Boyne, *in the Neighbourhood of Oldbridge.* Presented by Lieutenant Newenham, R. N.

An ancient Bronze Spear-head, from Roscommon. The Boss of a Shield. A small Bronze Celt. A decade Ring and Cross. A copper Fibula. Presented by Abraham White Baker, Esq.

A perforated Stone, found in the Grave-yard at Kille Derry, Derry Art. Presented by Lord George A. Hill.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1844–45. No. 49.

January 13, 1845.

SIR Wm. R. HAMILTON, LL.D., President, in the Chair.

Robert Scott Bradshaw, Esq., Thomas Davis, Esq., Sir Richard Franklin, M. D., Edmund Getty, Esq., George A. Hamilton, Esq., M. P., Henry G. Hughes, Esq., Capt. Henry James, R. E., Francis L'Estrange, Esq., Edward Lucas, Esq., John Phillips, Esq., Thomas N. Redington, Esq., M. P., Marmion W. Savage, Esq., and Richard Sharp, Esq., were elected Members of the Academy.

It was moved by Sir Wm. Betham, and seconded by P. D. Hardy, Esq., "That a Committee, not members of the Council, be appointed to examine the Returns laid on the Table of the Academy on the 30th of November last, to inquire into the facts therein stated, and to report thereon; and that the Committee consist of Sir William Betham, Lieut.-Col. Harry D. Jones, R. E., Thomas Hutton, Esq., William Hogan, Esq., and Aquilla Smith, Esq., M. D."

Which motion, having been put, was negatived without a division.

Mr. Robert Ball read a notice on the original use of certain Golden Ornaments, and other articles, in the Museum of the Royal Irish Academy.

Mr. Ball having urged on those who study antiquities the importance of applying observation and analogy to the solving of antiquarian difficulties, shewed how these instruments of inquiry should be applied, by the study of races of mankind at present existing, whose state may be supposed somewhat similar to that in which the people were into whose history inquiry is proposed to be made; it being fair to look for like effects from similar causes. He referred to a former paper, read January, 1844, in which he shewed how metal celts, identical in form with those found in Ireland, were used at the present day on the east coast of Africa; and how stone celts, also similar to those of Ireland, were used in Mexico. Applying the same reasoning to explain the object and use of the golden ornaments called by some diadems, by others gorgets or collars, he mentioned that in the Sandwich Islands the natives used stone celts precisely similar to those found in Ireland; they also had those curious lentilform discs of stone, precisely identical with those found in Ireland, and to which sundry fanciful uses have been ascribed, but which Cook and others found to be used as bowls in a favourite game of the natives, who had bone bodkins, &c., similar to those of olden time in Ireland; it was, therefore, little more than was to be expected, to find analogies to the golden ornaments found associated with the celts and bowls to which Mr. Ball referred. This, he maintained, he had done, in one case at least, that of the golden ornament referred to, which has its representative in the Sandwich Isles, where gold is not known. Sharks' teeth, mother of pearl, feathers, and basket work, are so put together, as was shewn by the figures exhibited, as in all but material to resemble the ornaments of gold in the most striking manner.

From the way in which the ornaments of the Sandwich

Islanders are stated to be worn, Mr. Ball declared he could not doubt the golden ornaments were worn in a similar manner. The Sandwich Island articles to which he alluded formed a part of the fine collection made in Cook's voyages, and deposited in the Museum of the University. He trusted he would be able to make many of the weapons and ornaments therein contained useful in throwing light on Irish antiquities. He referred to several curious instances, where the use of hypothesis had misled antiquaries, and where observations of existing people had set their opinions aside. He mentioned that he had recently proved, that an article long existing in the University Museum, and known as the best example of an old form of a trumpet, had, by the discovery of its remaining parts, proved to be a chemical instrument for burning gas, or inflammable vapour; and he concluded by stating, that the article figured in the seventeenth volume of the Transactions of the Royal Irish Academy, as an astronomical instrument of the ancient Irish, proved to be a piece of chain armour. These two last mistakes he gave as examples of a want of exactness of observation, and of the mischief of hypothesis.

The Secretary read a paper by Professor Young of Belfast, on Diverging Infinite Series, and on certain Errors in Analysis connected therewith.

The subject of diverging series is one of considerable perplexity in analysis, and has given occasion to theories of explanation involving views and statements entirely opposed to the general principles of algebraical science. It has, for instance, been affirmed of such series—when they present themselves as developments of finite expressions—that, though algebraically true, they may, nevertheless, be arithmetically false. By some they are considered to justify conclusions palpably erroneous and absurd, as, for example, that

$$1 + 2 + 4 + \ldots = -1,$$

while by others they are regarded as meaningless results, and have thus been altogether rejected from analysis.

It is impossible to avoid the occurrence of these series: they present themselves at a very early stage of algebra, in the form of geometrical progressions and binomial developments; and thenceforward are continually met with by the analyst up to the remotest applications of the integral calculus. The existing vagueness and indecision, as to the proper mode of interpreting such series, is thus a matter of some concern, as calculated to retard the progress of science, to diminish our confidence in some of the truths of analysis, and to give currency to results involving error and contradiction.

In the present communication it will be my endeavour to ascertain the causes of the perplexities and discrepancies above adverted to, and to discover the legitimate interpretation of diverging infinite series; from which it will, I think, follow that certain expressions received into analysis as the sums of several of these, are erroneous. The fact that Poisson, Cauchy, Abel, and indeed most of the modern continental writers, reject diverging infinite series, and pronounce them to have no sums, does not render such an endeavour the less necessary; inasmuch as the analytical operations, in virtue of which finite values have been attributed to extensive classes of these series by Euler and subsequent investigators, remain, I believe, unimpugned. Widely different methods appear to concur in furnishing the same numerical results for such series; as, for instance, the method of definite integrals, and that deduced from the differential theorem, both so frequently applied by Euler to effect the summations of series of this kind; and the numerical results obtained by him have often, apparently, been verified by later computers; some of whom have employed methods quite distinct from those of Euler; as, for instance, Horner, who arrived at Euler's results by aid of considerations drawn from the theory of continued fractions.*

So long, therefore, as the admitted operations of analysis thus conduct to conclusions—and conclusions, too, mutually confirmatory of one another, though arrived at by very different paths—we are surely not authorized in summarily rejecting them as meaningless or absurd, merely on account of any inherent difficulties involved in them. The only ground for such rejection, that can generally be considered as sufficiently cogent by analysts, must be errors in the reasoning by which those conclusions are reached. In attempting, therefore, now to point out the existence of these errors, it will be perceived that I proceed on the assumption that nothing has as yet been advanced, by the rejectors of diverging infinite series, against the *reasonings* of Euler, Lacroix, and others, in reference to this matter; more especially that the method of definite integrals, and that depending on the differential theorem, have not as yet been shewn to be erroneous. I may be wrong in this supposition; if so, I should feel most anxious to withdraw this Paper, rather than obtrude upon the attention of the Academy the discussion of a topic already disposed of—and, doubtless, in a more complete and satisfactory manner—elsewhere.

I.—As noticed above, the first step in the general theory of series occurs under the head of geometrical progression; the form of the series proposed for summation being

$$a + ax + ax^2 + ax^3 + \&c. \qquad (1)$$

where it is to be observed that the "&c." implies the endless progression of the terms beyond ax^3, according to the law exhibited in the terms which precede; excluding, however, every thing in the form of supplement or correction. The general expression for the sum of n terms of this series is known to be

* Annals of Philosophy: July, 1826, p. 50.

$$\text{s} = \frac{a}{1-x} - \frac{ax^n}{1-x}. \qquad (2)$$

Now it is customary to write the development of $\frac{a}{1-x}$ as follows, viz.

$$\frac{a}{1-x} = a + ax + ax^2 + ax^3 + \&c. \qquad (3)$$

and then to commit the mistake of confounding this with the series (1) above; overlooking the fact that the "&c." in the one, except under particular restrictions as to the value of x, is very different, as to the meaning involved in it, from that in the other.

If we dispense with the "&c." in the series (1), we may write that series thus:

$$a + ax + ax^2 + ax^3 + \ldots + ax^\infty, \qquad (4)$$

the sum of which will be truly expressed by the formula (2), by making n infinite; as that formula is perfectly general. But this same formula gives for $\frac{a}{1-x}$ the development

$$\frac{a}{1-x} = a + ax + ax^2 + ax^3 + \ldots + ax^\infty + \frac{ax^{\infty}}{1-x}, \qquad (5)$$

* shewing that the "&c." in (3) differs from that in (1) by a quantity which is infinitely great, whenever x is not a proper function: except in the single case of $x = -1$. When x *is* a proper fraction, the two series become identical by the evanescence of $\frac{ax^\infty}{1-x}$.

It thus appears that $\frac{a}{1-x}$ is *not* the fraction which generates the series (1), x being unrestricted: what this fraction really generates is exhibited in (5) above, an equation which is always true, whatever arithmetical value we assign to x; and to obtain the general expression for the sum of (1), we

* As the exponent in this last expression is infinite, it seems unnecessary to write it $\infty + 1$.

must connect to $\frac{a}{1-x}$ the correction $-\frac{ax^{\infty}}{1-x}$: a correction which is ambiguous as to sign, when x is negative.

When x is > 1, the series, omitting this correction, is ∞; the correction itself is also ∞, and opposite in sign: it is the difference of these two infinites which is the finite undeveloped expression.

There is thus no discrepancy between a geometrical series and the expression which generates it: nor is it the case that by connecting the two by the sign of equality, we shall have an equation algebraically true, but in certain cases arithmetically false, as has been frequently affirmed of late. The reverse of this affirmation is the more correct statement; inasmuch as by interposing the sign of equality between $\frac{a}{1-x}$ and the series (1), instead of the series (5), we have an equation algebraically false, though, within certain limits, arithmetically true: this last circumstance arising from the fact that the omitted correction, which renders the equation algebraically defective, would have vanished of itself, between the arithmetical limits adverted to, had it been introduced. Thus, the series noticed at the commencement of this paper, viz.

$$1 + 2 + 4 + 8 + 16 + \&c.,$$

and which is intended to represent the development of $\frac{1}{1-2}$, arises from expressing the general development of $\frac{1}{1-x}$ in the defective form

$$1 + x + x^2 + x^3 + x^4 + \ldots + x^{\infty},$$

instead of, in the accurate form,

$$1 + x + x^2 + x^3 + x^4 + \ldots + x^{\infty} + \frac{x^{\infty}}{1-x},$$

which defective form introduces arithmetical error only when x exceeds unit. When $x = 2$, the error arising from this defect is infinitely great; the true form giving, in that case,

$$-1 = 1 + 2 + 4 + 8 + 16 + \ldots - 2^{\infty} = \infty - \infty,$$

which involves no error or contradiction.

It hence appears that when the geometrical development is a converging series, for an arithmetical value of the common ratio, no error can arise from the omission of the supplementary correction, which is always necessary for the completion of the *algebraic* form of that development; but that when the arithmetical value of the ratio is such as to render the series divergent, the algebraic error necessarily introduces an arithmetical error infinitely great: the correction of the algebraic form furnishes, in such a case, the expression $\infty - \infty$, that is the difference of two infinites, for the finite undeveloped numerical value: and in this there is nothing inexplicable or peculiar.

We see, therefore, that in passing from the convergent to the divergent state of a geometrical series, we have no occasion for any new principle, such, for instance, as the *sign of transition*, introduced by Dr. Peacock, in the discussion of this subject, in his very valuable and instructive Report on Analysis, presented at the third meeting of the British Association. If there only be strict algebraic accuracy between the finite expression and its developed form, there will necessarily be equally strict *numerical* accuracy, whatever arithmetical values be given to the arbitrary symbols: a truth which must indeed universally hold in all the results of analysis.

II.—The developments of the binomial theorem, as well as those considered above, have also been the source of much perplexity and misinterpretation, when they have assumed a divergent form. In contemplating these developments, the fact has been overlooked, that although, when interminable, they each involve an infinite series, whose terms succeed one another, according to a certain uniform law, yet that series alone is not the complete algebraical equivalent of the undeveloped expression: a supplementary function of the symbols employed is always necessary to such completeness. This has already been seen in the development of $\frac{1}{1-x}$ or $(1-x)^{-1}$, which is a particular case of the binomial development: besides the series, the supplementary expression $\frac{x^m}{1-x}$ is necessary to the complete algebraical equivalence of the two members of the equation. And it is plain, from the nature of common division, that a like supplementary addition must be made to the infinite series furnished by the development of $\frac{1}{(1-x)^n}$ or $(1-x)^{-n}$. In the extraction of roots, too, as in $(1-x)^{\frac{1}{2}}$, $(1-x)^{\frac{1}{3}}$, &c., it is equally plain that, however far the extraction be extended, we approach no nearer to the actual exhaustion or annihilation of the algebraic remainder; and therefore we are not authorized to dismiss this remainder and to account it zero, when general *algebraic* accuracy is to be exhibited; although, as in geometrical series, we may do this in those particular numerical cases in which the remainder, if retained, would vanish. It thus appears that, calling the remainder after n terms, whether n be finite or infinite, $f(x)$, the ordinary binomial series, to n terms, will be the complete development, not of $(1-x)^{\frac{1}{m}}$, but of $(1-x-f(x))^{\frac{1}{m}}$; and therefore that, if this series be equated to $(1-x)^{\frac{1}{m}}$ merely, it will require a supplemental correction to produce strict algebraical equivalence; which correction must be such as to vanish for those numerical values of x, which cause $f(x)$ to vanish.

These values are all those which render the series divergent: for, as well known, we can, in every such case, approach by the series alone as near to the numerical value of the undeveloped expression as we please. It is thus only when the series ceases to be convergent, that the correction adverted to has any arithmetical existence, adjusting the equality of the two sides of the equation, and precluding the inconsistency so frequently affirmed to have place between them.

From these simple considerations, it is easy to explain and reconcile such results as

$$(a^2-x)^{\frac{1}{2}} = a - \frac{x}{2a} - \frac{x^2}{2.4a^3} - \frac{3x^2}{2.4.6a^5} - \frac{3.5x^4}{2.4.6.8a^7} - \&c.$$

for all arithmetical values of x; the "&c." being regarded as comprehending all that is necessary to render the second member of the equation a complete algebraical equivalent of the first. When x exceeds a^2, the series becomes divergent; and the first member of the equation becomes imaginary: and since it is impossible that any imaginary quantity can enter the series, it follows that it is in the supplementary correction under the "&c." that such quantity must occur, when in that correction a value greater than a^2 is given to x.

From what has now be shewn, it may, I think, be legitimately inferred—as far, at least, as geometrical and binomial series are concerned—

1. That whenever any such series becomes divergent for particular arithmetical values, what has been called above the supplementary correction becomes arithmetically effective, and cannot be disregarded without arithmetical error.

2. And that so far from such series being, as usually affirmed, always algebraically true, though sometimes arithmetically false, on the contrary, they are always algebraically false, though sometimes arithmetically true:—true in those cases, namely, and in those only, in which the proper algebraic correction becomes evanescent.

III.—Let us now pass to the consideration of other classes of diverging series.

There are two ways of investigating the differential of $\sin x$, or of $\sin mx$: one by proceeding, as Lagrange has done, by actual algebraic development; and the other by employing the method of limits, independently of development. According to Lagrange, we must proceed upon the assumption that

$$\sin mx = \text{A}x + \text{B}x^2 + \text{C}x^3 + \&c.$$

justifying this assumption on the ground that x and $\sin mx$ vanish together; which can be considered valid only so long as $m = \infty$ is excluded. In fact, whether we seek the development of $\sin mx$ after the manner of Lagrange, or by the theorem of Maclaurin, it is essential to the very nature of the investigation that the unknown coefficients A, B, C, &c. be all assumed to be finite. We cannot conclude, therefore, from Lagrange's reasoning, that $\frac{d \sin mx}{dx} = m \cos mx$, when m is infinite: and similar considerations forbid the conclusion that $\frac{d \cos mx}{dx} = -m \sin mx$, in like circumstances. The method of limits equally militates against such a conclusion; thus, if the function were $\sin x$, we should have

$$\sin(x+h) - \sin x = 2 \sin \tfrac{1}{2}h \cos(x + \tfrac{1}{2}h),$$

or

$$\frac{\sin(x+h) - \sin x}{h} = \frac{\sin \frac{1}{2}h}{\frac{1}{2}h} \cos(x + \tfrac{1}{2}h);$$

and since $\frac{\sin \frac{1}{2}h}{\frac{1}{2}h} = 1$, in the limit, or when $h = 0$, we should safely infer that $\frac{d \sin x}{dx} = \cos x$. But, by proceeding in like manner with $\sin mx$, we should have

$$\frac{\sin(mx + mh) - \sin mx}{h} = m \frac{\sin \frac{1}{2}mh}{\frac{1}{2}mh} \cos(mx + \tfrac{1}{2}mh),$$

from which, if m be infinite, it could not be inferred that $\frac{d \sin mx}{dx} = m \cos mx$; since we have no right to affirm that $\frac{\sin \frac{1}{2}mh}{\frac{1}{2}mh}$ tends to 1, as h diminishes, and finally terminates in that value when $h = 0$; nor that, in like circumstances, $\cos(mx + \frac{1}{2}mh) = \cos mx$. We have nothing to justify the assertion that $\frac{\sin \frac{1}{2}h}{\frac{1}{2}h}$ and $\frac{\sin \frac{1}{2}mh}{\frac{1}{2}mh}$ are the same at the limits when m is infinite: and it should create no surprise if conclusions, deduced from this assumption, prove to be absurd.

Bearing this in remembrance, let us take the series

$$\frac{x}{2} = \sin x - \tfrac{1}{2}\sin 2x + \tfrac{1}{3}\sin 3x - \&c.$$

first given by Euler, and which is known to be rigorously true for all values of x below π.*

From this series the following results have been deduced by differentiation, and they have been pretty generally received into analysis:

$$\tfrac{1}{2} = \cos x - \cos 2x + \cos 3x - \cos 4x + \&c.$$
$$0 = -\sin x + 2\sin 2x - 3\sin 3x + 4\sin 4x - \&c.$$
$$0 = -\cos x + 2^2\cos 2x - 3^2\cos 3x + 4^2\cos 4x - \&c.$$

and, generally,

$$0 = \cos x - 2^{2n}\cos 2x + 3^{2n}\cos 3x - 4^{2n}\cos 4x + \&c.$$
$$0 = \sin x - 2^{2n+1}\sin 2x + 3^{2n+1}\sin 3x - 4^{2n+1}\sin 4x + \&c.$$

so that putting $x = 0$ in the first of these, and $x = \frac{\pi}{2}$ in the second, we have

$$0 = 1 - 2^{2n} + 3^{2n} - 4^{2n} + \&c.$$
$$0 = 1 - 3^{2n+1} + 5^{2n+1} - 7^{2n+1} + \&c.$$

results which are all inadmissible; because, from the outset, it is assumed that

$$\frac{d\sin mx}{dx} = m\cos mx, \text{ and } \frac{d\cos mx}{dx} = -m\sin mx;$$

though m be infinite.

In reference to the preceding results, Abel justly asks: "Peut-on imaginer rien de plus horrible que de débiter

$$0 = 1 - 2^{2n} + 3^{2n} - 4^{2n} + \&c.$$

où n est un nombre entier positif?"†

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* It will be shewn, towards the close of this Paper, that it is true for all values up to π inclusive.

† Œuvres Completes, tome ii. p. 266.

It is plain that, however far such a series as this be ex-extended, a supplementary correction is always necessary to complete the equation; which correction must be infinite in value if the series be infinitely extended: and the analytical considerations offered above fully accord with this statement, the contrary of which could never have been entertained had not analysis seemed to justify the strange conclusion. All that analysis really authorizes us in saying, in reference to the extreme cases here considered, is—as the French analysts express it—that "la méthode ordinaire est en défaute."

Having mentioned the name of Abel in connexion with this subject, it may not be out of place to notice here, that that distinguished genius seemed inclined to trace the erroneous results above to another cause: "On applique aux séries infinies toutes les opérations, comme si elles étaient finies; mais cela est-il bien permis? Je crois que non. Où est il démontré qu'on obtient la différentielle d'un série infinie en en prenant la différentielle de chaque terme?" And he then adduces the result,

$$\tfrac{1}{2} = \cos x - \cos 2x + \cos 3x - \&c.$$

which he pronounces to be "résultat tout faux."*

But I submit that no such results of differentiation can ever be absurd, unless the absurdity attaches to one or more of the individual terms.

In the former part of this paper the examination was restricted to those classes of diverging series which arise from the development of fractions into geometrical series, and from the expansion of a binomial: but it is plain that the reasonings, in reference to the former developments, equally apply to those which arise from any fraction $\dfrac{f(x)}{\phi(x)}$; and the reasoning, in reference to the latter, equally applies to any root or power of (x). And, in what is shewn above, we see how divergent

* Œuvres, tome ii. p. 268.

trigonometrical series, arising from differentiating convergent forms, are to be understood.

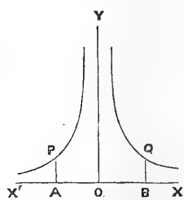
IV.—It remains now to be noticed that in some of the more advanced parts of analysis—especially in the doctrine of definite integrals—conclusions have been reached which seem to contradict the proposition endeavoured to be established in this Paper, viz. that convergent infinite series have no finite sum. But all such conclusions will be found upon examination to originate in mistake. I proceed to examine the more important of these.

The following has been recently offered, by a very cautious writer, in support of the statement that "$1 + 2 + 4 +$ &c. *ad infinitum,* is an algebraic representative of -1, though it only gives the notion of infinity to any attempt to conceive its arithmetical value":

$$\int x^{-2}dx = -x^{-1}, \int_a^b x^{-2}dx = a^{-1} - b^{-1}, \text{ which is finite;}$$

$$\int_{-m}^{0} x^{-2}dx = +\infty, \int_0^m x^{-2}dx = +\infty, \int_{-m}^{+m} x^{-2}dx = -\frac{2}{m}.$$

If, then, we construct the curve whose equation is $y = x^{-2}$, and if $\text{OA} = -m$, $\text{OB} = +m$, we find the areas PAOY... and QOBY... both positive and infinite, which agrees with all our notions derived from the theory of curves. Again, if we attempt to find the area PYQB, by summing PAOY and YOQB, we find an infinite and positive result, which still is strictly intelligible. But if we want to find the area by integrating at



once from P to Q, we find, as above, $-\frac{2}{m}$, a *negative result,* for the sum of two positive infinite quantities. The integral then, y being infinite between the limits, takes an algebraic character, standing in much the same relation to the required arithmetical result, which must have been observed in divergent series. "Thus, &c.," as quoted above.* The analogy thus apparently established is traceable to an oversight, of very easy detection, in the preceding integrations; which, in the correct form, will stand as follow:

$$\int_{-m}^{0} x^{-2}dx = +\infty - \frac{1}{m}$$

$$\int_{0}^{m} x^{-2}dx = +\infty - \frac{1}{m}$$

∴ adding,

$$\int_{-m}^{m} x^{-2}dx = 2\infty - \frac{2}{m}.$$

Or thus,

$$\int_{a}^{b} x^{-2}dx = \left(-\infty + \frac{1}{a}\right) - \left(-\infty + \frac{1}{b}\right)$$

$$\therefore \int_{-m}^{m} x^{-2}dx = \left(+\infty - \frac{1}{m}\right) - \left(-\infty + \frac{1}{m}\right)$$

$$= 2\infty - \frac{2}{m}.$$

But errors of a much more important kind occur in all the applications of definite integrals to the summation of diverging series: a mode of summation first, I believe, adopted by Euler, and very generally employed by subsequent analysts. A single example of this method will be sufficient to shew the character of the errors adverted to; which, though so glaring as almost to obtrude themselves upon the attention, have not hitherto, so far as I know, been noticed by any writer. Any one of the examples given by Euler (Institutiones Calc. Diff.), and afterwards by Lacroix (Traite du Calcul. &c., tome iii.), will answer the present purpose: I shall take that at page 573 of the English edition of the smaller work of Lacroix, viz.

$$s = 1.t - 1.2t^2 + 1.2.3t^3 - \&c. \qquad (6)$$

which, Sir John Herschel remarks, is such that "however

* De Morgan's Differential and Integral Calculus, p. 571.

small a value we attribute to t, the series must always diverge after a certain number of terms."*

The reasoning by which a finite sum is determined for s, when $t = 1$, is as follows:

$$\frac{sdt}{t} = 1.dt - 1.2tdt + 1.2.3t^2dt - \&c. \tag{7}$$

$$\therefore \int\frac{sdt}{t} = t - 1.t^2 + 1.2t^3 - \&c. \tag{8}$$

$$= t - st \tag{9}$$

$$\therefore \frac{sdt}{t} = (1-s)dt - tds,$$

or,

$$\frac{ds}{dt} + \frac{1+t}{t^2}s = \frac{1}{t};$$

and from this is found, for s, the definite integral

$$s = \frac{1}{t}e^{\frac{1}{t}}\int_0^t e^{-\frac{1}{t}}dt;$$

from which it is inferred that "if $t = 1$, or the above integral be taken from $t = 0$ to $t = 1$, we have the expression for the value of the series

$$1 - 1.2 + 1.2.3 - \&c."$$

Now several objections lie against the preceding reasoning: in the first place it is assumed, in the final step, that s vanishes, for $t = 0$, notwithstanding that "however small a value we attribute to t the series must always diverge," and thus at length furnish terms infinitely great: and in the next place it is assumed—and the assumption is somewhat similar to that

* If, however, t be indefinitely near to zero, the "certain number of terms" adverted to in the text, will be indefinitely great; that is, the divergency will be indefinitely postponed: the series therefore cannot be considered as divergent up to the limit $t=0$; yet, as the statement in the text seems to imply this, I have considered it to be comprehended in the hypothesis; although, as I have shewn, the point is of no moment in the matter under discussion.

already animadverted upon at page 35—that the series (7) is strictly the differential of the series (8) which involves the term $1.2.3 \ldots (n-1)t^n$, n being infinitely great, and for the differential of which the calculus seems to make no provision. But, waiving these objections, the deduction (9) is palpably erroneous, and altogether fatal to the final conclusion. For the series s is evidently coextensive with the series (8), and so, of course, is st; that is, if (8) contain n terms, so also must st: if therefore a new term t be prefixed to $-st$, in order that $t-st$ may commence with the same terms as the series (8), the series $t-st$ will contain $n+1$ terms; that is, however great n may be, $t-st$ will contain, besides the whole of the series (8), an additional term still more remote: so that if n be infinite, and we assume, as above, that the two series are equal, we commit an error infinitely great. And this is the error, thus introduced, which will be found to vitiate all Euler's processes for summing divergent series by definite integrals: an error which obviously has no existence for the convergent cases of those series; since the additional term, noticed above, is, in such cases, not infinite, but zero. We may safely infer, therefore, that the results so often quoted in analysis, viz.

$$1-1+1.2-1.2.3+\ldots\ldots = \cdot 596347362324$$
$$1-1.2+1.2.3-\ldots\ldots\ldots = \cdot 621449624236$$
$$1-1.2.3+1.2.3.4.5\ldots\ldots = \cdot 343279002556$$
$$\&c. \qquad\qquad \&c.$$

all involve errors infinitely great; and this, as it ought to be, is quite consistent with the common-sense view of diverging infinite series.

V.—There is another method of investigation by which these erroneous results appear to be established: the method suggested by the well-known differential theorem. But, as in the processes already considered, so here, that theorem will be found upon examination to be applicable only to convergent series. This will be manifest from what follows.

The differential theorem may be satisfactorily established by conducting the investigation thus:

Let

$$a - bx + cx^2 - dx^3 + \&c. = s$$

$$\therefore \quad - bx + cx^2 - dx^3 + \&c. = s - a \qquad (10)$$

$$\therefore - b + cx + dx^2 + ex^3 - \&c. = \frac{s-a}{x}. \qquad (11)$$

Consequently, by adding these two equations together, and representing the numerical differences $b-c$, $c-d$, $d-e$, &c. by Δ, Δ', Δ'', &c., there will result the equation

$$- b - \Delta.x + \Delta'.x^2 - \Delta''.x^3 + \&c. = \frac{x+1}{x}(s-a) \qquad (12)$$

$$\therefore -bx - \Delta.x^2 + \Delta'.x^3 - \Delta''.x^4 + \&c. = (x+1)(s-a) = s'$$

$$\therefore s = \frac{s'}{x+1} + a;$$

that is,

$$s = a - \frac{bx}{x+1} + \frac{x}{x+1}[0 - \Delta.x + \Delta'.x^2 - \Delta''.x^3 + \&c.] \qquad (13)$$

And by treating the series within the brackets as the original was treated, and so on, we shall finally obtain the transformation

$$s = a - \frac{bx}{x+1} - \frac{\Delta.x^2}{(x+1)^2} - \frac{\Delta^2.x^3}{(x+1)^3} - \&c.$$

or putting $a = 0$, and dividing by $-x$, we have

$$b - cx + dx^2 - ex^3 + \&c. =$$

$$\frac{b}{x+1} + \frac{\Delta.x}{(x+1)^2} + \frac{\Delta^2.x^2}{(x+1)^3} + \frac{\Delta^3.x^3}{(x+1)^4} + \&c.$$

which is the usual form of the theorem.

Now the preceding reasoning is inadmissible except the proposed series be convergent; that is, except rx^n approaches to zero as n approaches to infinity, rx^n standing generally for the n^{th} term of (10). For in (12), which results from the sum of (10) and (11), this n^{th}, or final term, is regarded as zero, and is neglected; inasmuch as it is by this term that the series (10) extends beyond the series (11) to the right; a fact which is of no moment when this term merges in zero, but of infinite consequence when it merges in infinity. In such a case therefore, a numerical error, of infinite amount, is committed at this step of the reasoning. Again, if the series within the brackets at (13), have its terms, like those of the original, tending to infinity, another numerical error of infinite amount comes to be introduced; and so on. In fact, just as in the method of definite integrals, before discussed, it is assumed, at each step of the reasoning, that terms infinitely great are excluded; and not only so, but that the terms ultimately diminish to zero. In the contrary case, therefore, the differential theorem is altogether inapplicable, leading to results which are equally inadmissible, whether the terms of the series increase without limit, or remain stationary in value: forming what has been called a neutral series. In this latter case the error committed will be finite; in the former it will be infinite. That an error is really committed in the application of this theorem to neutral series, will be more explicitly shewn presently.

Notwithstanding the imperfections noticed above, it should create no surprise that, in the applications of the differential theorem to particular diverging series, we so often obtain the algebraic function whose development really gives rise to the series, although no numerical approximation to the diverging series itself. The function, whose development gives rise to the series, being represented by $f(x)$, the series itself may be represented by $f(x) - \phi(x)$, agreeably to what has already been shewn in the former part of this Paper: it is the neglect of the function $\phi(x)$, in the particular application considered, that introduces the infinite numerical error into (13); leading us to conclude that, for the proposed value of x, $f(x) = \mathrm{s}$, instead of $f(x) - \phi(x) = \mathrm{s}$. Now if there exist a convergent case of s, that is a case in which $\phi(x) = 0$, the differential theorem will compute it, furnishing the proper function of x, $f(x)$, which accurately expresses the series in all its convergent cases, and of which the development *gives rise* to the series in its general form. When no such function $f(x)$ really exists, then it is only to the numerical value of an approximate function that our computation tends in particular numerical cases; as, for instance, in such a case as that considered at p. 39.

It may be worth while to notice here, as an immediate inference from the differential theorem, that when a series, proceeding according to the powers of x, and extending to infinity, has its coefficients such that their differences at length become zero, that series is always the development of a rational fraction whose denominator is some integral power of $(1 \pm x)$.

There is, I think, a mistake committed in always attributing this theorem to Euler. It was published by Stirling, in his Methodus Differentialis, so early as 1730; and I believe no mention of it occurs in the writings of Euler till long after this date.

VI.—As far as I know, there is but one other general analytical principle that has been affirmed to give countenance to doctrines opposed to those attempted to be established in the present Paper: the principle, namely, that when an algebraic expression, for continuous numerical values of the variable, approaches continuously to a certain finite numerical value, this value properly expresses the ultimate, or limiting state of that expression. In virtue of this principle, it has been stated* that, "Poisson would admit $1^2 - 2^2 + 3^2 - 4^2 + \ldots = 0$, since there is no question that, g being less than unity, the mere arithmetical computer might establish, to any number of decimal places, the identity of $1^2 - 2^2 g + 3^2 g^2 - \ldots$ and $(1-g)(1+g)^{-3}$."† But I submit that the series here

* Transactions of the Cambridge Philosophical Society, Part II. 1844.

† In order that the series $1^2 - 2^2 g + 3^2 g^2 - \ldots$ may become convergent after n terms, there must evidently exist the condition

$$\left(\frac{n+1}{n}\right)^2 g < 1, \text{ whence } g < \left(\frac{n}{n+1}\right)^2;$$

proposed exceeds the powers of computation more and more as g approaches to 1; involving at length terms infinitely great, and thus tending to no finite limit. In other words, however many terms of this series be summed, the results would diverge more and more from zero as g approaches to 1; and would actually become infinite when g reaches this limit. The conclusion, therefore, that $1^2-2^2+3^2-\ldots=0$ is, as in the other instances discussed in this Essay, erroneous to an infinite extent: and it thus affords one more example of the truth of the doctrine here advanced.

The general analytical principle announced above has been misapplied, or improperly neglected, in many important inquiries connected with series. It may not be uninstructive to advert more particularly to some instances of this.

At page 267 of the second volume of his works, Abel has the following remark: "On peut démontrer rigoureusement qu'on aura, pour toutes les valeurs de x inférieures à π,

$$\frac{x}{2}=\sin x-\tfrac{1}{2}\sin 2x+\tfrac{1}{3}\sin 3x-\&c.$$

Il semble qu'on pourrait conclure que la mème formule aurait lieu pour $x=\pi$; mais cela donnerait

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and as $\left(\frac{n}{n+1}\right)^2$ is itself less than 1 for every finite value of n, however great, it follows that g may approach so near to 1 as to postpone the point of convergency beyond any finite limit; which is tantamount to saying that this point can never actually be reached. The series, therefore, cannot tend to merge into zero as g approaches to 1; so that zero is *not* the limit to which the series continuously approaches as g approaches continuously to 1; and therefore the general principle stated in the text does not countenance the conclusion that $1^2-2^2+3^2-\ldots=0$.

I cannot help regarding the criterion of convergency proposed by Cauchy (Cours d'Analyse, p. 152) as open to objection; since, according to it, we should pronounce a series to be convergent under circumstances in which the point of convergency would be postponed beyond any finite limits: moreover, what security have we that *neutrality* may not have place before divergency commences?

$$\frac{\pi}{2} = \sin\pi - \tfrac{1}{2}\sin 2\pi + \tfrac{1}{3}\sin 3\pi - \&c. = 0,$$

résultat absurde."

Now the formula, agreeably to the general principle here affirmed to be in fault, does really comprehend the limiting case $x = \pi$, as well as all the cases up to this; for when x reaches this limit all the signs of the series become *plus;* and as it is known that

$$1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \&c. = \infty,$$

the series presents a particular case of $0 \times \infty$; which it is wrong to declare to be 0, in contradiction of its legitimate interpretation, $\frac{\pi}{2}$, on the left. This error has led Abel into other mistakes of consequence: thus, at page 90 of his first volume, he says that the function

$$\text{“} \sin\phi - \tfrac{1}{2}\sin 2\phi + \tfrac{1}{3}\sin 3\phi - \&c.$$

a la propriété remarquable pour les valeurs $\phi = \pi$ et $\phi = -\pi$ d'être discontinue." And at page 71 the same erroneous view has induced him to animadvert upon a certain principle of Cauchy, which the true interpretation of the matter would have tended to confirm.

Fourier, Poisson, and many other modern analysts, have also made similar mistakes in their general investigations respecting series. Thus, to quote Professor Peacock as to the views of the former,

$$\text{“} \cos x = \frac{4}{\pi}\left[\frac{2}{1.3}\sin 2x + \frac{4}{3.5}\sin 4x + \frac{6}{5.7}\sin 6x + \&c.\right]$$

a very singular result, which is, of course, true only between the limits 0 and π, excluding those limits."*

The series is, however, true including the limits: for when $x = 0$, the signs are all *plus;* and, as it is easily shewn that

$$\frac{2}{1.3} + \frac{4}{3.5} + \frac{6}{5.7} + \&c. = \infty,$$

* Proceedings of the Third Meeting of the British Association, p. 257.

we here again have a case of $0 \times \infty$, correctly interpretable by the left hand member of the equation; that is, the right hand member, when $x = 0$, is accurately 1. When $x = \pi$, the signs of the series all become *minus:* therefore the true value in that case is -1.

Before concluding this subject it may be proper to observe, that the investigation, whence the series for $\frac{x}{2}$ is usually deduced, is deficient in generality. Whenever logarithms are employed in connexion with imaginary quantities, the imaginary forms of the logarithms, as well as the real, ought always to be introduced into the investigation: hence the logarithmic expression, from which the series alluded to is derived, should be written thus:

$$\log u = u - u^{-1} - \frac{u^2 - u^{-2}}{2} + \frac{u^3 - u^{-3}}{3} - \frac{u^4 - u^{-4}}{4} + \&c. + 2k\pi\sqrt{-1}$$

By substituting in this $e^{x\sqrt{-1}}$ for u, and then dividing the result by $2\sqrt{-1}$, we shall have the correct and general form,

$$\frac{x}{2} = \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \&c. + k\pi,$$

where k is any whole number, positive or negative, determinable in any particular case, so as to conform to the first member of the equation: regarding that first member, x not exceeding π, as indifferently either $\frac{x}{2}$, or $k\pi + \frac{x}{2}$.

I have here used the limited logarithmic forms of Euler, and not the more general ones furnished by Mr. Graves's theory of imaginary logarithms,* since these limited forms are sufficient for all the *real* values in the general result.

It now merely remains to be shewn that, as briefly stated at page 43, the differential theorem is inapplicable, not only when the proposed series is divergent, but also when it ceases to be convergent, and becomes what Hutton has called a neu-

* Philosophical Transactions, Part I. 1829.

tral series. Thus—although the contrary has often been affirmed—we cannot legitimately infer from this theorem, without the aid of an additional principle, that

$$1 - 1 + 1 - 1 + 1 - 1 + \&c. = \tfrac{1}{2}.$$

For, as already shewn, the series within the brackets at (13) is deficient by a quantity, which in this case is ± 1. Introducing this, (13) gives for s the ambiguous result $\frac{1}{2} \pm \frac{1}{2}$; that is, 1 or 0. The additional principle adverted to, and which is absolutely essential to the received conclusion, is that already stated at page 44; or, as Dr. Whewell briefly expresses it, "that what is true *up to* the limit, is true *at* the limit."

The differential theorem, therefore, can never be employed with success to sum either a divergent or a neutral series; or to convert either into a convergent series.

There has been supposed to exist a perfect analogy between $1-1+1-1+$ &c., as the limiting case of $1-g+g^2-g^3+$ &c., and $1^2-2^2+3^2-4^2+$ &c., as the limiting case of $1-2^2g+3^2g^2-4^2g^3+$ &c., and that, in consequence of this analogy, we have as much right to affirm that $1^2-2^2+3^2-4^2+$ &c. is accurately expressed by 0, the limiting case of $(1-g)(1+g)^{-3}$, the fraction which generates $1^2-2^2g+3^2g^2-4^2g^3+$ &c., as that $1-1+1-1+$ &c. is accurately expressed by $\frac{1}{2}$, the limiting case of $\dfrac{1}{1+g}$, the fraction which generates $1-g+g^2-g^3+$ &c. But there is a total absence of analogy between these two instances: the series $1-g+g^2-g^3+$ &c. presents a series of convergent cases from $g=0$, up to $g=1$; and whatever rule or formula enables us to find the summation in all cases must necessarily enable us to find it in the extreme positive limits 0 and 1; for no values, short of those limits, can be the first and last of the admissible cases. But this rule or formula of summation, whatever it be, is constructed conformably to certain hypotheses; viz. that the convergent series expressed by it, commences, *in all cases*, with a finite quantity, such that the terms of the series, by continual diminution, tend to zero.

The circumstances are very different with respect to $1^2 - 2^2g + 3^2g^2 - 4^2g^3 + \&c.$ As observed in the foot-note at p. 44, the commencement of convergency, in the limiting case, is at a term infinitely distant from the origin of the proposed series, and infinitely great. What analogy can there be between the *general* converging series—if it may be so called—of which this is a limiting case, and ordinary convergent series? And can it be affirmed, of any one of its cases, that the terms necessarily tend to zero? The answers to these questions will, I think, destroy all idea of analogy in such examples as those adduced above.

I have been compelled, in several parts of the present Paper, to dissent from certain doctrines and opinions promulgated by some very distinguished writers on analysis. In developing the principles and views here submitted to the Royal Irish Academy, I could not easily avoid a reference to these. I trust, however, that I have done so in no captious or uncandid spirit: I have only been anxious to arrive at truth in an inquiry of acknowledged perplexity, and of interest, perhaps, in the estimation of some, sufficient to justify the attempt. There are one or two points of analytical delicacy involved in this inquiry, which may perhaps be open to further discussion: if I have myself fallen into error in my treatment of these, I hope I shall be indulged with the same candour and consideration which I have endeavoured to exercise towards others.

Professor Mac Cullagh made a communication on the subject of Total Reflexion.

In the case of total reflexion the vibrations which take place in the rarer medium are in general elliptical, and when this medium is a crystal, the equations by which the ellipse of vibration is determined are very complicated. The projection of this ellipse upon the plane of incidence may, however, be easily found by the remark in p. 102 of the present volume; the projecting cylinder is therefore known, and as the ellipse of vibration is a section of this cylinder, the question of determining the ellipse is reduced to that of determining its plane. For this purpose Mr. Mac Cullagh gave the following rule. Having constructed the ellipsoid of *indices* (that whose axes are parallel to the axes of elasticity, and inversely proportional to the three principal velocities of propagation in the crystal) let its two planes of circular section intersect the aforesaid cylinder. The curves of intersection will be ellipses, which shall be supposed to have a common centre O in the axis of the cylinder. Let OP, OP′ be the greater semiaxes of these ellipses, and OQ, OQ′ the less semiaxes; the lengths of the two former being denoted by p, p', and the lengths of the two latter by q, q'. Join the extremities P, P′ of the greater semiaxes, and the extremities Q, Q′ of the less semiaxes; and divide each of the right lines PP′, QQ′, in the ratio of $\sqrt{p^2 - q^2}$ to $\sqrt{p'^2 - q'^2}$. Then a plane drawn through the centre O and the two points of division will be the plane of vibration. In the application of this rule some precautions are to be observed, but they need not here be insisted on.

The foregoing rule was deduced (in the year 1843) from the general equations by a peculiar use of imaginary quantities, after the author had several times tried in vain to obtain a geometrical interpretation of those equations by considerations of a more obvious and ordinary kind. This use of imaginaries is founded on a remarkable theorem relative to the ellipse, by which it appears, that the *plane* of an ellipse and its *species* (that is, the directions and the ratio of its axes) may be expressed by two imaginary constants, just as the direction of a right line in space is expressed by two real constants. By means of this theorem—which it is unnecessary to repeat, as it has been published in the University Calendar (Examination Papers of the year 1842, p. lxxxiv.)—we may find such properties of elliptical vibrations as are analogous to those of rectilinear vibrations; and it was in this way that the above rule was discovered. It is analogous (though it scarcely appears so at first sight) to the rule by which, in the theory of Fresnel, the direction of rectilinear vibrations is determined, when the plane of the wave is given.

The Rev. Charles Graves read the first part of a paper on Algebraic Triplets.

The object which he proposes to himself is to frame, for the geometry of three dimensions, a theory strictly analogous to that by which Mr. Warren has succeeded in representing the combined lengths and directions of right lines in a plane. In carrying out this design Mr. Graves has necessarily been led to the consideration of new imaginaries.

For the sake of clearness it will be desirable to take, in the first instance, a brief survey of the fundamental properties of algebraic couplets, depending, as they do, upon the nature of the symbol $\sqrt{-1}$. The correspondence between received notions and the views now put forward will thus be made more apparent.

If we take the binomial or couplet $x + \sqrt{-1}\,.\,y$, in which x and y are real quantities, and multiply it by a similar couplet $x_1 + \sqrt{-1}\,.\,y_1$, the product will likewise be a binomial of the same kind, $x_2 + \sqrt{-1}\,.\,y_2$; and between the constituents of the three couplets there exists the relation

$$(x^2 + y^2)\ (x_1^2 + y_1^2) = x_2^2 + y_2^2. \qquad (a)$$

But couplets may be more readily compared after undergoing a simple transformation. Such an expression as $x + \sqrt{-1}\,.\,y$ may be reduced to the form $re^{\sqrt{-1}.\theta}$ by making $r = \sqrt{x^2 + y^2}$, and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. Hence it appears that if we agree to call r the *modulus* and θ the *amplitude* of a couplet, the following theorems will be true:

1. *If two couplets be multiplied together, the modulus of the product will be equal to the product of the moduli of the factors.*

2. *The amplitude of the product will be equal to the sum of the amplitudes of the factors.*

One of the most important analytical properties of the couplet $x+\sqrt{-1}.y$ consists in this, that the equation

$$x+\sqrt{-1}.y=0$$

is equivalent to two, viz. $x=0$ and $y=0$.

As regards the geometric interpretation of the foregoing results, it is sufficient to observe that the symbol $\sqrt{-1}$ has been explained as denoting rotation through a right angle; whilst the couplet $x+\sqrt{-1}.$ has been taken to represent both the length and the direction of the right line drawn from the origin to the point whose rectangular coordinates are x and y: the length of this right line is obviously r; and it is inclined to the axis of x at an angle equal to θ.

The problem now proposed by Mr. Graves is to assign two distributive symbols, ι and κ, of such a nature that (1) the sum or product of two triplets, $x+\iota y+\kappa z$ and $x_1+\iota y_1+\kappa z_1$, shall be itself a triplet of the same form: that (2) there shall be theorems concerning the moduli and amplitudes of triplets, similar to those already enunciated for couplets: that (3) the equation $x+\iota y+\kappa z=0$ shall be equivalent to the three, $x=0$, $y=0$, $z=0$: and that (4) the symbols ι and κ shall admit of a geometric interpretation analogous to that which has been provided for the symbol $\sqrt{-1}$.

The preceding conditions will be complied with, if we assume ι and κ to be distributive symbols of operation, which, when combined, are subject to the following laws:

$$\iota\kappa(a)=a:\ \kappa\iota(a)=a:\ \kappa^2(a)=\iota(a):\ \iota^2(a)=\kappa(a).$$

We must, at the same time, agree to regard $\iota(1)$ and $\kappa(1)$ as units absolutely differing in kind from each other and from the real unit. This, in fact, satisfies the third condition. As $\iota^2(a) = \kappa(a)$ we may, for the future, dispense with the symbol κ, and write the triplet in the form $x + \iota y + \iota^2 z$, or more shortly thus (x, y, z).

In the first place, it is evident that the sum or product of two triplets is itself a triplet.

Next, supposing $(x, y, z) \cdot (x_1, y_1, z_1) = (x_2, y_2, z_2)$ we shall have the modulus of the product equal to the product of the moduli of the factors, if we call the expression $x^2 + y^2 + z^2 + (\iota + \iota^2)(xy + yz + zx)$ the modulus of the triplet (x, y, z). And this *modular* theorem involves in it two others of the same kind, concerning the purely real moduli, $x + y + z$, and $x^2 + y^2 + z^2 - xy - yz - zx$. According as we bring the triplet into different forms by changing the variables in it, there will be either two theorems relating to moduli, and one relating to amplitudes, or one modular theorem, and two concerning amplitudes.

For the purpose of geometric interpretation let us suppose the three positive portions of the axes of rectangular coordinates to meet the surface of a sphere, whose centre is at the origin, in the points X, Y, Z, through which a small circle of the sphere is described, and let us give the name of *symmetric axis* to that diameter of the sphere, which passes through the poles of this circle. Now if we conceive the real unit placed on the axis of x, $\iota(1)$ on the axis of y, and $\iota^2(1)$ on the axis of z, we may interpret the symbol ι by saying that it denotes a conical rotation round the symmetric axis through an angle of 120 degrees. Three such operations, executed successively on the real unit, will bring it back to its original position on the axis of x. This is in accordance with the equations

$$\iota\kappa(a) = \kappa\iota(a) = \iota^3(a) = a:$$

in virtue of which we may regard $\iota(1)$ as a purely imaginary cube root of positive unity.

Mr. Graves mentioned that, since he had obtained permission to read the present paper, Sir William R. Hamilton had kindly communicated to him the abstract and the proof sheets of a memoir by Professor De Morgan, on Triple Algebra. That paper contains the discussion of a system of triplets, which is most closely connected with the one now proposed: the only difference being that Professor De Morgan uses what are in fact new imaginary cube roots of *negative* unity.

Mr. Graves thinks that in the interpretation and generalization of his results he has met with greater success; but he fully concedes to Professor De Morgan the prior possession of what must be looked upon as fundamental in this theory, the conception of symbols which act upon each other in the same manner as the imaginary cube roots of unity. Mr. Graves also stated that his brother, John T. Graves, Esq., had anticipated him in the idea of using cube roots of positive unity in the constitution of algebraic triplets.

The remaining portion of the paper, having reference chiefly to the interpretation of the formulæ obtained in the multiplication of triplets, was postponed until the next meeting of the Academy.

Mr. George Yeates presented a tabular Return of the Observations made by him with Barometer, Thermometer, and Rain Gauge, at his residence, near Portobello, County of Dublin, during the year ending 31st December, 1844.—(See *Appendix*, No. II.)

DONATIONS.

An Essay on Aerial Navigation. By Joseph M'Sweeny, M. D. Presented by the Author.

Archæologia, Vol. XXX., and *Index* to Vols. XVI. to XXX., inclusive. Presented by the Society of Antiquaries of London.

J. H. R. Mott's Advice and Instructions for playing the

Piano with Expression and brilliant Execution. 2 vols. Presented by the Author.

Philosophical Transactions for 1844. Part 2. Presented by the Royal Society.

Memoires de l'Institut de France. (*Morales et Politiques.*) Tome IV. Presented by the Institute.

Journal of the Royal Asiatic Society of Great Britain and Ireland. No. XV. Part 2. Presented by the Society.

Memoires Couronnés de l'Academie Royale de Bruxelles. Tome XVI. (1843).

Recherches Statistiques, par A. Quetelet, H. M. R. I. Academy.

Observations des Phénomènes périodiques.

Resumé des Observations magnétiques. (1843).

Instructions pour l'Observation des Phénomènes periodiques.

Annales de l'Observatoire Royal de Bruxelles. (1844).

Annuaire de l'Observatoire Royal de Bruxelles. (1843–44).

Bulletin de l'Academie Royal de Bruxelles. (*From* 5*th August*, 1843, *to* 3*rd August*, 1844).

Presented by the Academy.

Laws of the Philosophical Society of Leeds. 1841.

Report of the Philosophical Society of Leeds. 1825–6, and 1831–1844.

An Account of an Egyptian Mummy, presented to the Museum of the Philosophical Society of Leeds.

Transactions of the Philosophical Society of Leeds. Vol. I. Part 1. Presented by the Society.

1. *Proceedings of the American Philosophical Society, from April to December*, 1844. Nos. 30 and 31.

2. *A Discourse in Commemoration of Peter S. Du Ponceau, LL. D., late President of the American Philosophical Society, delivered before the Society, on the* 25*th October*, 1844. By Robley Dunglison, M. D. Presented by the Society.

1. *Report of Her Majesty's Commissioners of Inquiry into the State of the Law and Practice in respect to the Occupation of Land in Ireland.*

2. *Evidence taken before the Land Commission in Ireland.* Parts 1 and 2. Presented by the Lord Lieutenant.

Contributions towards a Fauna and Flora of the County of Cork. By Dr. Harvey, J. D. Humphreys, and Dr. Power. Published by, and presented by the Cuvierian Society of Cork.

Obits and Martyrology of Christ Church. Presented by the Irish Archæological Society.

Facts connected with the social and sanitary Condition of the working Classes in the City of Dublin. 1841. By Thomas Willis, F. S. S. Presented by the Author.

Observations made at the Magnetic and Meteorological Observatory, at Toronto, in Canada. Vol. I. for 1840, 1841, 1842. Presented by Her Majesty's Government.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1844–45. No. 50.

January 27, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

The Correspondence (see *Appendix*, No. I.) was read from the Minutes of Council.

The Rev. Charles Graves read the continuation of his paper on Algebraic Triplets.

The triplet $x + \iota y + \iota^2 z$, or (x, y, z), being employed to represent the right line drawn from the origin to the point whose rectangular coordinates in space are x, y, and z, it becomes a matter of interest to determine the position of the right line which represents the product of two such triplets. For this purpose it will be convenient to lay down some definitions.

1. The *symmetric axis* is the right line drawn from the origin, so as to make equal angles with the three positive portions of the axes of coordinates.

2. The *symmetric plane* is a plane passing through the origin, and perpendicular to the symmetric axis.

3. The *modular plane* is the plane containing the axis of x and the symmetric axis.

4. The *radius*, r, of the triplet (x, y, z) is the right line drawn from the origin to the point x, y, z.

5. Its *vector angle*, ρ, is the angle between the radius and the symmetric axis.

6. The *amplitude*, ω, of the triplet (x, y, z) is the angle between the modular plane and the plane containing the radius and the symmetric axis.

We are able to determine the radius, vector angle, and amplitude, of the triplet (r_2, ρ_2, ω_2), which is formed by multiplying together two triplets (r, ρ, ω) and (r_1, ρ_1, ω_1), by means of the following equations, in which s stands for the angle between the symmetric axis and the positive portion of any one of the axes of coordinates.

$$rr_1 \cos\rho \cos\rho_1 = r_2 \cos s \cos\rho_2 \tag{1}$$

$$rr_1 \sin\rho \sin\rho_1 = r_2 \sin s \sin\rho_2 \tag{2}$$

$$\omega + \omega_1 = \omega_2. \tag{3}$$

It appears from the first two of these equations that whether we call $\frac{r\cos\rho}{\cos s}$, or $\frac{r\sin\rho}{\sin s}$, the modulus of the triplet, it will be true to say that *the modulus of the product is equal to the product of the moduli of the factors.*

The third equation asserts, that *the amplitude of the product is equal to the sum of the amplitudes of the factors.*

As the real unit is supposed to be placed on the axis of x, we shall obtain the following theorem by dividing the second of the preceding equations by the first:

The tangents of the vector angles of the real unit, of the two factors, and of the product, form a proportion.

Mr. Graves stated that Sir Wm. Hamilton had been the first to announce that if the real unit line, the factors, and the product line, be projected upon the symmetric axis, the projections will form a proportion in the simple sense of that term: whilst the projections of the same lines on the symmetric plane form a proportion, according to the higher sense in which Mr. Warren uses the same word.

The former of these theorems is merely the geometric interpretation of equation (1): the latter of equations (2) and (3).

The following cases deserve special attention:

If either of the factor lines coincides with the symmetric axis, the product line must also coincide with it.

If either of the factor lines is contained in the symmetric plane, the product line must also be contained in it.

But if one coincides with the symmetric axis, and the other with the symmetric plane, the product line will vanish.

Startling as these consequences may appear, they are to be explained by reference to the geometric meaning of the symbols ι and ι^2; both of which, according to the interpretation assigned to them, are inoperative to move a right line out of either the symmetric plane or the symmetric axis.

The analytical difficulty raised by the last case seems to force us to admit that the vanishing of a product does not necessarily imply the vanishing of a factor. In the present instance it is caused by the vanishing of one of the moduli of multiplication belonging to each of the factors.

The length of the product line is equal to the product of the lengths of the factor lines only in the case where the vector angle of one of the factors is equal to the vector angle of the real linear unit (where ρ or $\rho_1 = s$).

Having thus interpreted the results of multiplication by means of the existing trigonometry, Mr. Graves proceeded to show how the use of a new kind of trigonometry gives increased symmetry and flexibility to the present theory of algebraic triplets.

The foundations of this new calculus are thus laid. Using the exponential development we shall find

$$e^{\iota\phi} = \lambda + \iota\mu + \iota^2\nu$$

where

$$\lambda = 1 + \frac{\phi^3}{1.2.3} + \frac{\phi^6}{1.2.3.4.5.6} + \&c.$$

$$\mu = \phi + \frac{\phi^4}{1.2.3.4} + \frac{\phi^7}{1.2.3.4.5.6.7} + \&c.$$

$$\nu = \frac{\phi^2}{1.2} + \frac{\phi^5}{1.2.3.4.5} + \frac{\phi^8}{1.2.3.4.5.6.7.8} + \&c.$$

Again,

$$e^{\iota^2\chi} = \lambda_{,} + \iota^2\mu_{,} + \iota\nu_{,}$$

where $\lambda_{,}$, $\mu_{,}$, $\nu_{,}$ are the same functions of χ that λ, μ, ν are of ϕ. Hence we have

$$e^{\iota\phi + \iota^2\chi} = \text{A} + \iota\text{M} + \iota^2\text{N},$$

A standing for $\lambda\lambda_{,} + \mu\mu_{,} + \nu\nu_{,}$, M for $\lambda\nu_{,} + \mu\lambda_{,} + \nu\mu_{,}$, and N for $\lambda\mu_{,} + \mu\nu_{,} + \nu\lambda_{,}$.

The three functions, A, M, and N depending, each of them, on two variables, ϕ and χ, hold the same place in the present calculus that *cosine* and *sine* hold in the received trigonometry. And as the sum of the squares of *cosine* and *sine* is always equal to unity, so the equation

$$\text{A}^3 + \text{M}^3 + \text{N}^3 - 3\,\text{AMN} = 1$$

holds good, no matter what be the amplitudes ϕ and χ.

The importance of these formulæ in our theory of triplets is most obvious. For a triplet $x + \iota y + \iota^2 z$ may in general be thrown into the form $m(\text{A} + \iota\text{M} + \iota^2\text{N})$, which, as we have seen, is equivalent to $me^{\iota\phi + \iota^2\chi}$. So that if m be called the modulus, and ϕ and χ the amplitudes, of the triplet, we shall find, on multiplying two triplets together, the following theorems to be true:

The modulus of the product is equal to the product of the moduli of the factors.

Either of the two amplitudes of the product is equal to the sum of the two corresponding amplitudes of the factors.

The modulus m is connected with the constituents of the triplet (x, y, z) by the following equation,

$$m^3 = x^3 + y^3 + z^3 - 3\,xyz,$$

with respect to which it is to be observed that the right hand member is the product of

$$x+y+z \text{ and } x^2+y^2+z^2-xy-yz-zx,$$

the two real moduli which have previously been shewn to belong to the triplet $(x, y, z.)$—See page 53.

Mr. Graves stated that he had obtained a multitude of formulæ concerning the functions A, M, and N, analogous to the fundamental formulæ of trigonometry. Amongst the more remarkable of these he pointed attention to one corresponding to the well-known theorem of Moivre.

By pursuing a similar course we may frame a theory of multiplets, admitting a like interpretation. In order to accomplish this we must assume a symbol κ, such that $\kappa^n(1)=1$, whilst $1, \kappa(1), \kappa^2(1), \kappa^3(1) \ldots$ &c. are looked upon as units absolutely differing in kind as much as unity differs from $\sqrt{-1}$. The development of $e^{\kappa\phi}$ into the form $\alpha + \kappa\beta + \kappa^2\gamma +$ &c. will give us a set of n functions, α, β, $\gamma \ldots\ldots$ each depending upon one variable ϕ: and again, the expansion of $e^{\kappa\phi + \kappa^2\chi + \kappa^3\psi + \ldots}$ furnishes us with a series of n functions, A, B, Γ, &c, each depending upon $(n-1)$ variables ϕ, χ, ψ, &c. The multiplet $a + \kappa b + \kappa^2 c +$ &c. being now written in the form $m(\text{A} + \kappa\text{B} + \kappa^2\Gamma +$ &c.), which is equivalent to $me^{\kappa\phi + \kappa^2\chi + \kappa^3\psi + \&c. \ldots}$, it is evident that, if we call m the modulus, and ϕ, χ, ψ, &c. the amplitudes of the multiplet, we shall have the same theorems concerning moduli and amplitudes that have been already established in the case of the multiplication of couplets and triplets.

If, for instance, we form a quadruplet (w, x, y, z) by the aid of the symbol κ, which is a pure imaginary fourth root of positive unity, we shall find that the quantities $[w+x+y+z]$, $[(w+y)-(x+z)]$, and $[(w-y)^2 + (x-z)^2]$ are all moduli of multiplication. The product of the three is equal to m^4.

Mr. Graves mentioned that his elder brother, Mr. John T. Graves, had been the first to conceive the notion of employing the functions λ, μ, and ν in the interpretation of this theory of triplets; but as they involve only one variable it is not possible to bring a triplet in general into the form

$$m(\lambda + \iota\mu + \iota^2\nu).$$

The President stated that the remarkable researches respecting algebraic triplets, made lately by Professor De Morgan and John T. Graves, Esq. in England, and here by the Rev. Charles Graves, had led him to perceive the following theorem:

If the three symbols ξ, η, ζ, or rather their squares and products, be supposed to satisfy the three following "equations of signification:"

$$\xi\,(b\eta + c\zeta) = a\,(\eta^2 + \zeta^2),$$
$$\eta\,(c\zeta + a\xi) = b\,(\zeta^2 + \xi^2),$$
$$\zeta\,(a\xi + b\eta) = c\,(\xi^2 + \eta^2);$$

and if, by the help of these three equations, we eliminate any three of the six quadratic combinations ξ^2, η^2, ζ^2, $\xi\eta$, $\eta\zeta$, $\zeta\xi$, from the development of the "formula of multiplication,"

$$(u\xi + v\eta + w\zeta)\,(x''\xi + y''\eta + z''\zeta)$$
$$= (x\xi + y\eta + z\zeta)\,(x'\xi + y'\eta + z'\zeta),$$

and then treat the three remaining combinations of the same set (ξ^2, &c.) as three entirely arbitrary and independent multipliers: the three separate equations thus obtained between the fifteen real quantities $a\ b\ c\ u\ v\ w\ x\ y\ z\ x'\ y'\ z'\ x''\ y''\ z''$ will be such, that whether we project the four lines (u, v, w), (x, y, z), (x', y', z'), (x'', y'', z''), on the axis $(a, b, c,)$ itself, or on the plane perpendicular to that axis, *the four projections thus obtained will in each case form a proportion;* the proportionality of the projections on the axis being of the kind considered in ordinary algebra, and the proportionality of the projections on the plane perpendicular to the axis being of the kind considered by Mr. Warren; that is to say, the lengths of these last projections are proportionals in the usual sense, and the rotation from the first to the second is equal to the rotation from the third to the fourth.

Sir W. Hamilton has been able to prove this theorem by treating the three real equations between the fifteen rectangular co-ordinates a, b, c, &c. according to the known methods of algebraical geometry. He has also arrived at simplifications of this proof by introducing the ordinary imaginary $\sqrt{-1}$, treated by the ordinary rules; but his first investigation, and the one which he prefers, has been founded on the rules of the calculus of quaternions, and consists in resolving by those rules the system of the two equations:

$$\frac{a\rho'' + \rho''a}{a\rho' + \rho'a} = \frac{a\rho + \rho a}{av + va}; \quad \frac{a\rho'' - \rho''a}{a\rho' - \rho'a} = \frac{a\rho - \rho a}{av - va};$$

in which

$$\begin{aligned} a &= ia + jb + kc, \\ v &= iu + jv + kw, \\ \rho &= ix + jy + kz, \\ \rho' &= ix' + jy' + kz', \\ \rho'' &= ix'' + jy'' + kz'', \end{aligned}$$

$i, j, k,$ being (as in former communications respecting quaternions) three imaginary units connected by the nine non-linear relations

$$i^2 = j^2 = k^2 = -1; \quad ij = k, jk = i, ki = j; \quad ji = -k, kj = -i, ik = -j.$$

The phrase "equations of signification" is borrowed from Mr. De Morgan. If the theorem be particularized, so as to correspond to that gentleman's system of triplets, by making

$$u = 1, \quad v = w = 0, \quad a = -b = -c,$$

then the equations of signification reduce themselves to

$$\xi^2 = \eta\zeta, \quad \eta^2 = -\zeta\xi, \quad \zeta^2 = -\xi\eta,$$

and the formula of multiplication resolves itself into the three relations

$$\begin{aligned} x'' &= xx' + yz' + zy', \\ y'' &= xy' + yx' - zz', \\ z'' &= xz' + zx' - yy'. \end{aligned}$$

On the other hand, some of the results of the systems of the two Messrs. Graves may be reproduced by taking the same *unit-line*, $u = 1$, $v = w = 0$, but employing that other *axis* for which $a = b = c$. The equations of signification give then, more simply,

$$\xi^2 = \eta\zeta, \quad \eta^2 = \zeta\xi, \quad \zeta^2 = \xi\eta;$$

and the terms in x'', y'', z'', are all to be taken with positive signs. Sir William Hamilton pretends to no farther merit in the matter than to that of having sought to illustrate, by generalizing in one direction, the foregoing points of the theories of his friends.

February 10, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Robert Forster, Esq., William Le Fanu, Esq., Reverend George Longfield, John M. Neligan, M. D., William Justin O'Driscoll, Esq., Nicholas P. O'Gorman, Esq., Algernon T. Preston, Esq., and James Emerson Tennant, Esq., M. P., were elected Members of the Academy.

The President read a paper on Quaternions.—See *Appendix*, No. III.

A stone celt was presented by the Rev. Dr. Walsh, H.M., from Captain Walsh.

February 24, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Matthew Baker, Esq., Patrick Joseph Blake, Esq., John D'Arcy, Esq., Rev. Nicholas John Halpin, Samuel Haughton, Esq., F. T. C. D., William Hogan, Esq., C. E., James Macdonnell, Esq., C. E., Right Hon. David R. Pigot, Matthew R. Sausse, Esq., Walter Sweetman, Esq., R. William Townsend, Esq., Alexander Taylor, M. D., and George Yeates, Esq., were elected Members.

Lieutenant-Colonel Jones (R. E.), on the part of the Shannon Commissioners, presented to the Museum a collection of ancient bronze weapons, ornaments, and other articles, found recently in the excavations made in the bed of the Shannon. And he offered, on the part of the Commission, to assist the Committee of Antiquities in conducting any antiquarian researches they might propose in the neighbourhood of that river. Whereupon it was

Resolved,—That the Committee of Antiquities be requested to put themselves in communication with the Shannon Commissioners, with reference to their obliging proposal.

John Anster, LL.D., read the conclusion of the paper by the Rev. James Wills, " On Dugald Stewart's Explanation of certain Processes in the Human Understanding."

Mr. Wills commenced by referring to his former paper, in which he had endeavoured to rectify the elementary principle, from which he considered the true theory of the intellectual powers as being deducible. As in that paper he had explained the origin and formation of those fixed associations which were the results of habit; so in the present paper it was his purpose to deduce from the same primary law the class of unfixed and accidental associations.

Before entering upon this statement, the Author dwelt at some length on the nature of the method by which he had arrived at his results. This he described as being, in the strictest sense, a method of observation. He had, he said, excluded all consideration of the metaphysical writers; and though he might coincide with many of their opinions, he would disclaim any authority, or ground of inference, but the simplest and most scrupulous investigation of the facts, which he would endeavour so to exemplify as to convey his results to the Academy. He also made some remarks on the difficulty of this method, by reason of the complexity of the mental operations, and the impossibility of otherwise attesting the results than by the exposition of the inquirer's own thoughts, and the appeal to the consciousness of others.

The Author then went on to explain the elementary law of simple apprehension, and shewed that however numerous or varied might be the objects presented in any single instant to the apprehension, the result must be one single idea—being a whole in itself, and of which all the components must be apprehended strictly *as parts*. From this fact he observed that the entire process of memory could be deduced. For this purpose he stated a case framed with a view to explain the ordinary process by which incidents and circumstances are recalled, by means of their association as parts or as wholes; so that either the part might recal the whole, or the whole the part.

The Author then explained how this operation might be accidental, or the effect of design. The first would be the case of mere passive remembrance; the other the active recollection of the will. The first would mostly be the result of the actual presence of some portion of the combined ideas; the other must be arrived at by an effort in which the mind succeeds in placing itself in some position of relation to the required idea, so as to fulfil the condition of the theory. To meet the only difficulty which might be suggested by this exposition, the Author pointed out, that the occasion which might require such an effort must necessarily present a first link in a chain of associations, from which every subsequent step would be a result similarly occurring in its order, so far as the operation might happen to be rightly directed, and the materials within the compass of the mind engaged in it.

The Author observed that the various causes of memory described by philosophers, are merely the circumstances attending the process here described. He observed that every means of association by which ideas can be combined, might serve the same purpose in the same manner, and that the same reasoning would apply; though he only considered it necessary to follow the inquiry with regard to the large class of accidental associations which are the main ground of the ordinary work of memory.

He next entered into an inquiry as to the comparative durability of the different classes of associations. This, he observed, must depend on their distinctness, and their coherency: and hence he inferred that, generally, visible objects are the most easily recalled; and that certain combinations of sound might be next in order: after these might be reckoned the class of professional associations. He remarked upon the expedients used in the systems of artificial memory, as founded on this principle; and strongly condemned the methods, as being a substitution of false combinations, instead of those founded on the real relations of things.

The Author then went on to apply his theory to some known phenomena in derangement, dreaming, &c.: and concluded with some general statements upon the sum of his results, and the progress he had made in his inquiry.

A paper was read by Dr. Allman, containing a notice of two undescribed algæ, which clothe the face of the rock over which the water trickles in the "Dripping Well" of Knaresborough. They are referrible to the genus *Microcystis*, Menegh.

Dr. Allman also noticed the occurrence in Ireland of the genus *Phylactidum* of Kutzing, which thus becomes an addition to the British Flora. The beautiful little alga now noticed, which appears specifically distinct from *P. elegans*, Kutz., was discovered by Dr. Allman, during the autumn of 1843, in a small, rapidly running rivulet near Bandon, in the County Cork.

The following algæ were also now, for the first time, recorded as additions to the Irish cryptogamic Flora:

Calothrix mirabilis, forming little tufts attached to stones and mosses, in a small stream trickling over a rock near Clonmel.

Nostoc muscorum, on a little-used foot-path, Castleknock, County Dublin.

Nostoc sphæricum, on the surface of a moist rock, Clonmel.

Euaster rota, in a small pond near Bandon, entangled with other algæ.

Euaster oblonga, along with Desmidium mucosum and D. Borreri, in a peat pit, Sheaghy, County Cork.

Desmidium Swartzii, in an old peat pit, Howth.

Desmidium cylindricum, in an old peat pit, Howth.

Desmidium mucosum, in a peat pit, Sheaghy.

Desmidium Borreri, in a peat pit, Sheaghy.

Leptomitus pisidicola, growing on a dead Caddis-worm in Lough Bray, County Dublin.

The President read a note from the Rev. Dr. Robinson, respecting Lord Rosse's telescope.

The Secretary read a letter from Mr. Charles Bourns, giving an account of the excavation made in the interior of the Round Tower of Lusk.

Mr. W. R. Wilde made some observations on the human skulls which were presented by Mr. Bourns, and which had been found within the tower.

March 15, 1845. (Stated Meeting.)

REV. JAMES H. TODD, D. D., Vice-President, in the Chair.

This being the day appointed by Charter for the Stated Meeting and annual election, the following Officers and Members of Council were chosen for the ensuing year:

President—Professor Sir Wm. Rowan Hamilton, LL.D.
Secretary—James Mac Cullagh, LL.D.
Secretary to the Council—Robert Kane, M. D.
Secretary of Foreign Correspondence—Rev. Humphrey Lloyd, D. D.
Librarian—Rev. Wm. H. Drummond, D. D.
Clerk and Assistant Librarian—Edward Clibborn.

Committee of Science.

Rev. Franc Sadleir, D. D., Provost; Rev. Humphrey Lloyd, D. D.; James Apjohn, M. D.; James Mac Cullagh, LL.D.; Robert Ball, Esq.; Robert Kane, M. D.; G. J. Allman, M. B.

Committee of Polite Literature.

His Grace the Archbishop of Dublin; Samuel Litton, M. D.; Rev. William H. Drummond, D. D.; Rev. Charles Graves, A. M.; Rev. Charles W. Wall, D. D.; John Anster, LL.D.; Rev. Samuel Butcher, A. M.

Committee of Antiquities.

George Petrie, Esq., R. H. A.; Rev. James H. Todd, D. D.; Henry J. Monck Mason, LL.D.; Samuel Ferguson, Esq.; J. Huband Smith, A. M.; Captain Larcom, R. E.; William R. Wilde, Esq.

A volume of Mr. Petrie's work on the Round Towers of Ireland, was laid upon the table.

The Secretary read the following Report from the Council, which was adopted, and ordered to be entered on the Minutes:

"Among the occurrences of the past year, the Council have to notice the acquisition by the Academy of a considerable collection of Irish manuscripts. This collection was purchased from Messrs. Hodges and Smith for 1250 guineas; of which sum so large a share as £600 was very liberally granted for the purpose by Her Majesty's Government. The remainder, with the exception of £100 contributed from the funds of the Academy, was raised by subscription from the Members of the Academy and the Public generally; and the Council are gratified in referring to this new instance of the right feeling and proper spirit which have, of late years, been manifested in the case of national remains; but, at the same time, they have to express their regret that the known existence of this feeling has had the effect of greatly increasing the price of such relics in the market.

"For the purpose of arranging and displaying the large collection of Antiquities now in the possession of the Academy, the room in which the meetings of the Academy used to be held has been converted into a Museum; and, at the trifling expense of £150, a new Board Room has been fitted up, in which the meetings of the present Session have been held. For the economy with which these arrangements have been made, the Academy are mainly indebted to the suggestions of their excellent Assistant Librarian, Mr. Clibborn, who cheerfully sacrificed part of his own accommodation, in order that the proposed changes might be carried into effect. The regulation of the Museum, and the adoption of such measures as may be necessary for the security of the valuable collections therein deposited, are objects which the Council recommend to the immediate attention of their successors.

"The Council are informed that a volume by Mr. Petrie, on the Round Towers of Ireland, will be ready in a few days, and will be laid on the table of the Academy at the Stated Meeting. This work will of course be submitted to the new Council before it is received as a part of the Transactions; and it will be the duty of the Council to report upon it to the Academy, with reference to the engagement entered into with Mr. Petrie.

"The number of Members elected during the past year has been unusually great. The following are their names:

"LIST OF MEMBERS OF THE ROYAL IRISH ACADEMY, ELECTED SINCE THE 16TH MARCH, 1844.

1. W. S. O'Brien, Esq., M. P.
2. The Marquis of Kildare.
3. W. H. Harvey, M. D.
4. Charles Hanlon, Esq.
5. Maxwell M'Master, Esq.
6. Thomas Oldham, Esq.
7. Philip Reade, Esq.
8. Henry Roe, Esq.
9. Robert Wilson, Esq.
10. Duke of Leinster.
11. Marquis of Downshire.
12. Lord Farnham.
13. Lord Wallscourt.
14. Robert S. Bradshaw, Esq.
15. Thomas Davis, Esq.
16. Sir Richard Franklin.
17. Edmund Getty, Esq.
18. G. A. Hamilton, Esq., M. P.
19. Henry G. Hughes, Esq.
20. H. James, Esq. (Capt. R. E.)
21. Francis L'Estrange, Esq.
22. Edward Lucas, Esq.
23. John Phillips, Esq.
24. T. N. Redington, Esq., M.P.
25. Marmion W. Savage, Esq.
26. Richard Sharp, Esq.
27. Robert Forster, Esq.
28. William Le Fanu, Esq.
29. Rev. George Longfield.
30. John M. Neligan, M. D.
31. W. Justin O'Driscoll, Esq.
32. N. P. O'Gorman, Esq.
33. Algernon T. Preston, Esq.
34. J. E. Tennant, Esq., M. P.
35. Matthew Baker, Esq.
36. P. Joseph Blake, Esq.
37. John D'Arcy, Esq.
38. Rev. N. John Halpin.
39. Samuel Haughton, Esq.
40. William Hogan, Esq., C. E.
41. James Macdonnell, Esq., C. E.
42. Right Hon. David R. Pigot.
43. M. R. Sausse, Esq.
44. Walter Sweetman, Esq.
45. R. W. Townsend, Esq., C. E.
46. Alexander Taylor, M. D.
47. George Yeates, Esq.

"Total elected, from 16th of March, 1844, to 16th March, 1845, —forty-seven.

" The following are the Members deceased within the year :

Life Members.	*Annual Members.*
* Rev. Charles Boyton, D. D., 1822.	William Henry, Esq., 1844.
* Hugh Ferguson, M. D., 1824.	Matthew O'Conor, 1837.
* Rev. Thomas Goff, 1802.	*Honorary Member.*
* William Webb, Esq., 1808.	Francis Baily, Esq., LL. D., F. R. S., &c., 1826.

" The distinguished name of M. Arago, Perpetual Secretary of the Academy of Sciences of Paris, has been added to the list of Honorary Members.

" The list of Honorary Members has lost a name well known to science, that of Francis Baily, Esq., late President of the Astronomical Society of London, of which he was one of the principal founders. On the worth of his character, and on the value of his services to science, it is needless to dwell, as Sir John Herschell has already done justice to them, in a Memoir read to the Society over which he presided, and which, up to the period of his death, was the object of his unceasing care and attention."

Mr. Robert Ball, on the part of the Dean of Lismore, presented the seal of the late Archbishop of Cashel.

The thanks of the Academy were voted to the Dean of Lismore.

April 14, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Joseph Allen Galbraith, Esq., James Jameson, Esq., and John Francis Waller, Esq., were elected Members of the Academy.

READ,—The Resolution of Council passed at the meeting of 7th April, viz.

"That the Volume which has been printed by Mr. Petrie, as the twentieth volume of the Transactions, be received as such; and though it cannot be regarded as a complete Work, that, nevertheless, the Council do recommend it to be taken, as acquitting Mr. Petrie of his engagements to the Academy.

RESOLVED UNANIMOUSLY,—That the recommendation of Council be received and adopted.

READ,—The following letter from Messrs. Hodges and Smith:

"104, GRAFTON-STREET,
"*April* 14*th*, 1845.

"SIR,—Having been informed that a person of the name of O'Brien, in the County of Clare, had some curious ancient documents which he would dispose of, we went down to that county for the purpose of examining them. We found his collection to consist chiefly of family documents and papers of little moment, with the exception of four pages of the celebrated work known by the name of Leabhar Breac. Those pages contain a beautiful Latin Hymn to St. Patrick, by Secundinus, and other interesting matter. They have been missing for upwards of fifty years, and we have now much pleasure in restoring them to the Academy, whose property the Leabhar Breac has been for many years; and we also beg to present two copies of Mac Firbis's work on the Genealogy of ancient Irish Families, written in 1650, and another volume containing an abstract of the same great work, written by James M'Guire, in 1721; and, fifthly, a very good copy of Keating's History of Ireland, written in 1712.

"We have the Honour to be, Sir,
"Your very obedient Servants,
"HODGES AND SMITH.

"*To the Secretary of the Royal Irish Academy.*"

RESOLVED,—That the thanks of the Academy be given to Messrs. Hodges and Smith for their donation.

Rev. Dr. Todd presented, in the name of the Rev. James Spencer Knox, a curious book, found near Maghera, composed of tablets made of wood, and covered with wax.

The President, under his hand and seal, appointed the following Vice-Presidents:

Rev. Charles Wm. Wall, D. D.; George Petrie, Esq.; James Mac Cullagh, LL.D.; Thomas A. Larcom (Captain, R. E.)

His Excellency the Lord Lieutenant having attended as a visitor,

The Rev. Dr. Robinson proceeded to give an account of the construction and results obtained with the six foot reflecting telescope of the Earl of Rosse.

April 28, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

The Treasurer presented the following abstract of the Annual Account, as audited, for the year ending 31st March, 1845:

ROYAL IRISH ACADEMY.

ABSTRACT OF TREASURER'S ACCOUNT FOR YEAR ENDED 31st MARCH, 1845.

THE CHARGE.

	£	s.	d.
Balance on last year's Account	210	5	6
Government Annual Grant	300	0	0
Government Special Grant for MSS.	600	0	0
Treasury warrants	146	17	8
Donations to purchase Irish MSS.	658	5	6
Dividends on Stock	76	8	0
Rent of Stable	21	0	0
Annual Subscriptions	331	16	0
Entrance Fees	210	0	0
Life Compositions	241	10	0
Total Charge	£2796	2	8

THE DISCHARGE.

	£	s.	d.	£	s.	d.
Books and Manuscripts:						
MSS.	1358	11	0			
Books	124	7	3			
Woodcuts	9	8	6			
				1492	6	9
Repairs on Board-room	131	1	6			
Other repairs	74	11	7			
				205	13	1
Contingencies and Postage				47	9	7
Salaries				271	18	8
Antiquities				453	17	6
Stationery				15	12	9
Rent and Taxes				134	0	4
Fuel and Lighting				61	6	1
Total Expenditure				2682	4	9
Balance in favour of the Academy				113	17	11
Total Discharge				£2796	2	8

Signed, R. BALL,
Treasurer.

Rev. Dr. Todd presented, from Lord Granville Somerset, a copy of the Charters of the Duchy of Lancaster.

Colonel Jones, R. E., presented a printed description and drawing of an antique tombstone, of the natural size, found at Athlone.

Mr. J. Huband Smith read an account of the present state of the ancient ecclesiastical remains of Iona.

After alluding to the difficulty of procuring accommodation in this remote island, and the impossibility of obtaining more than a mere glance at the details of the group of ecclesiastical buildings, in the way in which they are now usually seen by tourists, Mr. Smith went into a very minute account of the Nunnery, with its grave-yard, containing some interesting monuments. He then described two ancient causeways, and the church of St. Oran, which he supposes to be the most ancient edifice on the island, though he declined to express any definite opinion as to its age. Of the doorway of this church Mr. Smith exhibited a very careful drawing, on a large scale, as well as others of some of the most remarkable tombstones in the Releg Oran. He then proceeded to give some account of the grave of St. Columba, adjoining to the great western door of the cathedral, the nave, chancel, and aisles of which, together with its noble square tower, he described in detail, with measurements which he had carefully taken.

The numerous drawings made by Mr. Smith, from his own sketches on the spot, contributed not a little to convey a more satisfactory idea of various particulars than any verbal description could have done; and, with many rubbings taken from tombstones and crosses, enabled him to correct some errors into which Pennant and other writers had fallen, in the deciphering and translation of certain Irish inscriptions yet remaining at Iona. Mr. Smith lastly noticed the Cross of Inverary, which is said to have been brought from Iona at the time of the Reformation, and produced a very careful rubbing of the entire Cross, giving the elaborate patterns on both sides, and the curious inscription along its edge, which he translated; and he then concluded by expressing his conviction, that notwithstanding that during the five days he remained at Iona he was unceasingly occupied, he had, nevertheless, left much unnoticed, with regard to the numerous group of buildings in the vicinity of the cathedral, as well as of many interesting localities in the interior of the island, connected with the history of the celebrated St. Columbkille and the Culdees.

May 12, 1845.

CAPTAIN LARCOM, R. E., Vice-President, in the Chair.

Charles Bournes, Esq., W. C. Dobbs, Esq., Wm. Henn, Esq., D. B. Starkey, Esq., and Benjamin Wilme, Esq., were elected Members of the Academy.

Mr. William Roberts, F. T. C. D., read a paper on some Geometrical Theorems relative to Elliptic Functions.

It is well known that the arc of a spherical ellipse is expressed by an elliptic function of the third order, with a circular parameter: and it has been also proved (see M. Liouville's Journal of Pure and Applied Mathematics, vol. ix.) that the arc of the intersection of a sphere with a cone of the second degree, one of whose external principal axes is a diameter, and whose vertex is situated on the surface, may be made to supply a general representation of the three elliptic transcendants; at least with the aid of the *modular* transformations, which have been given by Lagrange and M. Jacobi. The object of the present communication is to shew that these results are but particular cases of the general theorem, that the arc of any symmetrical intersection of a sphere with a cone of the second order (i. e. when the centre of the sphere lies upon one of the principal axes of the cone) may be expressed by elliptic functions. This is easily deduced from the following formulæ for the rectification of the class of spheric curves represented by the polar equation,

$$\sin^2\rho \mathrm{F}(\omega) + \sin 2\lambda \cos\rho = 1 \tag{a}$$

where λ is a constant. Let s_1, s_2, denote the two arcs which correspond to the same value of the polar angle ω, and we will have

$$s_1 + s_2 = \cos\lambda \int \sqrt{\left\{\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4\mathrm{F} + \sin^2 2\lambda}{\mathrm{F} - \cos^2\lambda}\right\}} \frac{d\omega}{\mathrm{F}}$$

$$s_1 - s_2 = \sin\lambda \int \sqrt{\left\{\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4\mathrm{F} + \sin^2 2\lambda}{\mathrm{F} - \sin^2\lambda}\right\}} \frac{d\omega}{\mathrm{F}} :$$

F' denoting the derived function of $\mathrm{F}(\omega)$.

Now in the case of the intersection of a cone and sphere, such as we have described, $\mathrm{F}(\omega)$ is a linear function of $\cos 2\omega$, and each of the foregoing integrals is reducible to an elliptic function of the third kind.

In the case of the spherical ellipse, the two arcs s_1 s_2 are equal; and when the vertex of the cone is upon the surface, the second arc s_2 vanishes altogether.

The same relation between the principal angles of the cone, and the distance of its vertex from the centre of the sphere, will cause the parameters in both the functions Π to vanish. And it is remarkable that, in this case, when the sum and difference of the arcs s_1, s_2, are each expressible by a transcendant of the first order, the curve will coincide with the locus of the vertex of a spherical triangle, the base of which is given, and of which the product of the sines of the semi-sides is constant, and less than the square of the sine of the fourth part of the base.

This result is strikingly analogous to M. Serret's theorem on the rectification of the Cassinian curve, published in M. Liouville's Journal, vol. viii. p. 145.

If the moduli of the two functions which express the values of $s_1 \pm s_2$ be complementary, the curve will coincide with the locus of the vertex of a spherical triangle whose base is given, and of which the product of the tangents of the semi-sides is constant, and less than the square of the tangent of the fourth part of the base.

Equation (a) may be transformed into

$$\tan^4 \tfrac{1}{2}\rho - 2\tan^2 \tfrac{1}{2}\rho \, \mathrm{F}(\omega) + \frac{1-\sin 2\lambda}{1+\sin 2\lambda} = 0,$$

where $\mathrm{F}(\omega)$ simply is written for $\dfrac{1-2\mathrm{F}(\omega)}{1+\sin 2\lambda}$. The class of plane curves whose polar equation

$$r^4 - 2r^2\mathrm{F}(\omega) + c^4 = 0$$

is analogous to the above, may be rectified by similar formulas. These are

$$s_1 + s_2 = \frac{1}{\sqrt{2}}\int\sqrt{\left(\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4c^4}{\mathrm{F} - c^2}\right)}\, d\omega$$

$$s_1 - s_2 = \frac{1}{\sqrt{2}}\int\sqrt{\left(\frac{4\mathrm{F}^2 + \mathrm{F}'^2 - 4c^4}{\mathrm{F} + c^2}\right)}\, d\omega.$$

Each of these integrals will be reducible to an elliptic function of the third order, provided that $\mathrm{F}(\omega)$ be a linear function of $\cos 2\omega$. The following curves, among others, possess this property: first, the locus of a point, such that, tangents being drawn from it to two equal non-intersecting circles, their rectangle is constant, and less than the square of the tangent drawn to either from the point midway between their centres: secondly, the locus of the intersection of tangents to an ellipse which include a given angle. This curve is composed of two closed branches concentric with the ellipse, and satisfying the given condition by angles which are supplemental.

The well-known property of the lemniscate may be extended to the locus of the orthogonal projections of the centre of an ellipse or hyperbola in general upon its tangents. First, in the case of the ellipse, the curve may be derived by taking the locus of a point, such that tangents being drawn from it to two equal intersecting circles, their rectangle may be constant, and equal to the square of half the common chord.

If the circles do not intersect, and if the rectangle under the tangents be equal to the square of the tangent to either from the middle point between the centres, the locus will give the curve derived from an hyperbola, whose real axis is greater than the imaginary.

To get the curve when the imaginary axis is greater than the real, we must take the locus of a point, such that lines being drawn from it to meet the circumferences of two equal, non-intersecting circles, and subtending right angles at their centres, their rectangle may be constant, and equal to the sum of the squares of the radius, and of half the distance between the centres.

There exist in spherical geometry numerous properties of an analogous character.

The Rev. Charles Graves made a communication relative to the new functions employed by him in the interpretation of his theory of Algebraic Triplets.

The three functions A, M, N, defined in p. 60, possess so many interesting properties that they seem to deserve distinctive appellations. The first is symmetrical with respect to its two amplitudes, ϕ and χ; and its properties are in the main analogous to those of the trigonometrical function *cosine*. On the other hand, M and N are not symmetrical functions of ϕ and χ; but either of them may be obtained from the other by interchanging the two amplitudes; and they correspond in many respects to the trigonometrical *sine*. Mr. Graves proposes then to call A the *cotresine*, and M and N the *tresines* of ϕ and χ: and he designates them respectively by the symbols $\operatorname{cotr}[\phi, \chi]$, $\operatorname{tres}[\phi, \chi]$, $\operatorname{tres}[\chi, \phi]$.

As in trigonometry $\cos\phi = \cos(2i\pi + \phi)$, and $\sin\phi =$

$\sin(2i\pi+\phi)$, i being any integer number; so also in this calculus,

$$\operatorname{cotr}[\phi, \chi] = \operatorname{cotr}[\phi+2i\tau,\ \chi-2i\tau]$$
$$\operatorname{tres}[\phi, \chi] = \operatorname{tres}[\phi+2i\tau,\ \chi-2i\tau]$$

where $\tau = \frac{\pi}{\sqrt{3}}$. The quantity τ holding in the present theory the same place that π does in the calculus of sines. From the theorem of Moivre we learn that

$$(\cos\phi+\sqrt{-1}.\sin\phi)^m = \cos m(\phi+2i\pi)+\sqrt{-1}.\sin m(\phi+2i\pi).$$

The corresponding theorem already announced by Mr. Graves may be written as follows:

$$(\operatorname{cotr}[\phi,\chi]+\iota\operatorname{tres}[\phi,\chi]+\iota^2\operatorname{tres}[\chi,\phi])^m$$
$$=\operatorname{cotr}[m(\phi+2i\tau), m(\chi-2i\tau)]+\iota\operatorname{tres}[m(\phi+2i\tau), m(\chi-2i\tau)]$$
$$+\iota^2\operatorname{tres}[m(\chi-2i\tau),\ m(\phi+2i\tau)].$$

Among the most important consequences flowing from the preceding theorem is a mode of resolving an equation of the form

$$z^{3n}-pz^{2n}+qz^n-r=0$$

into its cubic factors. For this purpose we must first reduce the equation to the form

$$y^{3n}-3\operatorname{cotr}(\mathrm{A},\mathrm{B})y^{2n}+3\operatorname{cotr}(-\mathrm{A},-\mathrm{B})y^n-1=0. \qquad (b)$$

Now it will be found that if we multiply together the three expressions

$$x-\operatorname{cotr}[\phi,\chi]-\operatorname{tres}[\phi,\chi]-\operatorname{tres}[\chi,\phi]$$
$$x-\operatorname{cotr}[\phi,\chi]-a\operatorname{tres}[\phi,\chi]-a^2\operatorname{tres}[\chi,\phi]$$
$$x-\operatorname{cotr}[\phi,\chi]-a^2\operatorname{tres}[\phi,\chi]-a\operatorname{tres}[\chi,\phi]$$

in which a stands for $\frac{-1\pm\sqrt{-3}}{2}$, the cube root of $+1$, the product will be

$$x^3-3\operatorname{cotr}[\phi,\chi]x^2+3\operatorname{cotr}[-\phi,\ -\chi]x-1.$$

Hence it may be proved that each of the cubic factors of (b) will be of the form

$$y^3-3\,\mathrm{cotr}\left[\frac{1}{n}(\mathrm{A}+2i\tau),\ \frac{1}{n}(\mathrm{B}-2i\tau)\right]y^2$$
$$+3\,\mathrm{cotr}\left[-\frac{1}{n}(\mathrm{A}+2i\tau),\ -\frac{1}{n}(\mathrm{B}-2i\tau)\right]y-1.$$

From the result just obtained Mr. Graves deduces a singular geometrical theorem analogous to the celebrated one of Cotes.

But it must be observed here that the geometrical propositions to which we are led in interpreting formulæ involving tresines, do not, in general, relate to the *lengths* of lines. If x, y, z and x', y', z' be respectively the rectangular coordinates of two points in space, the distance between them is expressed by the quantity $\sqrt{(x'-x)^2+(y'-y)^2+(z'-z)^2}$. But this is not the function of x, y, z, x', y', z' that the formulæ in question commonly bring before us: they most frequently introduce, instead of it, the function

$$\sqrt[3]{(x'-x)^3+(y'-y)^3+(z'-z)^3-3(x'-x)(y'-y)(z'-z)}$$

which Mr. Graves proposes to call the *cubic modulus* of the line joining the points x, y, z and x', y', z'. This cubic modulus may be written in a form which suggests important consequences.

If we put

$$\begin{aligned} x &= m\,\mathrm{cotr}\,[\phi,\chi] & x' &= m'\,\mathrm{cotr}\,[\phi',\chi'] \\ y &= m\,\mathrm{tres}\,[\phi,\chi] & y' &= m'\,\mathrm{tres}\,[\phi',\chi'] \\ z &= m\,\mathrm{tres}\,[\chi,\phi] & z' &= m'\,\mathrm{tres}\,[\chi',\phi'] \end{aligned}$$

it becomes

$$\sqrt[3]{m'^3-3m'^2m\,\mathrm{cotr}[\phi-\phi',\chi-\chi']+3m'm^2\mathrm{cotr}[\phi'-\phi,\chi'-\chi]-m^3}$$

Thus we have the modulus of the line joining two points in space expressed by means of the differences of their corresponding amplitudes and the moduli of the right lines drawn to them from the origin:—a result analogous to the fundamental proposition in plane trigonometry, by which the length of one side of a triangle is found from the two remaining sides and the included angle. Having now got the notion of the modulus of a right line in space, we may easily advance to the conception of the modulus of a curved line: for we may regard it as the sum of the moduli of the elements of the curve: so that to find m, the modulus of the curve itself, we employ the formula

$$m = \int \sqrt[3]{dx^3 + dy^3 + dx^3 - 3\,dx\,dy\,dz}.$$

The right line drawn from the point x, y, z to another point, z', y', z', may be looked upon as having amplitudes as well as a modulus. In order to determine them we put

$$x' - x = m\,\mathrm{cotr}[\phi, \chi] : y' - y = m\,\mathrm{tres}[\phi, \chi] : z' - z = m\,\mathrm{tres}[\chi, \phi]:$$

m being the modulus of the right line.

For the purpose of illustrating his views, Mr. Graves has discussed the surface whose equation in rectangular coordinates is

$$x^3 + y^3 + z^3 - 3\,xyz = 1.$$

As might be expected, this surface possesses numerous properties which admit of being stated in such a manner as to exhibit a striking similarity to those of the circle. The three coordinates x, y, z belonging to any point on it, may be put equal to $\mathrm{cotr}[\phi, \chi]$, $\mathrm{tres}\,[\phi, \chi]$, and $\mathrm{tres}\,[\chi, \phi]$; and we may call ϕ and χ the amplitudes of the point. It will also be convenient to designate the point, whose amplitudes are $-\phi$ and $-\chi$, as *reciprocal* to the point whose amplitudes are ϕ and χ.

Amongst other theorems relating to this surface, which Mr. Graves proposes to name *the surface of tresines*, the following may be considered as deserving attention:

1. *The angle between the tangent planes at any two points on the surface is equal to that between the radii drawn to the two reciprocal points.*

2. *The measure of curvature at any point is equal to* $-r_1^{-4}$: r_1 *denoting the radius of the reciprocal point.*

3. *The surface is one of revolution: and it is its own polar reciprocal with reference to a sphere whose centre is at the origin of coordinates and whose radius is unity.*

The curve of double curvature, or *curve of tresines*, defined by the equations

$$x = \operatorname{cotr}[\phi, o]$$
$$y = \operatorname{tres}[\phi, o]$$
$$z = \operatorname{tres}[o, \phi]$$

appears likewise to be most fertile in properties analogous to those of the circle. The following were stated by Mr. Graves:

1. *The angle between the tangents at any two points on the curve is equal to that between the two corresponding radii.*

2. *The angle between the osculating planes at any two points on the curve is equal to the angle between the radii drawn to the two reciprocal points.*

3. *The angle of contact at a point on the curve is therefore equal to the torsion at the reciprocal point.*

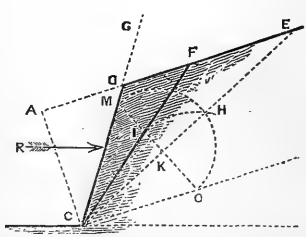
4. *The element of the curve described by the reciprocal point is double the elementary area described by the radius vector.*

5. *The polar reciprocal of the developable surface formed by the osculating planes is itself* the curve of tresines.

6. *The product of the radii of curvature at any point and at its reciprocal is equal to unity.*

7. *The radius vector traces a logarithmic spiral upon a plane parallel to the symmetric plane.*

Mr. John Neville read a paper on the maximum Amount of Resistance required to sustain Banks of Earth and other Materials.



Let CDE be any bank of earth, sand, or other material, and CE the position of the line of repose with respect to the horizon and bank; then if we put CA, the perpendicular from C on DE produced $= h$, the $\angle$ DEA $= b$; the angle ECA $= c$; and the $\angle$ ACF, which the fracture CF makes with CA, $= \phi$; then when the horizontal resistance R, necessary to support the wedge FCD, is a maximum,*

$$\tan\phi = \frac{\sec.c\sqrt{\tan b\tan c+1}-1}{\tan c} \tag{1}$$

and

$$\text{R} = \frac{h^2\,\text{w}}{2}\left(\frac{\sec c - \sqrt{\tan b\tan c+1}}{\tan c}\right)^2, \tag{2}$$

where W $=$ the weight of a cubical unit of the stuff in FDC. Equation (2) may be changed into the following,

$$\text{R} = \frac{h^2\text{w}}{2\tan c}(\sqrt{\tan c+\cot c} - \sqrt{\tan b+\cot c})^2 \tag{3}$$

a simple form to calculate the values of R from.

Also if CO is drawn parallel to DE, and we put

$$\theta = \angle\,\text{GDE} = \angle\,\text{DCO};\quad \delta = \angle\,\text{OCE} = \theta - (c-b),$$

and H $=$ CD the slant height of the face: then

$$\tan(c-\phi) = \tan\angle\,\text{FCE} = \sqrt{\tan^2\delta+\tan d\tan\delta} - \tan\delta \tag{4}$$

and

$$\text{R} = \frac{\text{H}^2\,\text{w}}{2}\times\frac{\tan\theta}{\tan\theta-\tan d}\times\tan^2(c-\phi), \tag{5}$$

where $d = c - b = \angle$ ECD.

When $b = 0$, CD coincides with CA, and we get from equation (1)

$$\tan\phi = \frac{\sec c - 1}{\tan c} = \tan\tfrac{1}{2}c,$$

which shews that the equation of Prony holds good, whatever the inclinations of CD and DE to the horizon may be, if these lines continue at right angles to each other. We also get from equation (2) or (5) in this case,

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* Generally we have R $=$ W $\times$ DCF $\times$ tan. FCE, for any fraction CF; a very simple expression for finding the value of the resistance when the position of the fracture is given.

$$R = \frac{WH^2}{2} \tan^2 \tfrac{1}{2} c, \qquad (6)$$

which shews the value of R retains the same form whatever the inclination of CD may be, if DE continues at right angles to it, as may also be seen, in the more general case, from equation (2) or (5).

When $\theta = c - b$, DE stands at the angle of repose, and when infinite, from equation (4), the fracture CF becomes parallel to it; and from equation (2) or (5) may be had, by reduction,

$$R = \frac{WH^2}{2} \times \sin^2 (c - b). \qquad (7)$$

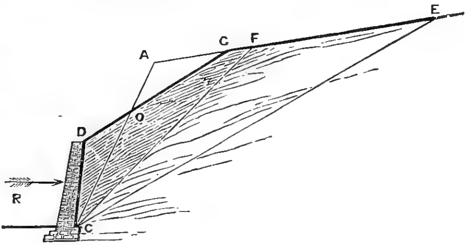
As in loose stuff DE can never stand at a steeper inclination than the angle of repose, equation (7) gives the greatest value R can ever attain; the height of the face H being constant. When $b = 0$, H vertical, and $c = 34°$, which corresponds to a slope of repose of $1\frac{1}{2}$ to 1 nearly, we get, by comparing equations (6) and (7),

R in (6) : R in (7) : : 283 : 687 : : 3 : 7, nearly.

In fluids the ratio will be as 1 to 1, and when the angle of repose is 90°, the ratio will be as 1 to 4; for $\tan^2 \frac{1}{2} c : \sin^2 c : : \frac{1}{2} c^2 : c^2 : : 1 : 4$: hence no instance can occur wherein, the face H being vertical, the value of R can exceed four times the value when the top DE is horizontal.

Equation (4) gives the following simple geometrical construction for finding the line CF. Draw any line MO, cutting the line of repose CE at right angles in K, and terminating in the face CD at M, and in the line CO parallel to DE, the top, in O. On MO describe a semicircle, cutting CE, the line of repose, in H; from O as centre, with OH as radius, describe an arc to cut OM in I; join C to I, and produce CI to F; then the wedge FDC will require the maximum resistance. When DE is at right angles to DC, the angle ECD is bisected by EF.

The equations apply also to finding the value of R and position of CF for "dwarf walls" CD, at the toes of cuttings or embankments; but when the fracture CF lies inside the top of the slope DG, the value of R and position of CF are found as follows:



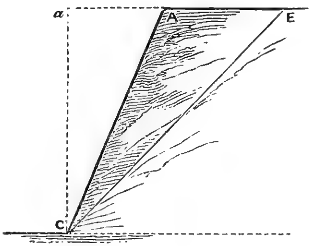
Produce EG to A, and draw CA so that ΔCOD = ΔGOA, and as, therefore, ΔFAC = FGDC, we have only to determine R, and draw CF as if the bank to be supported was CAE.

In the Encyclopædia Britannica, seventh edition, article "*Masonry*," by Tredgold, there are given the following equations:

$$\text{"(23)} \quad \tan a = \frac{-1 + (\tan c \tan^3 i + \tan^2 i + \tan c \tan i +)^{\frac{1}{2}}}{\tan i}\text{"}$$

$$\text{"(25)}\ R = \frac{h^2 s}{2 \tan i}\left[\tan i + \tan c + \frac{2}{\tan i} - 2\left(\tan c \tan i + \frac{\tan c + 1}{\tan i} + 1\right)^{\frac{1}{2}}\right]\text{"}$$

where a = $\angle$ ECa, which the plane of fracture CE makes with the vertical line Ca; c = $\angle$ ACa; i = the complement of the angle of repose; h = AC the vertical height, and S = the weight of a cubical foot of the material in ACE, AE being *horizontal:* and the following table of calculated values of R from equations (26) and (25):



		Angle of Repose.	Weight of S.	Value of R. when $c = 0°$	Value of R when $c = 10°$
1	Water, ...	0°	62·5lbs.	$R = 31\frac{1}{4} h^2$	$R = 31\frac{1}{4} h^2$
2	Fine dry sand, .	33°	92 —	$R = 13·8 h^2$	$R = 4·8 h^2$
3	Do. moist .	——	119 —	$R = 17·85 h^2$	$R = 6·2 h^2$
4	Quartz sand, dry,	35°	102 —	$R = 13·77 h^2$	$R = 4·6 h^2$

Now, using the same data, and calculating from equations (6) and (2) here given, the values will be as follow:

		Angle of Repose.	Weight of W.	Value of R. when $b = 0$, and C D vertical: equation (6).	Value of R when $b = 10°$ and D E horizontal: equation (2).
1	Water, ...	0°	62 5lbs.	$R = 31\frac{1}{4} h^2$	$R = 31\frac{1}{4} h^2$
2	Fine sand, dry, .	33°	92 —	$R = 13·5 h^2$	$R = 9·7 h^2$
3	Do. moist, .	——	119 —	$R = 17·5 h^2$	$R = 12·6 h^2$
4	Quartz sand, dry,	35°	102 —	$R = 13·8 h^2$	$R = 9·7 h^2$

The values in the last column have been calculated from equations (2) and (5) respectively, which gave the same results when the latter is multiplied by $\sec^2 b$, as $h^2 \times \sec^2 b = H^2$: they are fully double those given by Tredgold, and made use of by him in calculating the dimensions of retaining walls given in another Table. The differences in the Tables, when $b = 0$, are immaterial. Equation (25) is, however, incorrect, as will be seen by squaring the second part of equation (3) here given, actually, when we get

$$R = \frac{h^2 W}{2\tan c}\left(\tan c + \tan b + \frac{2}{\tan c} - 2\sqrt{\tan c \tan b + 1 + \frac{\tan c \tan b + 1}{\tan^2 c}}\right)$$

Whence it appears that the term $\frac{\tan c + 1}{\tan i}$, in Tredgold's equation (25) under the square root, should be $\frac{\tan c \tan i + 1}{\tan^2 i}$; this being the case, his equation is then reducible to equation (3). Equation (23) can also be reduced to the form of equation (1), which is general, whether DE is horizontal or inclined.

Mr. Thomas Bergin read the following communication from Dr. Thomas Woods of Parsonstown, on a new photographic process, which he calls the Catalysotype. Mr. Bergin also read two letters from H. Fox Talbot, Esq. (M. P.) to Dr. Woods, and his answer.

While investigating the property that sugar possesses, in some instances, of preventing precipitation, I noticed that when syrup of ioduret of iron was mixed in certain proportions with solution of nitrate of silver, the precipitate was very quickly blackened when exposed to the light, and I thought that, if properly used, it might be employed with advantage as a photographic agent. If not entirely without profit, it would hardly repay the trouble of reading the history of all the experiments I tried in order to prove whether or not this idea were correct, for there were many difficulties to be overcome, and unexpected hindrances to be surmounted, before I could be certain of success. However, the results at which I have arrived make me hope that my trouble has not been thrown away, and that a photographic process has been discovered, which is more manageable, and more satisfactory, than any which has before been used; and I think that the pictures produced by it are more minutely and delicately brought out, and the time for their production at least not longer than is required by any other method.

To enter very minutely into the particulars, or to explain the *rationale* of the process would be too tedious; however, it is so simple, that those who will feel any pleasure in trying it will, I am sure, easily succeed, and to attempt any explanation of its theory would, in the present state of our knowledge of light, be advancing a mere hypothesis; I will, therefore, only state generally the method in which the paper is prepared, and then briefly giving my reasons for such parts of the process as are not at first sight obvious, will thereby enable the experimenter to be guarded against the failures that these precautions are intended to overcome.

Let well-glazed paper (I prefer that called wove post) be steeped in water to which hydrochloric acid has been added in the proportion of two drops to three ounces. When well wet, let it be washed over with a mixture of syrup of ioduret of iron half a drachm, water two drachms and a half, tincture of iodine one drop. When this has remained on the paper for a few minutes, so as to be imbibed, dry it lightly with bibulous paper, and being removed to a dark room, let it be washed over evenly, by means of a camel-hair pencil, with a solution of nitrate of silver, ten grains to the ounce of distilled water. The paper is now ready for the camera. The sooner it is used the better; as when the ingredients are not rightly mixed, it is liable to spoil by keeping. The time I generally allow the paper to be exposed in the camera varies from two to thirty seconds; in clear weather, without sunshine, the medium is about fifteen seconds. With a bright light, the picture obtained is of a rich brown colour; with a faint light, or a bright light for a very short time continued, it is black. For portraits out of doors, in the shade on a clear day, the time for sitting is from ten to fifteen seconds. If the light is strong, and the view to be taken extensive, the operator should be cautious not to leave the paper exposed for a longer period than five or six seconds, as the picture will appear confused from all the parts being equally acted on. In all cases, the shorter the time in which the picture is taken the better.

When the paper is removed from the camera no picture is visible. However, when left in the dark, without any other preparation being used, for a period which varies with the length of time it was exposed and the strength of the light, a negative picture becomes gradually developed, until it arrives at a state of perfection which is not attained, I think, by photography produced by any other process.* It would seem

* The picture, when developed, is not readily injured by exposure to moderate light; it ought, however, to be fixed, which may be done by washing it with a solution of bromide of potassium, fifteen or twenty grains to the ounce, or iodide of potassium, five grains to the ounce. It may either be applied with

as if the salt of silver, being slightly affected by the light, though not in a degree to produce any visible effect on it if alone, sets up a catalytic action, which is extended to the salts of iron, and which continues after the stimulus of the light is withdrawn. The catalysis which then takes place has induced me to name this process, for want of a better word, the Catalysotype. Sir J. Herschell and Mr. Fox Talbot have remarked the same fact with regard to other salts of iron, but I do not know of any process being employed for photographic purposes which depends on this action for its development, except my own.

My reason for using the muriatic acid solution, previous to washing with the ioduret of iron, is this: I was for a long time tormented by seeing the pictures spoiled by yellow patches, and could not remedy it, until I observed that they presented an appearance as if that portion of the nitrate of silver which was not decomposed by the ioduret of iron had flowed away from the part. I then recollected that Sir J. Herschell and Mr. Hunt had proved that iodide of silver is not very sensitive to light, unless some free nitrate be present. I accordingly tried to keep both together on the paper, and after many plans had failed, I succeeded by steeping it in the acid solution, which makes it freely and evenly imbibe whatever fluid is presented to it. I am sure that its utility is not confined to this effect, but it was for that purpose that I first employed it. My reason for adding the tincture of iodine to the syrup is, that having in my first experiments made use of, with success, a syrup that had been for some time prepared, and afterwards remarking that fresh syrup did not answer so well, I examined both, and found in the former a little free iodine; I therefore added a little tincture of iodine

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a camel-hair pencil, or by immersion. The picture must then be well washed in water, to remove the fixing material, which would cause it to fade by exposure to light.

with much benefit, and now always use it, in quantities proportioned to the age of the syrup.

The following hints will, I think, enable any experimentor to be successful in producing good pictures by this process. In the first place, the paper used should be that called "wove post," or well-glazed letter-paper. When the solutions are applied to it, it should not immediately imbibe them thoroughly, as would happen with the thinner sorts of paper. If the acid solution is too strong, it produces the very effect it was originally intended to overcome; that is, it produces yellow patches, and the picture itself is a light brick-colour, on a yellow ground. When the tincture of iodine is in excess, partly the same results occur; so that if this effect is visible, it shews that the oxide of silver which is thrown down is partly re-dissolved by the excess of acid and iodine, and their quantities should be diminished. On the contrary, if the silver solution is too strong, the oxide is deposited in the dark, or by an exceedingly weak light, and in this case blackens the yellow parts of the picture, which destroys it. When this effect of blacking all over takes place, the silver solution should be weakened. If it be too weak, the paper remains yellow after exposure to light. If the ioduret of iron be used in too great quantity, the picture is dotted over with black spots, which afterwards change to white. If an excess of nitrate of silver be used, and a photograph immediately taken before the deposition of the oxyde takes place, there will be often, after some time, a positive picture formed on the back of the negative one. The excess of the nitrate of silver makes the paper blacker where the light did not act on it, and this penetrates the paper, whereas the darkening produced by the light is confined to the surface. The maximum intensity of the spectrum on the paper, when a prism of crown glass is used, lies between the indigo and blue ray. The difference of effect of a strong and weak light is beautifully shewn in the action of the spectrum: that part of the paper which is exposed to the indigo ray is coloured a reddish brown, and this is gradually darkened towards either extremity until it becomes a deep black.

I have not had many opportunities of experimenting with the Catalysotype, but it certainly promises to repay the trouble of further investigation. The simplicity of the process, and the sensibility of the paper, will, no doubt, make it be extensively used. It has all the beauty and quickness of the Calotype without a tenth of its trouble, and very little of its uncertainty; and, if the more frequent use of it by me, as compared with other processes, does not make me exaggerate its facility of operation, I think it is likely to be practised successfully by the most ordinary experimenters.

SUPPLEMENT TO THE PRECEDING PAPER.

P. S.—Since the preceding Paper was written I have been experimenting with the Catalysotype, and one day having had many failures, which was before quite unusual with me, I am induced to mention the cause of them, for the benefit of subsequent experimenters. The paper I used was very stiff, and highly glazed, so that the solution first applied was not easily imbibed. The blotting-paper was very dry and bibulous. When using the latter, I removed nearly all the solution of iron from the first, and, of course, did not obtain the desired result.

While varying the process in endeavouring to find out the cause just mentioned, I discovered that the following proportions gave very fine negative pictures, from which good positive ones were obtained:—take of syrup of ioduret of iron, distilled water, each two drachms; tincture of iodine, ten to twelve drops: mix. First brush this over the paper, and after a few minutes, having dried it with blotting-paper, wash it over in the dark (before exposure in the camera) with the following solution, by means of a camel-hair pencil:—take of nitrate of silver one drachm; pure water one ounce: mix. This gives a darker picture than the original preparation, and, consequently, one better adapted for obtaining positive ones;

it also requires no previous steeping in an acid solution. To fix the picture, let it be washed, first in water, then allowed to remain for a few minutes in a solution of hydriodate of potassa (five grains to the ounce of water), and washed in water again. The paper I use is the common unglazed copy paper, but such as has a good body. I have tried the same paper with the original preparation, and find it to answer exceedingly well; it does not require in this case, either, an acid solution. The same precautions and hints apply to the amended as to the original process: such as when it blackens in the dark, there is too much caustic used; when it remains yellow, or that it is studded with yellow spots, too much iodine; when marked with black spots, too much iron. It is necessary to mention these, on account of the varying strength of the materials employed.

The following is the correspondence laid before the Academy on the part of Dr. Woods:

"LACOCK ABBEY, CHIPPENHAM,
"11*th March*, 1845.

"SIR,—Excuse my addressing you on the subject of a Paper which you sent to the British Association at York, last September, containing the description of a photographic process.

"Some years ago I described a process for obtaining camera pictures without using any *second wash*. It was described nearly as follows in the specification of my English patent: Take *iodised paper*, wash it with gallic acid, dry it, and keep it in store for subsequent use. This is called *io-gallic paper* from its constituents. When wanted, take a sheet of io-gallic paper, wash it with nitrate of silver, and put it in the camera. The image obtained is generally, at first, invisible, but it rapidly *developes itself* when removed from the camera, requiring no further care, except ultimately to fix it. Instead of gallic acid, sulphate of iron answers the same purpose perfectly. The same effect is very often, but not always, produced in the ordinary Calotype process, which I described in 1841; indeed I discovered it in that way.

"The process which you have called Electrolysotype appears to me to be strictly analogous to the above. If I comprehend your description, you use an iodised paper in which iodide of iron is employed instead of iodide of potassium.

"You may be quite right in attributing the effects to Electrolysis, but then it follows that my Calotype process, with all its variations, must result from the same cause.

"I am, Sir,
"Your obedient Servant,
"H. Fox Talbot.

"*Dr. Woods.*"

"Lacock Abbey,
"18*th March*, 1845.

"Sir,—I have to acknowledge the receipt of your courteous letter of the 15th instant, upon which I beg leave to make a few observations. In my Calotype process, iodide of silver is decomposed by the joint influence of light and a deoxydising agent (gallic acid). Mr. Hunt has shewn that sulphate of iron may be substituted for gallic acid, and he calls the process so altered Energiatype. But since *tannin* and other substances may also be substituted for gallic acid, each of these variations in the process would require, on the same principle, to have a *separate name*, which would, surely, be inconvenient. In your method, iodide of silver is decomposed by the joint action of light and iron; the three reacting substances being *the same* as in Mr. Hunt's Energiatype; and therefore, imperfect as the theories of photography confessedly are, I cannot persuade myself that a *catalytic* action can take place in your process, *unless* it also takes place in the Energiatype and in my original Calotype process: I therefore cannot help considering these three processes as variations of the same, and not essentially different. I hope, however, you will not consider me as detracting in the least from your valuable labours: my remarks only refer to the *nomenclature* of the science.

If I am not mistaken, the three methods I have named produce pretty nearly identical *results*, though I speak from experience of only two of them, Mr. Hunt's and my own. Both of these are nearly certain in operation, very rapid, giving a camera picture of a bright object in a second of time, and requiring no second wash if enough of the deoxydising agent is employed in the first wash. It is customary to make the *positive* copies on a different paper, which I have called photogenic drawing-paper, consequently, the *final results* of the two processes cannot anyhow be distinguished.

I thank you for your courtesy in mentioning that you are about to send a Paper on the subject to the Royal Irish Academy by the hands of Dr. Robinson. May I request that this letter and my former one, with the permission of the Academy, may be read to them on the same occasion, if Dr. Robinson will kindly take charge of them. It may be left to their scientific judgment to say whether a new principle is involved or not in your experiments. If any new principle be involved, then a distinctive name, such as you have given, is, of course, desirable,—otherwise it would not be so. I would refer also to the instance of the Daguerreotype, now so differently managed from what it used to be at the time of its first promulgation. It is now at least a hundred times more rapid in its effects, but it still continues to be called the Daguerreotype. On the other hand, I believe it is not affirmed that any process on paper has been discovered more rapid or more certain than the Calotype; I am not aware of any such having been as yet described. We should certainly be very grateful to any one who discovered a more rapid process, depending on new combinations; but if I do not err in defining the Calotype process as depending on a combination of iodine, silver, and a deoxydising agent, your process would be included in that definition, unless good reasons to the contrary could be shewn, all which I willingly leave to the judgment of the scientific world: and, thanking you for your polite attention in so soon answering my last letter,

" I remain, Sir,

" Your's very truly,

" H. Fox Talbot.

" P. S.—If your process does anything which the Calotype cannot do, or does it *better*, I willingly admit its importance; but I apprehend that you are not aware of the facility and rapidity with which our Calotype operations are now conducted. Indeed, that was my chief reason for troubling you with a letter, as your Paper read at the York meeting mentioned the spontaneous development of photogenic images as something new, whereas it is a phenomenon of very frequent occurrence in the Calotype, and *always* occurs when we use the io-gallic paper."

"PARSONSTOWN,
"*March* 25, 1845.

"SIR,—I am sorry I did not receive your letter before Dr. Robinson left this town, which I should have done, it being dated March 18: I did not get it until March 24. I will, however, forward it to him, and I am sure he will, with his usual kindness, make whatever use of it he thinks right. I will ask him to have it laid before the Academy with my Paper. I agree with the observations you make on my Paper on *general principles*. There is no doubt at all that your Calotype, Mr. Hunt's Energiatype, and my Catalysotype, if I may be allowed, for the present, to call it so, are all pretty similar in their modes of action, and perhaps they all come to the same point in the end,—the decomposition of the salt of silver; but, as I said in my former letter, if new modes of producing this effect were not to be named, why call your process Calotype—why call Mr. Hunt's Energiatype, &c., as they all agree in their general results with the first experiments made with light on the nitrate of silver? Why not regard them all merely as *instances* of the same general principle, and not isolate them, as it were, by designating each by a particular name? You will say now that I agree with your ideas: I always did. I think that cumbering science with a multiplicity of hard names for every particular fact is very bad; but the christening of my process has been forced on me by a similar line of conduct in others; and when a nomenclature sufficiently good appears (a task which I wish you would undertake), I will be the first to blot out the word Catalysotype.

"I do not pretend to any discovery; nor do I think my process, in its chemical character, distinct from the general mass of facts in active chemistry. I merely regard it as a new combination, acting with great facility, very little complication, and, though not involving a new principle, being developed without requiring any second wash, which I looked on as characteristic until you mentioned the io-gallic paper, a process of which I was certainly ignorant before. You mention also Mr. Hunt's Energiatype as being similar to mine; but, as first published, it undoubtedly could have no claim to the advantages mine possesses, either in facility of execution or rapidity of result. I think he says it requires six or eight minutes to accomplish what mine does in two or three seconds. After my process was published by being read at the meeting of the Association at York, the sulphate of iron was applied to iodised paper, but not before. That proceeding has increased its sensibility, and made it approach in sensibility to mine; but it obviously does not interfere with my right to consider the Catalysotype my own child, and to call it what I please. However, I think all experimentors with light owe you a great debt, and should pay particular attention and respect to your opinion on a subject for which you have done so much; I will, therefore, not insist on adhering to the word Catalysotype, but leave the process to be dealt with as a fact in the general history of active chemistry. For the present the name must be borne with, as my Paper is written and given to Dr. Robinson; but if it ever should be again spoken of, which is perhaps not probable, we will not elevate it to the honour of a distinct prefix.

"I am, &c.

"THOMAS WOODS."

May 26, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

A sealed packet was opened, with the consent of Mr. R. Mallet, by order of the Academy, which he had deposited with the Academy at the meeting of the 13th of November, 1843. Mr. Mallet then stated the subject matter of the enclosed document to be certain propositions regarding improved methods of working atmospheric railways, and that his object in calling for the production of the packet upon this evening was to claim his priority of invention, as similar plans have been since proposed by French and English engineers.

A letter from Mr. Clibborn was read by the Secretary, relating to the discovery of certain gold antiquities near Naas.

"26*th May*, 1845.

"Sir,—It may be well to have a notice inserted in the minutes of the meeting of this evening, of the discovery of three gold antiquities of considerable value, which were found recently in the neighbourhood of Naas.

"The most interesting of these articles is a gold torquis, which I have purchased for the Marquis of Kildare's collection. It differs in form from the two in the Academy collection, which were found at Tara; and it is smaller than the largest of these, and larger than the smaller; its ends are also larger than those of the larger torquis. In form, the ends resemble those of the small torquis which the Academy purchased from the late Major Sirr. This torquis weighs 18 oz. 4 dwts. and 6 grs. The principle of its construction is quite manifest, for the four gold bands of which it is made are not perfectly connected together in several places: thus it exhibits a difference in its construction from the gold articles from Africa, which resemble those twisted gold ornaments found in Ireland.

"The other two articles were gold rings, or bent round bars of gold, one large enough to go round the *neck* of a man, and the other to go round his *wrist*. The larger weighs 31 oz. 14 dwts. and 16 grs.; and the smaller one weighs 7 oz. 5 dwts. and 19 grs., being a quarter of the weight of the larger. As the weights were ascertained with great care, they may be depended on.

"The weight of the torquis has not been yet verified. It is, for the present, deposited for inspection in the case with the gold ornaments belonging to the Academy.

"Your obedient Servant,

"Edward Clibborn,

"*Assist. Librarian, R. I. A.*"

The Rev. Dr. Todd read a paper on the ancient Wax Tablets which he had presented to the Academy on the 14th of April last, on behalf of the Rev. J. Spencer Knox.

From the words still legible on the tablets it is evident that they had belonged to some schoolmaster, who had employed them in the instruction of his pupils, or to some scholar, who had inscribed upon them his exercises in grammar and dialectics: and from the words "hoc ꞇꞃıuıum," which occur on one of the pages, it would seem that the owner was engaged in learning or teaching the *trivium*, i. e. the first three of the seven liberal arts, in which the first Degree is still taken in our Universities.

The characters inscribed on these tables, as far as Dr. Todd was able to determine, were of the fifteenth century, if not earlier. He shewed that the use of waxed tablets continued to the seventeenth century, and that there was no foundation for the opinion maintained by a learned French Dominican, Père Alexandre, that the use of tablets of this kind ceased in the fifth century.

Dr. Todd concluded by proposing that the special thanks of the Academy be presented to Mr. Knox for this valuable donation to the Museum; and recommended that it be referred to the Council to have drawings of the tablets immediately made, lest they should receive injury from the ordinary heat of the room, or otherwise.

Professor Allman made some observations on the wood composing the tablets, which he submitted to microscopical examination.

The Rev. Humphrey Lloyd read the following paper by the Rev. Thomas Knox, on the quantity of Rain which fell with different winds, at Toomavara, during five years since 1827.

I beg leave to lay before the Royal Irish Academy the following results of the rain-gauge kept at River Glebe, Toomavara, for five consecutive years.

The amount of rain is given separately for the eight principal points of the wind; and the curves in the accompanying plates are formed (as mentioned in a former communication) by taking on each of the eight points, distances from the centre respectively proportional to the amount of rain which had fallen when the wind was in that direction; by then connecting these spaces a curve is formed, which shews at a glance the character of the rain for that particular period. The plates are only drawn for the mean results, which are the only ones of any importance. The period of five years is rather shorter for very accurate mean results than could have been wished, but absence from home after that period had elapsed put a stop to the observations.

There are one or two points to which I wish to draw attention. First of all, taking the average monthly rain at three inches, the first six months of the year are below the average, the other six months above it. *November* and *July* are by far the two wettest months in the year; and in each the greatest amount is from S. W. *April* is much the driest month, and there is nearly as much rain in it from the northern portion of the compass as from the southern.

With regard to the gross amount which fell from each point in the entire year, that which fell from S. S. W. and W. is much above the average. From the other points it is below it.

There is a curious circumstance with regard to the curve of the entire year (Plate 6): that if it be divided by a line running N. E. and S. W., then the rain on either side of this line is equal all but a fraction of an inch. This is the more remarkable, as these two points had been fixed on by Professor Dove, in his Paper on the Winds, as being the points of greatest and least barometric pressure. That is to say, the wind supposed at S. W., any shift of it, either towards S. or W. produces a rise of mercury; and also any shift on either side of N. E., a corresponding fall.

Now in the rain the greatest amount is from S. W. (corresponding with the lowest state of mercury). The least is from N. E. (where the mercury is highest), and on either side of this line it varies regularly, as an inspection of the Plate

(No. 6) will shew. For instance the amounts from W. and S. are nearly equal, and both less than S. W.; N. E. and S. still less; N. and E. still decreasing; and N. E. the least of all. Whether this analogy between the barometric pressure and amount of rain is accidental or not remains to be proved.

The following table shews the numerical average of the rain. The detailed tables and curves will be published in the Transactions of the Academy.

		S.	S. W.	W.	N. W.	N.	N. E.	E.	S. E	Total.
YEARLY AVERAGE EACH POINT.		6.548	10.639	6.034	2.789	2.352	2.172	2·251	3.173	35.958
Average of each Month for each of the 8 Points during the five Years.	January.	.573	.721	.609	.171	.162	.096	.180	.306	2.918
	February	.655	.668	.397	.235	.136	.167	.175	.332	2.766
	March .	.635	.814	.679	.277.	.130	.064	.122	.237	2.957
	April .	.175	.375	·404	.192	.200	.153	.088	.118	1.704
	May . .	.359	.595	.280	.110	.184	.289	.251	.290	2.359
	June . .	.613	1.236	.285	.074	.058	.181	.062	.189	2.699
	July . .	.517	1.148	.568	.448	.398	.281	.242	.110	3.712
	August .	.588	1.440	.408	.262	.242	.187	.147	.152	3.427
	Septem.	.711	1.020	.597	.229	.237	.157	.058	.304	3.313
	October.	.387	.797	.651	.269	.412	.206	.117	.195	3.033
	November	.629	.856	.556	.283	.165	.234	.580	.550	3.844
	December	.706	.969	.500	.139	.038	.157	.229	.390	3.228
Monthly Mean during each Season for 5 ys.	Winter .	.645	·786	.535	.215	.112	.140	.195	.343	2.911
	Spring .	.390	·595	.454	.193	.171	.169	.154	.215	2.341
	Summer	.573	1·275	.420	.261	.233	.216	.150	.150	3.278
	Autumn	.576	·891	.601	.260	.268	.199	.352	.350	3.397
Average for five years, total of	Winter .									8.912
	Spring .									7.020
	Summer									9.836
	Autumn									10.190
Average Year										35.958

There is one particular in which this separating the gross amount of rain into the eight portions, as brought by different winds, may be useful, viz., to ascertain the respective specific gravities, and the amount of saline matter brought from each direction; this may be useful in regard to agricultural matters. For instance, we could easily suppose a case of two portions of land, not many miles asunder, but on different sides of a high range of hills, getting very different amounts of salt from one being exposed to, and the other sheltered from that wind in which the greatest amount was found; but by this mode of collecting the rain, an accurate mode of estimating this is within our reach.

To this branch, namely, an examination of solid and gaseous matter brought in the rain from each direction, I hope, on a future occasion, to find time to turn my attention to.

Rev. H. Lloyd read an extract from a letter from Edward W. Chetwode, Esq., describing a remarkable lunar halo and paraselene, seen in the Queen's County, on the night of the 21st of May.

"I send a rough sketch of what struck me last night as a most beautiful and uncommon appearance, seen from our hall-door at twelve o'clock: the moon, with cruciform rays, surrounded by a halo; two bright spots *in directum* with the horizontal arm of the cross, on the periphery of the halo; a crescent light, not quite so intense as the horizontal spots, also on the periphery, *in directum* with the perpendicular axis of the cross; and at a considerable distance above it (perhaps the distance of halo-radius) another much larger crescent, looking as if it were the base of another halo circle. The sky had a good many of those electric sweepings of light through it at the time. No doubt there was a fourth bright spot on the halo, but it was hid by a dense mass of trees. The two horizontal spots, which were very bright, had decided rainbow colours, strongly marked."

The second figure in the lithographic plate at the end of the part represents the phenomenon described by Mr. Chetwode.

The phenomenon was likewise seen in the neighbourhood of Dublin, although not in so developed a form. The following are the notes of its appearance, as observed at Sandy-Cove, by Digby Starkey, Esq.

"Ten minutes after eleven, P.M. Wind N.W. Mist across the sky to the North, East, and South, in striæ, as represented above. The line passing through the moon, and the eastern and western mock moons, dipped a little to the

South West. The illuminated line from the western mock moon was not traceable to any distance, nor did a mock moon appear in the North, opposite to the moon. The red rays were next the moon. The phenomenon was faint and ill-defined; the eastern and upper moons scarcely discernible. It disappeared in about half an hour after it was first observed.

" No observation being taken at these hours at the Magnetic Observatory, there is no record of the phenomenon as observed there. But the observer at ten, P. M. has noted as follows:—Sky all covered with very fine long cirrostrati, extending across the entire sky from North to South. Haze visible about the moon. The barometer, at this hour, stood at 29.978; the thermometer at 48.4; wet thermometer, 47.1.

Professor Allman made the following observations on the same phenomenon.

Observed between the hours of 10 and 11 o'clock, P. M., on the 21st May, 1845, a thin haze spreading over the sky. The arm of the cross which approached more nearly to the horizontal was slightly inclined to the horizon, at an angle which, as far as the unassisted eye could determine, might have been about 15°. The other arm was perpendicular to this. The diameter of the circle included within the halo was equal to about forty diameters of the moon. At the points where the cross, if produced, would have intersected the halo, were paraselene. The two lateral paraselene were more constant in their appearance than the superior, which was visible only two or three times during the observation. The lower part of the halo was not visible. The moon itself was immediately surrounded by an ordinary blur-like halo, which was more intensely luminous than the cross, and within the margins of which it was wholly confined. The extremities of the paraselene, which were placed upon the halo, were obscurely prismatic; but I could determine nothing satisfactory as to the order of the colours. The arms of the cross were intermittent in visibility, that which was opposite to the left hand of the spectator being the most so.

After 11 o'clock the appearance became more and more obscure, until the cross was replaced by an obscure blur. The paraselenæ also disappeared, while the halo, though diminished in brightness, continued some time longer.

June 9, 1845.

CAPTAIN LARCOM, R. E., Vice-President, in the Chair.

James Claridge, Esq., Adolphus Cooke, Esq., Windham Goold, Esq., Charles Croker King, M. D., and Charles Wye Williams, Esq., were elected Members of the Academy.

Mr. Richard Sharpe read a notice of a new electric clock, on the principle of Mr. Wheatstone's telegraphic instruments.

The Rev. Charles Graves made a further communication relative to Algebraic Triplets, and their Geometric Interpretation.

Besides the system of algebraic triplets developed in former communications to the Academy, Mr. Graves has conceived another, which appears to admit of an interpretation in some respects more closely analogous to Mr. Warren's geometrical representation of imaginary quantities.

As the symbol $\sqrt{-1}$ may be taken to indicate a rotation in the plane of x from the axis of x to the axis of y, it seems natural to conceive another symbol representing a rotation in the plane of xz, from the axis of x to the axis of z. The repetition of either of these operations would reverse the direction of a right line originally placed on the axis of x: and therefore they are both equally fitted to serve as geometric representations of the square root of negative unity. But what is more, there is an infinite number of geometric operations of which this is equally true. For instance, rotation through two right angles in any plane passing through the axis of x would reverse the direction of a line placed upon that axis.

Let us take then two symbols, i and j, denoting *distinct* distributive operations, such that

$$i^2(1) = j^2(1) = -1 : ij(1) = ji(1),$$

and form with them and the three real magnitudes x, y, z the expression

$$x + iy + jz + ij\frac{yz}{x}.$$

As it depends upon three quantities, it may be looked upon as a *triplet;* whilst it is, in some sense, a *quadruplet,* being made up of units of four different kinds: for there is reason to regard $ij(1)$ as an imaginary unit, differing both from $i(1)$ and $j(1)$.

Before we proceed to consider the results arrived at in the multiplication of such triplets, it will be convenient to change their form. For this purpose let us put

$$x = m\cos\phi\cos'\chi, \quad y = m\sin\phi\cos\chi, \quad z = m\cos\phi\sin\chi;$$

the expression $x+iy+jz+ij\dfrac{yz}{x}$ will thus be transformed into $m(\cos\phi\cos\chi + i\sin\phi\cos\chi + j\cos\phi\sin\chi + ij\sin\phi\sin\chi)$, which is evidently equivalent to $me^{i\phi+j\chi}$.

If then we call m the modulus, and ϕ and χ the amplitudes of the triplet, it will appear that *the modulus of the product of two triplets will be equal to the product of the moduli of the factors: and each amplitude of the product will be equal to the sum of the corresponding amplitudes in the factors.*

The modulus and amplitudes of the triplet (x, y, z) are derived from its constituents by the equations

$$m^2 = x^2 + y^2 + z^2 + \frac{y^2 z^2}{x^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) \;:\; \chi = \tan^{-1}\left(\frac{z}{x}\right).$$

Hence if x, y, z be the rectangular coordinates of a point in space, the modulus of the right line drawn to it from the origin is a fourth proportional to the projections of this line upon the axis of x and the planes of xy and xz. And the amplitudes are the angles between the axis of x and these two last projections.

The construction thus obtained for the product of two right lines obviously coincides with Mr. Warren's in the case where $z = 0$.

The nullity of the triplet $x + iy + jz + ij\,\frac{yz}{x}$ involves the three equations, $x = 0$, $y = 0$, $z = 0$.

But, however well these triplets fulfil the requisitions of multiplication, we find they will not stand the test of addition. The sum of two such triplets is not necessarily a triplet; nor can we add two of them together, unless they happen to have a common amplitude.

What has been here said may readily be extended, for we might develope the expression $e^{i\phi + j\chi + k\psi + \&c.}$, in which i, j, k, &c. represent $(n-1)$ distinct square roots of negative unity, into a series of terms, such as

$$\begin{aligned}&\text{A} + i\text{B} + j\text{C} + k\text{D} + \ldots\\ &+ jk\,\frac{\text{CD}}{\text{A}} + ki\,\frac{\text{BD}}{\text{A}} + ij\,\frac{\text{BC}}{\text{A}} + \ldots\\ &+ ijk\,\frac{\text{BCD}}{\text{A}^2} + \ldots\end{aligned}$$

and, conversely, we may reduce a multiplet

$$\begin{aligned}&a + ib + jc + kd + \ldots\\ &+ jk\,\frac{cd}{a} + ki\,\frac{bd}{a} + ij\,\frac{bc}{a} + \ldots\\ &+ ijk\,\frac{bcd}{a^2} + \ldots\end{aligned}$$

depending upon n constituents a, b, c, d, &c., to the form

$$me^{i\phi+j\chi+k\psi+\cdots},$$

The modulus m, and the amplitudes ϕ, χ, ψ, ... of the multiplet, will be found by the equations

$$m^2 = a^2 + b^2 + c^2 + d^2 + \frac{c^2d^2}{a^2} + \frac{b^2d^2}{a^2} + \frac{b^2c^2}{a^2} + \frac{b^2c^2d^2}{a^4} + \cdots$$

$$\phi = \tan^{-1}\left(\frac{b}{a}\right), \quad \chi = \tan^{-1}\left(\frac{c}{a}\right), \quad \psi = \tan^{-1}\left(\frac{d}{a}\right) \ldots\ldots\ldots$$

The nullity of the multiplet involves the n equations

$$a = 0, \quad b = 0, \quad c = 0, \quad d = 0, \text{ \&c.}$$

What has been already proved in the case of distinct square roots of negative unity, may be applied, *mutatis mutandis*, to multiplets of the form $me^{i\phi+j\chi+k\psi+\cdots}$, in which i, j, k, &c. are used to denote wholly distinct geometric, or purely imaginary n^{th} roots of positive or negative unity.

Mr. Graves noticed that triple integrals such as $\iiint V\,dxdydz$ may sometimes be advantageously transformed, by putting $x = m\cos\phi\cos\chi$, $y = m\sin\phi\cos\chi$, and $z = m\cos\phi\sin\chi$: the element $dxdydz$ will then be replaced by $m^2\cos\phi\cos\chi\,dmd\phi\,d\chi$.

On the other hand, if we put

$$x = m\operatorname{cotr}[\phi,\chi], \quad y = m\operatorname{tres}[\phi,\chi], \text{ and } z = m\operatorname{tres}[\chi,\phi],$$

we should transform the same integral into $\iiint Vm^2dm\,d\phi\,d\chi$.

Mr. Petrie gave an account of an inscription on an ancient Irish tombstone at Athlone.

Mr. Mallet read extracts from letters by the Rev. Dr. Robinson and others, relating to suggestions for the improvements in working atmospheric railways.

A letter was read from Messrs. Hodges and Smith, stating that, contrary to their directions, 500 copies of Mr. Petrie's volume on the Round Towers had been printed, instead of

450, as volumes of the Transactions of the Academy, and offering the fifty copies which still remained on hands at the same price as the former ones, namely, thirty shillings a copy.

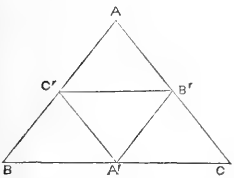
Resolved, on the recommendation of Council, that this offer be accepted.

June 23, 1845.

GEORGE PETRIE, ESQ., Vice-President, in the Chair.

James Strathearn, Esq., Daniel Conolly, LL. D., David Moore, Esq., Rev. Classon Porter, and James Talbot, Esq., were elected Members of the Academy.

The following notice, by the President, Sir William R. Hamilton, of a theorem derived from his Researches on Quaternions, was read.



Let AC′A′B′ be called a *spherical parallelogram*, if A′, B′, C′ bisect the sides BC, CA, AB of a spherical triangle ABC; and let it be said that the corner A of the triangle is the point which *completes the parallelogram* when A′B′ and A′C′ are given as two adjacent sides thereof.

Take any spherical quadrilateral, KLMN, and any point on the same spheric surface, P; draw the four arcs PK, PL, PM, PN, and complete, in four points, K′, L′, M′, N′, the four spherical parallelograms, of which the given pairs of adjacent sides are PK, PL; PL, PM; PM, PN; PN, PK. Then the four new points, K′, L′, M′, N′, form a new spheric quadrilateral, such that its four sides, K′L′, L′M′, M′N′, N′L′, touch a certain spherical conic, having the poles of the diagonals KM, LN of the old quadrilateral for its foci.

This theorem was stated to follow as an easy corollary from what Sir William Hamilton had already communicated to the Academy respecting quaternions.

Certain encaustic tiles and other antique remains were presented to the Academy by Richard Cane, Esq., of St. Wolstan's, they having been found on the site of that ancient abbey.

Resolved,—That the thanks of the Academy be given to Mr. Cane for his donation.

The Secretary read the recommendation of Council at their meeting of the 16th June:

"That Mr. Clibborn be appointed Curator of the Museum, with an increased salary; that an Assistant be supplied to Mr. Clibborn, and also a Porter."

Resolved,—That the recommendation of Council be adopted, and that it be referred back to Council, in order that the same may be carried into effect.

Captain Larcom, V. P., having taken the Chair, Mr. Petrie presented, upon the part of Dr. Stokes, to be deposited in the Museum for the present, an ancient reliquary, called the Fiacail Phadruig, or St. Patrick's Tooth.

Resolved,—That the thanks of the Academy be given to Dr. Stokes for this deposit.

July 14, 1845. (Extra Meeting.)

SIR Wm. R. HAMILTON, LL.D., President, in the Chair.

Resolved,—That an adjourned meeting be held on Monday next, 21st instant, at eight o'clock, in order to afford an opportunity to the Council to correct an informality in the last Minutes of Council connected with the summonses for this meeting, and that the Academy be duly summoned.

Captain Larcom, V. P., having taken the Chair, the President read a communication on the Applications of the new Imaginaries to some Dynamical Questions.—See *Appendix*, No. III.

July 21, 1845. (Adjourned Extra Meeting.)

SIR WM. R. HAMILTON, LL. D., President, in the Chair.

READ,—The following Resolution of Council, passed on 16th July, 1845:

"RESOLVED,—That at the Adjourned Meeting of the Academy, summoned for Monday, 21st July, it be recommended to the Academy to authorize the Treasurer to sell Stock to the amount of £400 sterling, for the purpose of discharging existing liabilities."

RESOLVED,—That the Academy do approve of and adopt the recommendation of Council, as now read; and that the Treasurer be authorized to sell Stock to the amount of £400, for the purposes described.

Mr. PETRIE, V. P., having taken the Chair, the President continued a paper on the Applications of Quaternions to some Dynamical Questions.—See *Appendix*, No. III.

The President having resumed the Chair, the Rev. Charles Graves read the following paper on two methods of solving Biquadratic Equations.

I.—An equation being supposed to have a pair of imaginary roots, $a + \sqrt{-1}.b$ and $a - \sqrt{-1}.b$, if we diminish all its roots by the quantity a, the transformed equation would plainly have two roots differing only in their signs. This consideration suggested the following mode of solving the biquadratic equation,

$$x^4 + A_2x^2 + A_3x + A_4 = 0. \tag{1}$$

Let its roots be diminished by a, a quantity to be determined by the condition that the transformed equation, found by substituting $y + a$ for x, shall have two roots which differ only in their signs. In order that this condition should be fulfilled, we must have, in the transformed equation, the sums of the terms containing the even and odd powers of y separately equal to 0; a must therefore be a root of the equation formed by eliminating y between the two equations thus obtained. In this manner, by a very simple process, we get the following equation in a,

$$64a^6 + 32 A_2 a^4 + 4(A_2^2 - 4A_4) a^2 - A_3^2 = 0, \qquad (2)$$

which is of a cubic form.

Equation (2) will be at once recognized as equivalent to the auxiliary cubic arrived at in Lagrange's, and indeed in every other known method of solving the biquadratic equation. Nor is it difficult to shew why the roots of Lagrange's auxiliary cubic are thus related to the different values of the quantity a, by which the roots of the equation (1) are diminished in the method here presented.

x_1, x_2, x_3, x_4, being the roots of equation (1), Lagrange seeks the equation whose roots are the expressions

$$\begin{array}{ll} x_1 + x_2 - x_3 - x_4 & x_3 + x_4 - x_1 - x_2. \\ x_1 + x_3 - x_2 - x_4 & x_2 + x_4 - x_1 - x_3 \\ x_1 + x_4 - x_2 - x_3 & x_2 + x_3 - x_1 - x_4 \end{array}$$

and finds it to be

$$u^6 + 8A_2 u^4 + 16(A_2^2 - 4A_4) u^2 - 64 A_3^2 = 0.$$

Comparing this with equation (2), we see that $4a = u$. If then we put

$$\begin{array}{ll} x_1 + x_2 - x_3 - x_4 = 4a_1 & x_3 + x_4 - x_1 - x_2 = 4a_4 \\ x_1 + x_3 - x_2 - x_4 = 4a_2 & x_2 + x_4 - x_1 - x_3 = 4a_5 \\ x_1 + x_4 - x_2 - x_3 = 4a_3 & x_2 + x_3 - x_1 - x_4 = 4a_6 \end{array}$$

and attend to the relation $x_1 + x_2 + x_3 + x_4 = 0$, which subsists, inasmuch as the equation (1) wants its second term, we shall find

$$\begin{aligned} x_1 - a_1 &= \tfrac{1}{2}(x_1 - x_2) \\ x_2 - a_1 &= \tfrac{1}{2}(x_2 - x_1) \\ x_3 - a_1 &= \tfrac{1}{2}(3x_3 + x_4) \\ x_4 - a_1 &= \tfrac{1}{2}(3x_4 + x_3) \end{aligned}$$

and there are five other similar systems of equations, in each of which, among the right-hand members, appear two expressions, differing only in their signs: accordingly, if we diminish the roots of the original equation (1) by any one of the six quantities, a_1, a_2, a_3 a_4 a_5, a_6, the transformed equation will have two roots differing only in their signs.

II.—The second method of solution referred to by Mr. Graves, was suggested by observation of the fact that the product of the four quadrinomials,

$$\begin{array}{l} w + ix + i^2y + i^3z \\ w + i^2x + i^4y + i^6z \\ w + i^3x + i^6y + i_9z \\ w + x + y + z \end{array}$$

in which i stands for $\sqrt{-1}$, is real, and equal to
$w^4 - 2(y^2 + 2xz)w^2 + 4y(x^2 + z^2)w + (y^2 - 2xz)^2 - (x^2 + z^2)^2$.
Now if we identify this expression with the left hand member of the biquadratic equations

$$w^4 + A_2w^2 + A_3w + A_4 = 0,$$

we shall have three equations, from which to determine x, y, and z. By the elimination of x and z, we readily deduce from these the reduct cubic ordinarily arrived at.

The President made some remarks on the solution of equations of the third, fourth, and fifth degrees.

[The following Report of the communications made to the Academy by Dr. Robinson, on the 25th of April, 1842, and the 14th of April, 1845, has been received since the Proceedings of these dates were printed.]

ON LORD ROSSE'S TELESCOPE.

Dr. Robinson, when giving, in November, 1840, to the Academy, an account of the three-feet telescope constructed by the Earl of Rosse, had announced to them the intention of that nobleman to attempt an instrument of double aperture and focal length. The attempt had succeeded even beyond expectation, and he hoped that a brief notice of its progress and results would not be uninteresting; more especially as he felt that the approbation with which they had received his former communication, and the importance which they attached to Lord Rosse's discoveries, had not been the least powerful cause of the triumph which their countryman has now achieved.

The speculum was cast on the 13th of April, 1842, according to the principles which had been so successfully applied to the smaller mirrors; but with several changes of the details, made necessary by the gigantic scale of the work.

It is well known to all who have experimented on specula, that the alloy must be formed in the first instance, and remelted for casting at a much lower heat: otherwise the mirror is full of pores. The fusion must, in both cases, be effected in covered crucibles, to preserve the definite proportions of the alloy, which would be *lowered* by oxidation of its tin if exposed to the draught of the furnace. It is also necessary that the speculum be of uniform composition and superficial density; and as it is impossible to fuse the requisite quantity of metal for one of six feet in a single vessel, the different portions must flow into the mould under circumstances as nearly as possible identical. Much thought and many experiments must have been expended before these conditions were so completely fulfilled. The crucibles are, of course, cast iron; no earthen one being able to bear the pressure of such a mass of fluid metal at so high a temperature. They are thirty inches internal depth, and twenty-four diameter, weighing about half a ton each, and manufactured with the precautions pointed out by Lord Rosse (Phil. Trans. 1840). Notwithstanding their great strength, they yield so much that it is obviously hazardous either to use them frequently or to increase their dimensions. Three were employed at once, each containing about one and a half tons of the alloy: they were placed in furnaces whose mouths were level with the ground, eight feet deep and four in diameter, disposed round a large stack or chimney, into which their flues vent. The fuel is turf, peculiarly fitted for this work, as giving a much more manageable heat than coke; about 2200 cubic feet of it are consumed in a casting. The furnaces were filled with fuel, and ignited at the top, on the preceding evening, that the crucibles might be gradually heated; and in about ten hours they were ready to receive the metal. This was unintentionally made of a lower standard than that of the three feet, in consequence of the atomic number for tin being taken as given in Turner's Chemistry, 57.9 instead of 58.9, causing a deficiency of about half per cent, too trifling to impair materially its reflective power, though it will certainly make it more liable to tarnish. That its uniformity might be insured, each ingot of it was broken into three pieces, as nearly equal as possible, and stored in three casks, each of which contributed equally to form the successive charges of the crucibles in an order regularly varying. They were charged at intervals of two hours, and the whole was fused in twelve: they were then withdrawn from the furnaces by a powerful crane, and transported to the iron cradles of pouring frames, arranged 90° asunder round the mould.

The essential part of the mould is its base, composed of hoop iron six inches broad, packed on edge in a strong frame seven feet diameter, and supported by strong transverse bars below. The upper surface was turned to a convex segment of a sphere, 108 feet radius, on the polishing machine, over which a self-acting slide rest was fixed, whose frame was of the same curvature. This process required several weeks, and it was then ground smooth by a frame filled with concave blocks of sandstone. The bed of hoops so prepared being set exactly level, and heated sufficiently to blue its surface (for the purpose of burning out the tallow with which its interstices are filled, when not in use, to protect it from oxidation), the wooden pattern of the speculum was placed on it, and founders' sand rammed round it to its top. The mould thus formed was about a foot deep, the thickness of the speculum, $5\frac{1}{2}$ inches in this instance, being determined by the quantity of metal melted. By this arrangement, as Dr. Robinson formerly explained to the Academy, the fluid metal which comes in contact with the hoops is *chilled* at once into a dense sheet about half an inch thick; the air which might be entangled with it in pouring, escaping through their interstices. The circumference sets much more slowly in consequence of the inferior conducting power of the sand; and the upper surface, which is only in contact with air, remains so long fluid, that the greatest part of the shrinkage occurs there; its tendency to crack the cast is prevented, and the coarse structure which it produces is confined to a place where it is unimportant.

On this occasion, besides the engrossing importance of the operation, its singular and sublime beauty can never be forgotten by those who were so fortunate as to be present. Above, the sky, crowded with stars and illuminated by a most brilliant moon, seemed to look down auspiciously on their work. Below, the furnaces poured out huge columns of nearly monochromatic yellow flame, and the ignited crucibles, during their passage through the air were fountains of red light, producing on the towers of the castle and the foliage of the trees, such accidents of colour and shade as might almost transport fancy to the planets of a contrasted double star. Nor was the perfect order and arrangement of every thing less striking: each possible contingency had been foreseen, each detail carefully rehearsed; and the workmen executed their orders with a silent and unerring obedience worthy of the calm and provident self-possession in which they were given.

It has been found that a good criterion of the time for pouring the alloy into the mould is afforded by stirring it with a pole of dry wood. This, as long as the temperature is above a certain point, reduces the film of oxide which covers its surface; and it becomes clean and bright, though a new film forms immediately. At length as it cools this reduction no longer occurs; and at a signal the three crucibles are emptied into the mould by means of levers connected with the pivots of their cradles. Though familiar with heavy castings, Dr. Robinson had never seen any thing so magnificent as the burning lake that was then produced; and for many minutes it rolled in heavy waves like those of quicksilver, which broke in a surf of fire on the sides of the mould, effecting the most perfect mixture of the metal. At last it became solid, and was examined as it cooled, till it barely yielded to pressure with an iron rod at its centre, which is the indication that it may be removed to the annealing furnace.

This furnace extends along the fourth side of the mould: it is a low square chamber lined with firebrick, with sides about thirty inches thick, strongly hooped, and covered by an arch, from the centre of which rises a flue. Its floor is convex, of the same curvature as the speculum, and is heated from beneath by nine arches, which communicate with lateral flues. It opens towards the mould by a low arch a little wider than the speculum; but behind has merely an aperture to admit an iron bar. For some weeks the chamber and arches had been kept full of burning turf, so that the whole interior was of a full red heat. The speculum, also red hot (at which temperature, it is to be remarked, the alloy has nothing of that brittleness which characterizes it when cold, but is as tough as malleable iron), was cleared from the sand, and encircled by a strong ring attached to the bar above mentioned. By connecting this bar with a powerful capstan, it was drawn from the bed of hoops, along strong beams covered with iron, to its place in the centre of the annealing furnace. The ring was then removed, and the rest of the chamber filled up with charcoal; the arches with fuel; all the flues and apertures were closed carefully with masonry, and it was left to cool gradually for sixteen weeks, during the first three of which the exterior of the building was sensibly warm.

In the course of this year considerable progress was made with various parts of the mounting; and when Dr. Robinson visited it in February, 1843, he found that the speculum had been ground (on a machine similar to the old one, but of strength proportioned to its work); that the foundations of the piers were laid, the tube was in preparation, and the massive frame-work and levers by which the speculum is supported in the tube, were cast. This elegant contrivance requires some explanation. Suppose the back of the mirror divided by two concentric circles into three portions, of which the central circle is cut by radial lines into six sectors, the middle zone into nine segments, and the exterior into twelve, and that all of these are equal. If each of these be supported by an equal force applied at its centre of gravity, the speculum is obviously in the most favourable condition as to flexure. The frame mentioned above is rectangular with a cross-piece cast in one, and weighs one ton and a half: it bears three strong triangles, also of cast iron, supported at their centres of gravity on hemispherical bearings. Each angle of each of these bears a similar triangle, the angles of which give the twenty-seven points of equilibrated bearing for the speculum. They do not, however, press directly, but carry platforms of cast iron of the shape of the areas which they are to bear, and made exceedingly stiff by flanches at their edges, and by edge-bars crossing them diagonally. A layer of felt is over these; strong uprights from the frame of a similar character prevent any lateral shifting; and the operation of the arrangement is found to be perfect at all altitudes.

The construction of the tubes, the piers, the mechanism of the counterpoises, occupied the remainder of this year and the beginning of 1844. In August, a partial polish was given to the mirror in order to verify its focal length (which was found exactly fifty-four feet); and the observing galleries and the apparatus for controlling the instrument in right ascension were proceeded with. All these gigantic constructions (of whose prodigious mass some idea may be formed from the fact that they contain, along with their other materials, more than one hundred and fifty tons of iron castings), have been executed in Lord Rosse's workshops, by persons taken from the surrounding peasantry, who, under his teaching and training, have become accomplished workmen, combining with high skill and intelligence the yet more important requisites of steady habits and good conduct. It may also be mentioned that (such was the clear and definite arrangement of the whole in its inventor's mind) nothing failed from first to last; and it was not necessary for him in any instance to retrace his steps.

At the beginning of February, 1845, the work being sufficiently advanced to permit the use of the instrument without personal danger, Dr. R. and his friend Sir James South were invited to enjoy the trial of it. Its appearance is certainly peculiar, and presents a striking contrast to the more complicated framing of the three-feet telescope which is placed beside it. At first sight, one wonders how it is to be moved, for nothing attracts notice except the massive piers and the tube; but a nearer approach shews that it is the perfection of mechanical engineering. To have mounted it on the plan of the three-feet would have been impracticable as well as useless. The speculum with its supports is seven times the weight of that in Herschel's four-feet, and both on this account, and the well-known principle that similar machines are weaker in proportion to their bulk, such a stand must have been so heavy as to present great obstacles to its motion. The vast surface exposed to the action of wind must have made it unsteady; and its durability could not be great. Lord Rosse, therefore, determined to confine the range of observation to the vicinity of the meridian. There the stars are at their greatest altitudes, and atmospheric influences affect them least; their places can be determined with most accuracy, and an equatorial movement, so essential to micrometer measures, can be easily obtained. With such optical power there will never be a scarcity of objects for examination; and the restriction will only be felt in the case of planetary bodies. The base of the actual mounting is a very massive joint of cast iron; its lower axis permitting motion in the meridian plane, its upper in a direction perpendicular to that circle. On this is firmly bolted a cubical wooden chamber, about eight feet wide, in which the speculum is placed, one of its sides opening for the purpose. This again carries the tube, which when vertical and viewed from the interior of the chamber, is more like one of the old round towers than any more ordinary object of comparison. It is fifty feet long, eight feet in diameter in the middle, but tapering to seven at the extremities: it is made of deal staves an inch thick, hooped with strong iron clamp-rings, and secured from collapse by iron diaphragms; and carrying at its upper extremity the apparatus of the Newtonian small mirror, which, from its great weight and bulk requires to be counterpoised. The telescope is moved in declination by a strong chain cable attached to its top and passing over a pulley fixed at a proper height to the north, down to a windlass on the ground which is wrought by two workmen. East and west, near the top of the piers, large iron pulleys are fixed, having free movement in azimuth, so that their planes may always be in those of the traction: chains suspending the counterpoise weights pass over these to the sides of the tube. The weights, however, are constrained to descend in quadrants of circles by chain guys attached to the frame which bears the declination pulley. It is easily seen that their action is a maximum when the tube is level, and nothing when it is vertical; but between these positions it decreases more slowly than the downward tendency of the tube. To correct this, the tube is connected with loaded levers (placed to its south), by chains of such lengths that one of them is not raised till it is at 40° altitude, and the other at 80°; the latter being necessary for the return of it after it has passed the zenith. The slow motion in declination was not yet applied, but the ordinary one was quite convenient, except for the difficulty of giving orders to the men, who were sometimes seventy or eighty yards from the observer. A two-feet circle, with a fine level and a pair of verniers, will also be attached to the tube to give the declination; its place was then supplied by a small protractor, five inches diameter, over which was a strong screen to protect the assistant who attended it from any such casualty as the fall of an eye-piece.

The eastern pier bears what may be called the meridian of the instrument: it is a strong semicircle of cast iron, about eighty-five feet diameter, and composed of several pieces accurately planed. Each of these is bolted to the pier and separately adjustable to a meridian line formed by straining a fine wire over notches in two cast iron chairs firmly secured at the north and south of the masonry. Sir James South took charge of this delicate operation, and performed it with such precision that when a transit instrument was adjusted by this line, it gave the passage of Polaris *to a small fraction of a second.* The telescope is compelled to move in the meridian, being connected with this circle by a strong bar provided with friction rollers, that it may traverse it easily; and thus it can be used as a transit instrument with considerable precision. But this bar is racked, and attached to the tube by wheelwork, so that a handle near the eye-piece enables the observer to move it on either side of the meridian, and thus examine it before its passage, or follow its motion. The movement is surprisingly easy; and a rough graduation on the bar supplies at present the place of an hour circle for finding objects, for which it is quite sufficient, except that the strong light required to set it disturbs the repose of the eye. The elder Herschel has not in the least exaggerated the importance of this when faint objects, especially nebulæ, are to be examined; and a better contrivance is to be applied. The rack being perpendicular to the meridian, gives a motion not strictly equatorial, but easily made so: had the declination pulley been in a parallel to the earth's axis, passing through the great joint, and had this latter been itself equatorial, this would have been the case; but the deviation is easily corrected by the addition of a second pulley altering the direction of the chain. Its range is half an hour on each side of the meridian for a star at the equator; and Lord Rosse intends to effect it by clock-work, as is now generally the case in large equatorials; though the problem is much more difficult than in those instruments.

The western pier supports the stairs and galleries destined to the observers. Up to 42° of altitude is commanded by the first of them: a strong and light prismatic framing slides between two ladders attached to the southern faces of the piers: it is counterpoised and is raised to any required position by a windlass; its upper plane affords support for a railway on which the observing gallery moves about twenty-four feet east and west, two of its wheels being turned by a winch near the observer. Three other galleries in succession reach to 5° below the pole; these are each carried by two beams which run between pairs of grooved wheels, and are drawn forward, when they are turned, by a mechanism of singular elegance. These are able to hold twelve people, but one man can easily work them; and though it is rather startling to a person who finds himself suspended over a chasm sixty feet deep, without more than a speculative acquaintance with the properties of trussed beams, all is perfectly safe. Every bearing part has been *proved* to ten times its utmost probable load, and the doors of the galleries open inward, and are kept close by springs. From this point too is obtained the most distinct perception of the telescope's prodigious bulk, which at a greater distance is not so striking, for want of a standard of comparison; yet, notwithstanding the hugeness of the masses to be moved, so effective are all the arrangements that Sir James South found it was possible to uncover the mirrors and find a given star in less than eight minutes.

Unfortunately the whole month of February was of the worst astronomical character; and though the great speculum had only the imperfect polish already noted, it was kept in the tube as long as there were any hopes of seeing the great nebula of Orion. That, however, was always clouded while within its range. A few clear minutes on the 13th allowed them to see some stars and clusters; but the only circumstances worth mentioning were, that it shewed the stars of Castor far apart *without an eye-glass*,* and that the stars of the cluster 67 Messier, which Sir J. Herschel describes as being from the eleventh to the twelfth magnitude, were many of them as bright as those of the first appear in a three and a-half feet achromatic.

At length, when all hopes of Orion were lost in the twilight, the mirror was removed from the telescope, and polished on March 3rd. Its frame is supported in the cubic chamber of the tube by three strong screws which give the adjustment of its optic axis. By unscrewing these, when the tube is vertical, four wheels with which the frame is furnished come to bear on a railroad fixed at the bottom of the chamber and communicating over a bridge (laid from its door when necessary) with another railway laid on an inclined plane of about sixty feet. It is drawn up this by the declination windlass, and at its top runs on a strong truck by means of which it is drawn a quarter of a mile, on a common road, to the laboratory.

The polishing machine differs in nothing but size from that

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* Only twenty-two inches of the mirror could act in this case.

described by Lord Rosse in the Philosophical Transactions. It makes the speculum revolve once for twenty-four and a-half strokes, and the eccentric once for eighteen. From seven to eight strokes of twenty-four inches are made in the minute, and the lateral movement of the speculum by the eccentric is fourteen inches.* A screw whose nut runs on a railroad above the machine lifts the speculum, with its frame and levers, from the truck, and deposits it on the revolving platform, where it is levelled, centred, and secured. The same apparatus serves to move the polisher during its preparation and to apply it to the speculum, so that it is even more manageable than that of the three-feet was. It was cast with the transverse grooves; the circular were cut in the lathe. The time required for polishing is about six hours; and Lord Rosse has found that this period cannot be exceeded without injury to the figure, in consequence of the soft pitch being squeezed out, and the harder and unyielding material coming into contact with the iron of the polisher: unfortunately, this occurred to some extent in the present instance. The ammoniacal solution of soap used towards the close of the process, happened to be made with ammonia prepared from gas liquor and containing some sub-

* These are the proportions which Lord Rosse prefers; but it must be kept in mind that they change with circumstances. *Probably* they will not answer for those specula which have an aperture larger than one-ninth of their focal length, and *certainly* not for those which are perforated in the middle. Dr. Robinson has made many experiments on one of the latter, fifteen inches aperture and nine feet focus, with a machine nearly the same as Lord Rosse's; and he finds that the nature of the polishing depends on the figure given in grinding. If the eccentric be regulated so as to make this hyperbolic, its action must be lessened in polishing so as to shorten the focus. In this way it is possible to obtain very good results. He, however, prefers the opposite course pointed out by Lord Rosse; grinding to an elliptic figure, he polishes with a very long primary stroke, and small action of the eccentric. A speculum thus polished shews ε Arietis well separated and defined with 940, and with 465 the fifth star in the trapezium of Orion's nebula is visible even when the acting surface is reduced to seventy-two circular inches.

stance which acted on the mirror and produced a dulness that was not removed till after three hours' additional work. Lord Rosse warned them that the figure must be imperfect, and wished to repolish; but they overruled this proposal, and it was replaced in the tube next day.

On examining it by diaphragm and discs, it was found that, as he anticipated, the edge was not quite perfect. All its zones showed ξ Ursæ Majoris very well with 560; but the exterior six inches manifested, when the star was thrown out of focus, that though of the same focal length, this portion was irregular. Few other double stars were observed, as most of the lucid interval from the 4th to the 13th of March was devoted to nebulæ, and after that it again became cloudy; but enough were seen to satisfy them that the instrument possessed a very high defining power. This, indeed, was evident from the admirable exhibition of Regulus, seen on March 5th, neat and round, without appendages or flare. Gamma Leonis, ξ Virginis, 2 Comæ, and Gamma Virginis, were also well shewn with powers of 400 to 800 on an unfavourable night; and the companions of ν Ursæ, and 245 of Struvé's second Catalogue, which appear in the Slough and Pulkova telescopes as of the eleventh and tenth magnitudes, seem in this large stars.

Of planetary bodies, none were visible except D'Arrest's Comet and the Moon. The former, when viewed March 10th, presented nothing remarkable: the brighter portion, towards the centre, shewed no abrupt change of light which might indicate a solid nucleus; there was no resolvable appearance in the Coma, and the very minute stars with which that part of the sky was dotted, were visible almost to its very centre. Only one view of the moon was obtained, March 20th, and it was shared with them by several visitors, who, when once in possession of the telescope, were by no means disposed to make way for the astronomers. The fascination of the sight is, indeed, such, that one can scarcely withdraw the eye:

Dr. Robinson, therefore, and his friend, had but little time for observation. He was, however, much interested by the vicinity of the craters named Hansteen and Mairan, in the map of Beer and Mædler, where, besides the crowd of hills described by them, these are an infinity of others not visible even in the three-feet, but looking in this with 560 like grains of sand. Are these fragments ejected from the crater? If so, and if they occur round others, it would explain what had always presented to him a great difficulty. The lunar craters differ widely from those of earth; and most in this, that their depression below the general surface is enormously greater than the elevation of their walls above it, while the area of the hollow is far greater than that of the latter. What, then, became of the materials which had once filled it? He had formerly supposed that they were in a fluid or gaseous state when ejected; but the fact just mentioned seems to give the true solution, and appears to account for them when combined with the consideration of the feeble gravity on the moon, which would permit the exploded fragments to be scattered over a far larger space than with us. Another beautiful object was the river-like valley that runs northward from the crater Herodotus: its raised banks, and their irregularities, were easily seen; the internal and external shadows could have been satisfactorily measured had a micrometer been applied. As it was, the much greater breadth of the former showed at a glance that this strange channel was sunk deep below the lunar surface. Taking as a standard the measures given there by Beer and Mædler, he had no doubt that they then saw *without difficulty* spaces of eighty or ninety yards. It is difficult to say *à priori* what should be the minimum visible at the moon in such a telescope. If we assume, as one extreme, the statement of Amici, that the non-coincidence of two black lines on paper can be seen at twenty-eight feet, when it amounts to one-twelfth of an inch, or subtends 51″, then 311 feet should be visible at the moon with 1000. On the other hand, Jurin states (Smith's Optics) that a piece of silver wire can be seen on white paper, when it subtends 3″, a result depending on the intensity of this metallic reflection. This would give eighteen feet! Dr. Robinson finds that he can see the spider lines of his circle without much contrast of light, when they subtend to him 16″. This gives ninety-seven feet; but it must be remembered that aperture influences visibility as well as magnifying power, though we cannot as yet estimate its effect numerically.

The most important part of their observations were made on nebulæ; and, besides establishing completely the prodigious superiority of this instrument over all yet constructed, they have added some facts to our knowledge of these mysterious objects. A list of them was formed from the invaluable catalogue of Sir John Herschel (Phil. Trans. 1833), comprising such as, from brightness or any other peculiarity, seemed deserving of notice; of which forty were examined by Dr. Robinson and also by Sir James South, except some which the latter lost while making the transit observations required for the meridian line.* They may be separated into three classes; those which are round and of nearly uniform brightness;† those which are round, but appear to have one or more nuclei;‡ and those which are extended in one direction, sometimes so much as to become long stripes or rays.§ Of the first class, all that were examined are easily resolved, even with a triple eye-piece of wide field and power 360, used for finding the objects. In 854 the stars were seen through haze; in 1929 during twilight; and 1833 was noted as "consisting

* Sir James South published an interesting and instructive notice of this telescope in the Times, April 16, 1845.

† Nos. 538, 739, 777, 844, 845, 854, 1797, 1833, 1907, 1929.

‡ Nos.564, 706, 711, 743, 748, 749, 805, 843, 846, 1146, 1385, 1456, 1622, 1881.

§ Nos. 536, 604, 668, 791, 792, 810, 859, 1066, 1132, 1148, 1352, 1357, 1368, 1466, 1926. The numbers and the figures cited in the text are those of Sir John Herschel's catalogue.

of rather coarse stars, and resembling Messier 13." Any increase of brightness towards the centre seemed to proceed from the greater depth of stars there rather than from any notable difference of their magnitude. But the second class presents much more interesting phenomena: the appearances which previous observers had described as *sudden condensation*, *nuclei*, or even single or multiple *central stars*, proving to be clusters of comparatively bright stars, surrounded by much larger collections of minute ones. A very beautiful example of this is 1456, fig. 41, M. 94, described in the catalogue as "very suddenly much brighter, almost up to a nipple-shaped nucleus:" it proved, however, to be "a vast circular cluster of stars, with ragged filaments, in which, and apparently central, is a globular group of much larger stars, power 400." The same system of arrangement (which seems very common) occurs also in 706, 748, 805, and many others: it is also found in the magnificent clusters 1663, M. 3; 1558, M. 53; and 1916, M. 5. In these, the splendour of which is not to be described, besides the stars visible in other instruments (which here seem of the first or second magnitude), the whole field is crowded with others much smaller, to such a degree that, had the first been absent, these would still have been noted as remarkable objects. The interior group is not, however, always central or symmetrical, but has knots of greater condensation, which sometimes (as in 1385) are alone visible in smaller telescopes, and then look like "twin nebulæ;" at others (as in 739), like stars. In 1622, fig. 25, M. 51, which is so well known from a sort of resemblance to Saturn, and from the more exact analogy which, as Sir John Herschel has well remarked, it bears to the Milky Way, we have another different development of this arrangement. Here also the central nebula is a globe of large stars; as indeed had been previously discovered with the three feet telescope: but it is also seen with 560 that the exterior stars, instead of being uniformly distributed as in the preceding instances, are condensed into a ring, although many are also spread over its interior. Were the centre absent, we should have a ring nebula;* and were the line of vision near the plane of this ring it would become one of those rays with a bright nucleus and parallel band or satellite nebulæ which occur so frequently in the catalogue. In comparing it with our own sidereal system, Dr. R. thinks we should consider the stars visible to the naked eye, and the larger telescopic classes as constituting the central cluster, while the Milky Way represents the exterior and minuter stars either disposed in an irregular ring or in a stratum, two of whose dimensions are much greater than the third. We have no reason for believing that the comparative brightness of stars depends *only* on their distances; 61 Cygni is not more remote than α Lyræ; much less can we assume that our stars are uniformly distributed: Orion, the Pleiades, Prœsepe, the clusters in Perseus, M. 36 and 37, with many others, are evidently mere knots of condensation in our immediate neighbourhood, our peculiar cluster; and it seems a mere arbitrary assumption to fancy that, were we transported to a remote part of the Milky Way, we should see any thing similar to our present sky.

The nebulæ of the third class which were examined seemed to differ from this type only by being seen obliquely, and therefore projected into ellipses sometimes almost linear. In this last case they proved much more difficult of resolution, probably from greater *optical* condensation, and yielded most easily towards their minor axes. In these the nucleus of brighter stars is sometimes extended like the exterior portion, as in 602, which is of considerable length and easily resolved: the central part has three knots, of which two are represented in fig. 70, all the rest having been invisible. 668 is similar,

* It is possible that the exterior part of M 94, may be merely a circular *disc* of stars: the absence of the central globe would make this a planetary nebula: but it is possible that these differ from the annular only in degree; all the latter which he has seen having faint nebulosity within them.

but the central part is of more uniform character. In general, however, the nucleus is globular, and remarkable from the comparative smallness of its diameter, and its very condensed appearance. Either the stars which compose it are few in number, or more closely compacted than is usual. 1132, M. 98, is a good example: "the long ray is resolved, except at the very extremities, with 560; the globular nucleus is seen with 1280 to consist of very close stars." 1148, described as "a nucleus with two branches, a star north following," appeared to Dr. R. as "an irregular ring of stars round a brighter group, but having an appendage like that of M. 51, in which is the bright star seen by H." 1357, fig. 37, is a similar object, both "the ray and appendage being full of stars, but the nucleus requires a higher power to resolve it than the night will bear." In 1466, fig. 84, *the nucleus projects on each side of the ray*, so that its diameter must be greater than the thickness of the exterior stratum.

He could not leave this part of his subject without calling attention to the fact that no REAL nebula seemed to exist among so many of these objects chosen without any bias: all appeared to be clusters of stars, and every additional one which shall be resolved will be an additional argument against the existence of any such. There must always be a very great number of clusters, which from mere distance will be irresolvable in any instrument; and if it prove to be the case that *all* the brighter nebulæ yield to this telescope, it appears unphilosophical not to make universal Sir J. Herschel's proposition, that "a nebula, at least in the generality of cases, is nothing more than a cluster of discrete stars."

These observations will suffice to show how much may be hoped from this telescope; but they are far from being a fair measure of its powers, being made at very low temperatures. Almost always the thermometer was at 22° or 20° when they ceased working; and on one occasion it was as low as 17° the lowest he remembered in Ireland. In such circumstances it is notorious that even small reflectors act very imperfectly: and he was therefore unprepared for any tolerable action of this gigantic speculum. In the day time it was of course colder than the air, and, if uncovered before that had sunk to its temperature, was covered with dew: when this went off it always defined sharply. The huge mass of metal cooled much more slowly than the atmosphere; and as the difference increased, the performance of the telescope was deteriorated. This arose from no change of figure, as he satisfied himself by throwing the stars out of focus; it was probably the result of currents in the tube occasioned by this difference of temperature. How far it will be possible to obviate this by mechanical means, remains to be tried; but it is certain that the inconvenience does not increase in a higher ratio than the power of the telescope, as he had formerly apprehended. On the same nights, it defined quite as well as the three-feet with a far lower power; and therefore, it is reasonable to expect that it can be used with advantage much more frequently than he once supposed.

Enormous as is its illuminating power, it might be increased one-third, by using it with the front view, supposing it can be properly figured for this oblique action. Without that, he fears that in an instrument where the aperture is so large compared with the focal length, the definition would be imperfect. He verified this by an experiment with the three feet, and found that though the light was increased quite as much as he expected, yet the perfection of the image was utterly destroyed for large stars. There was no exact focus, but merely two places where the sections of the cone of rays were smallest. One, the least exceptionable, shewed a flare in the direction of the slope like a comet's tail: at the other this disappeared, but the star became a sort of curved rectangle with rays from its corners. In the Newtonian form this speculum a few nights before had defined ζ Orionis very well with 500; but now γ Leonis could not be seen double with any power; the companion of Rigel (some way from the meridian however) was lost in the flare, and even that of Polaris, though perfectly visible, was sadly disfigured by it. It was of course useless to try more difficult tests, as even this degree of imperfection would make it utterly incapable of resolving such objects as the nuclei of the long nebulæ, be its illuminating power what it may. One thing, however, deserves notice, that in consequence of removing the second reflection, the colours of the stars come out with extraordinary splendour.* β Cygni, for instance, had a pureness and brilliancy of yellow, in the large star, which was new to him, though he had seen it in many first-rate telescopes. Lord Rosse does not apprehend any insurmountable difficulty in applying his method to give the form necessary for aplanatic oblique reflection: more than one plan for this has occurred to him; and Dr. R. believes it is his purpose, as soon as the six-feet has its machinery completed, to try them on one of the three-feet specula, and, if successful, to alter the great one.

As it is, Dr. R. congratulates the Academy and their country on the success of this matchless instrument; to which, as nothing at all approaching to its power has yet existed, so it is not probable that there will soon be any superior. It has been reported that the French Government, at the suggestion of M. Arago, are about to construct an achromatic of a metre aperture. Supposing homogeneous discs of glass can be obtained and wrought of that magnitude, there remain other difficulties. The optician who proposed to supply them stated that they would weigh at least four hundred pounds; now these, when mounted, must be supported by at most two lateral bearings; and it is known that a very moderate pressure produces in

* The lenses of achromatics have often a tinge of green or straw colour which modify the colour of objects seen through them. Something of this may perhaps cause the predominance of green and "cinereus" which exists in the Dorpat catalogue.

glass a double refraction, most injurious to its performance in an object glass. But supposing this and the equally probable change of curvature from the weight of the lenses obviated, still such an achromatic would be far below the six-feet in quantity of light. From Amici's experiment with an object glass of two and a half inches it follows that it equals a Newtonian when their acting surfaces are as six to ten: this would imply in the great one an aperture of fifty-six inches and a focal length of eighty feet. But the absorption certainly increases with the thickness of the medium, though neither the law of this, nor the loss by the reflections at the four surfaces, are accurately known. Mr. Potter found that a good object glass by Dollond of four inches aperture and six feet focus transmitted but 0.66 of the incident rays. This gives the ratio of the equivalent surfaces 0.74, and it will be still greater where the glass is three or four inches thick. It is said that the construction of a reflector still larger than this is contemplated by a northern Sovereign who has already shewn himself a most munificent patron of Astronomy. If so, none will rejoice more than Lord Rosse himself. It was not the mean desire of possessing what no other possessed, or seeing what no other had seen, that induced him to bestow so many precious years on this pursuit: had such been his motives, he would have kept to himself his methods, instead of opening his workshops without reserve to all who had the slightest desire of following his steps, and communicating in the most liberal manner the fruits of long and painful experience. His sole object is to extend the domain of astronomical knowledge: and the more common such instruments become, the more perfectly will it be fulfilled.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

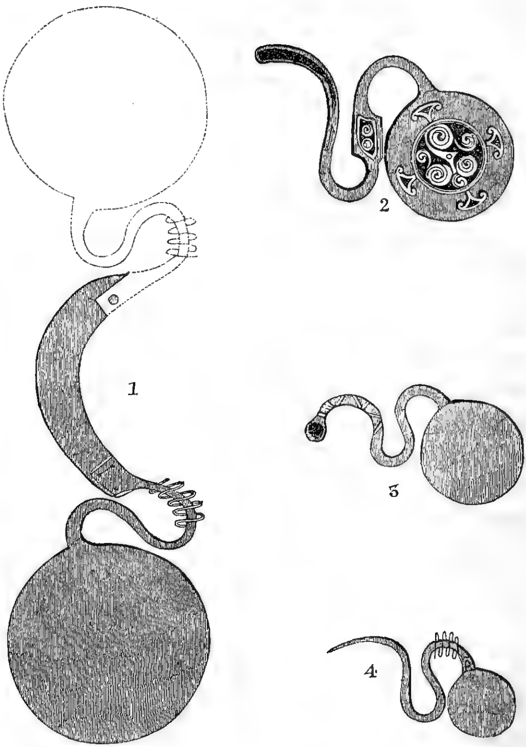
1845. No. 51.

November 10, 1845.

GEORGE PETRIE, Esq., Vice-President, in the Chair.

Mr. Ball made a communication, the object of which was to shew that the article called a crotal, of which there are three specimens in the Academy's Museum, had properly but one disc, and not two, as represented in Ledwich's Antiquities (plate xxiv. fig. 6), and Camden's Britannia (Gough's Ed. vol. iii. plate xxxiv. fig. 1). He founded his arguments upon the fact, that the three specimens in the Academy's Museum (Nos. 2, 3, 4, of the annexed cut) were each evidently perfect, while that figured by Ledwich and Camden, which still exists in the University Museum, is a compound of two specimens, most rudely and *recently* rivetted together with a common copper rivet. (See No. 1, annexed cut).

Mr. Petrie stated, that of the six specimens said to have been found at Slane, he had seen three which were certainly double, though he would not undertake to say that they had not been compounded, as that in the University Museum quite evidently is. A gentleman who had been recently in Persia, on seeing the specimens in the Academy Museum, stated, that in that country, at the present day, they were used in the manner of castanets for keeping time, and that they were not provided with double discs. The manner in which boys here hold and beat time with long bits of slate, may be a specimen of the practice of using crotals.



The Secretary of the Academy read a notice of an Ogham Stone found by Mr. Nevins in the County of Wexford. A rubbing of the inscription was also exhibited; but it was so imperfect, from defects in the stone, that its publication in a wood-cut would answer no end.

DONATIONS.

Statistical Return of the Dublin Metropolitan Police, for 1844. Presented by the Commissioners.

Comptes Rendus hebdomadaires des Seances de l'Academie des Sciences. No. 3. (21 *Juillet*, 1845.) Presented by the Academy.

Memorie di Matematica e di Fisica della Societa Italiana della Scienza residente in Modena. Tome XXIII. (*Parte Fisica*). Presented by the Society.

A Sermon, preached in the Chapel of Trinity College, Dublin, on 25th May, 1845. By the Rev. Charles W. Wall, D.D. Presented by the Author.

Memoirs of Francis Baily, Esq., F.R.S., &c. &c. By Sir John F. W. Herschell, Bart., &c. Presented by the Author.

Poems in Irish and English (MSS.) By Carolan. Presented by Robert Ball, Esq.

The Coinage of Scotland. By John Lindsay, Esq. Presented by the Author.

The Eneis (Books I. and II.) *rendered into English blank Iambic, with Notes, &c. &c.* By James Henry, M.D. Presented by the Author.

Journal of the Franklin Institute. Vols. VII. and VIII. Third Series. 1844. Presented by the Institute.

Three Reports, upon improved Methods of constructing and working Atmospheric Railways. By Robert Mallet, Esq. Presented by the Author.

Resultats des Observations Magnetiques faites a Geneve dans les Annés 1842 *et* 1843. By Mons. E. Planta. Presented by the Author.

Proceedings of the Geological Society of London, for Session 1843–44. Presented by the Society.

Archives du Museum d'Histoire Naturelle. Tome 1me et 2me Livraison. Presented by the Governors.

Archæologia Cambrensis. No. I. Presented by the Editor.

A Gold Fibula. Presented by the Marquis of Kildare.

An ancient Cannon Ball, made of Sandstone, found in an Excavation made near the Castle of Dalkey. Presented by Frederick W. Porter, Esq.

A Heel-ball Rubbing from a brass Plate in the Cathedral of Amiens. Also, *a Rubbing from an Inscription on an Oak Rood-screen in the old Church of Llanfair, near Kneighton.* Presented by Dr. Todd.

An ancient Wooden Tray, found at Hilltown, Co. Westmeath. By Arthur Webb, Esq.

November 29, 1845. (Stated Meeting.)

GEORGE PETRIE, Esq., Vice-President, in the Chair.

William Wordsworth, Esq., was elected an Honorary Member of the Academy.

It was resolved, on the recommendation of Council,

"That the sum of £50 be placed at the disposal of the Committee of Antiquities, for the purchase of articles of antiquarian interest for the Museum."

The following letter, from Edward J. Cooper, Esq., on the Zodiacal Light, was read:

Markree Castle, 13*th Nov.* 1845.

"SIR,—The phenomenon of the Zodiacal Light being but very rarely visible in these countries, I am induced to trouble you with this communication, to inform you of its appearance here this month. At ten minutes past four o'clock, A.M., on the 4th instant, a strong light, convex in its upper limits, was observed by me, and one of my assistants, at my observatory, in the horizon east by south. It was very similar to that of low Auroræ Boreales. We could not believe that it was crepuscular, as it was too early, nor that it was of the nature of what are commonly called the "Northern Lights." We watched it for a considerable time, during which it appeared to vary in brilliancy. However, it branched out to Regulus, and also towards Coma Berenicis, the edge of low fog, towards the south, being also illuminated. It faded first in the branch towards Coma Berenicis; and, lastly, under the advancing twilight, in that towards Regulus. Gamma Virginis was in the axis, near the horizon, and Kappa Crateris on the azimuthal limits towards the south. From east, through the north to west, stars were visible to the horizon, which but very seldom is the case here. A considerable number of shooting stars were streaming about this morning. Having little doubt that the branch of elliptic light which extended towards Regulus was the zodiacal light, although I had never before seen it in the morning (and, indeed, in Italy alone in the evenings of the months of March and April), I resolved to look out again for it. The weather was unpropitious until the morning of the 10th, when it was seen from the observatory, at ten minutes before four o'clock, and for some time afterwards, by my assistant, Mr. Magrath. I saw it from my house at a few minutes before five o'clock, when it shewed, with very tolerable definition, the elliptic outline which I have so often remarked in Italy in the spring evenings. There was no trace, on this occasion, of a branch of light towards Coma Berenicis. In less than a quarter of an hour it was almost entirely lost behind a rising fog, which left a sharp white frost upon the ground. The remarkable features of the phenomena we witnessed seem to be these, viz., 1st, that on the morning of the 4th there was a second branch of light, and also an illuminated edge to the fog in horizon; neither of which were visible on the morning of the 10th, nor have I ever previously observed any thing similar to accompany the evening exhibitions of zodiacal light. 2ndly, That there was on that morning a flux and reflux of the light. I cannot attempt to account for the former; but I suspect that the latter appearance arose from a rising and sinking of the imperceptible terrestrial vapour.

"Edward J. Cooper.

" *To the Secretary of*
" *The Royal Irish Academy, &c. &c.*"

Mr. William Hogan read the following notice of the storm of Sunday, 6th July, 1845:

"I was in Leamington at the time, and, though it did not rage there, I had an opportunity of witnessing the atmospheric phenomena, as the thunder-cloud passed at a small distance to the north, and I observed its course for an hour and a half.

"To shew what its aspect was to those over whom it passed, I extract the following particulars of its history twenty miles to the north of Leamington, from the published account of Mr. Onion, of the Philosophic Institution at Birmingham. After alluding to a violent storm of the preceding Thursday, he says: 'Birmingham has again been visited by a thunder-storm more terrible, and in its consequences more disastrous, than the former. On Saturday afternoon the thermometer varied at different times from 70° to 78°; *not a breath of air stirring*, the barometer being moderately high. About eight o'clock, P. M., a few heavy drops of rain fell, which were shortly afterwards followed by a complete deluge of water; the lightning was grand and awful in the highest degree, flash succeeding flash in rapid succession, and of that beautiful purple tint which betokens a large quantity of free electricity in the atmosphere. The thunder in the mean time rolled in nearly one continued peal; the wind, which had been varying from S. E. to E., suddenly shifted to S., and about the middle of the storm the vane *veered suddenly and with great rapidity through the whole points of the compass,* again setting in the south. For the space of three or four hours afterwards it continued very inconstant, changing in all directions, and eventually settling in the S. W. In the short space of half an hour 1.945 inches of rain fell.'

"Such were the phenomena at Birmingham as the thunder-cloud passed over it. Its arrival was preceded by oppressive heat and a dead calm, while a fresh breeze blew at Leamington. Its passage was marked by a whirlwind, with constant lighting, and the fall of nearly two inches of rain in half an hour.

"Particular circumstances led me to be an attentive observer of the state of the atmosphere from seven to half-past eight on the evening of the 6th of July. I saw the thunder-cloud appear in the south-west, and pass over Birmingham in its course towards the north-east; it was of great extent, and I should think very high; a rapid current of air in the same direction carried a light scud under the cloud, or at least between me and it, so as to appear under it. The heat was intense the whole evening, *but previous to and during the storm we had a constant fresh breeze from the south-east;* the cloud came from south-west. The lightning was very brilliant and constant, but I heard very little thunder, perhaps owing to the state of the wind. In shape and appearance the cloud might be represented by the map of Africa, from the Bight of Benin southwards, laid on its side, with the eastern coast and the island of Madagascar uppermost. A cloud of the relative size and position of this island kept constantly and steadily in advance of the larger one, and all the lightning which I saw until the cloud passed was from the upper surface, and generally played round the cloud represented by Madagascar, though it sometimes darted out from the latter in every direction. It was forked lightning, but its appearance was not that of bars of light, but such as one would observe were the electric current sent along a zig-zag chain in a darkened room.

" As the cloud passed on to the north-east, and I looked after it, the end next me had the appearance of a vast crater, emitting forked lightning and flashes of light; and it was from this crater-like opening that I suppose the lightning was emitted which was visible at Birmingham, after the storm had ceased and the cloud passed on towards Nottingham.

" All accounts say that meteors and lightning were observed at the rear of the cloud for an hour or two after the tempest. When the cloud had nearly passed over Birmingham, the quantity of ozone which saturated the air at Leamington was so great as to be very unpleasant, and I was obliged to close the window to exclude it. It is to be observed that no rain fell in Leamington at that time, and consequently there was no moisture to absorb the ozone and prevent its accumulation. The lightning always broke from the upper surface of the cloud while it passed before me; when I looked after it, it came from the inside of the crater-like opening in the rear, but never from the surface next the earth. The unpleasant effects of the storm on the invalids of my party soon afterwards occupied all my attention.

" This immense cloud, so heavily charged with water, appeared to be completely isolated; it did not attract the flying scud, nor did it break into masses; and the sky became serene and blue when it had passed. I observed its approach from the south-west; at eight it had reached Birmingham, and at nine it had passed. I traced it in the local newspapers in a straight line from Hereford to Nottingham, where it caused prodigious floods, passing over Kidderminster, Dudley, and Birmingham, in a direction from south-west to north-east. All the newspapers agreed in their general description of brilliant, constant lightning, and heavy rain; some also spoke of hail.

" About twenty miles to the south-west of Birmingham the storm began at seven P.M., and ceased about eight P.M.; at that time it had reached Birmingham, where it raged about an hour. I therefore conclude that the cloud was twenty miles long, and that it passed over its own length in an hour; or, in other words, that it only moved at the rate of twenty miles an hour, carrying the whirlwind along with it; for that the whirlwind was caused by the cloud, and had no connexion with the tranquil current of air which bore it, was evident.

"Where this immense cloud was formed, and charged with materials for such prodigious torrents of rain, and by what means that immense weight was supported, are problems for the meteorologist. Supposing that the rain fell as heavily in other places as in Birmingham—and every account makes the supposition probable—every square superficial foot of the under surface of the cloud must have given out two inches of water per hour; and calculating six gallons to the cubic foot, every superficial square foot must have deposited one gallon per hour during its course; and if that course only lasted six hours, each superficial foot must have given out 50lbs. of water in that time.

"Whether the cloud actually carried this weight of 50lb. on each square foot, and if it did, what was the sustaining power which enabled it to do so, would not be easily told; but another question suggests itself, was all, or the greater part of the water deposited during its course, generated from time to time in the cloud? One thing seems clear, constant lightning accompanied the release of the water from its aerial carriage, and a whirlwind seems to have been necessary to supply the consumption of atmospheric air.

"The great accumulation of heat for two or three days before such violent thunder-storms, seems to indicate that they are preceded by a cessation of some ordinary heat-absorbing atmospheric processes over the places where the storm afterwards passes. In the case of this storm Mr. Onion says, that the barometer had been in a very unsettled state for some days previously, often in the course of twenty-four hours rising or falling to a considerable amount, although not followed by a corresponding change in the weather. Perhaps the phenomena of a thunder-storm are due as much to the state of the atmosphere in each locality, as to the ominous black cloud which passes over it at the time of their appearance.

"A few days after the storm of the 6th of July, I had an opportunity of observing some of the effects of the whirlwind which accompanied the cloud, and thus tracing a portion of its course. It was at Hilhampton, adjoining Whitley Court, the residence of her Majesty the Queen Dowager; which lies on the direct line between Hereford and Birmingham—several large, full-grown elms, standing in the middle of a field, were torn up by the roots; other trees were stripped of their branches at one side only, while the stem and remaining branches had not been touched; a low brick wall, not a foot high, which supported some paling, was torn up. It did not proceed in a straight line, but in a zig-zag or curve, running in the direction of the course of the cloud; and what is a little remarkable, the dwelling-house, which was not injured, lay in one of the bends of the curve; a large fir tree in the front had all the branches on one side twisted off, and a walnut tree immediately behind the house suffered in the same way; neither tree was ten feet from the house which was between them, and it was in its course round the house that the low brick wall was torn up; it then passed amongst some hay-cocks, which it carried off and scattered."

Colonel Harry D. Jones gave the following account of recent excavations which he made in the Round Towers of Clonmacnoise:

"As some time must necessarily have elapsed before the vegetable material in the large tower could be removed to the level of the lower floor, it was determined to employ another party in sinking below the foundations of the smaller tower, called Teampull; the ground in the interior was level with the sill of the door and with the ground outside.

" Upon sinking nine inches, vegetable mould only was found, then sand mixed with small stones and mould, which became more compact as the excavation proceeded. At the depth of three feet nine inches the workmen discovered the ribs of a skeleton; they were then directed to proceed very carefully in their operation: it should be observed that the pieces of bone taken up would scarcely bear to be handled, falling into small fragments; upon clearing the earth away, the skull of a human body was found in a perfect state, firmly imbedded in the earth and gravel. Upon removing the earth to bare the skeleton, a second was found, one foot above that first discovered. Every endeavour was then made to lay bare this second skeleton, which was to a great extent effected, but no skull could be found; the legs were buried considerably under the foundation-walls of the tower; the bones found were very perfect until we attempted to raise them, when they were found to be very brittle, falling to pieces with the least pressure or strain. Having removed the upper skeleton, the intervening layer of earth was carefully raised, and thus the second or lower skeleton, with the exception of the ribs, which had been disturbed upon first finding it, upon being laid bare, were found as perfect as the day the body was placed in the grave; the bones of one leg were taken out uninjured, which measured one foot two inches, but not so the skull, which broke into several pieces upon the attempt to raise it from the soil, which was very firmly attached to it; the formation of the head appeared to be remarkable; the fragments have been carefully preserved. Having removed the bones of the lower skeleton, the excavation was then proceeded with, until the natural ground was found, which was coarse limestone gravel, mixed with boulder-stones of a middling size, rendering the progress of the work very difficult, and evidently shewing that it had never been disturbed. The lower of the two skeletons was laid upon the surface of the limestone gravel, apparently in a naked state, as there was not the slightest appearance of wood or linen, or anything to indicate that the body had been enveloped by a covering of any description.

"From the position of the two bodies, it would appear evident that they had been buried subsequent to the erection of the tower, inasmuch as the lower, and most perfect of the two, and the one on which any reasoning that may be made should be applied, was placed with the head two feet from the interior face of the tower, in consequence of the rubble-work of the foundation projecting that distance within the upright walls of the tower, and the feet were inserted under the foundation on the opposite side. The direction of the body was W. N. W. and E. S. E., the head at the west, and which appears to be the general direction of the graves in the adjoining yard.

"Upon commencing operations, it became necessary to remove the great quantity of vegetable matter, which was composed of pieces of wood, twigs, &c., evidently the debris of bird's nests, mixed with stones thrown in by idle persons.

"Upon entering the large tower by the doorway, which is eleven feet six inches above the upper footing-course of the foundation, the interior was found to be filled with decayed vegetable matter, bones of birds, sheep, pigs, &c., with a few human bones, all intermixed to the height of five feet above the level of the footing-course, or, as was subsequently ascertained, above the level of the lower floor of the tower. The first thing to be accomplished was the removal of such a large mass of material; this was done by fixing staging of planks across the interior, and hoisting it up and discharging it outside; this was a long and tedious operation; when accomplished, it was ascertained that the dressed stone was eleven feet six inches below the sill of the doorway, corresponding with the height measured outside. Upon sinking eight feet, the vegetable mould, &c., was found lying upon a rubble stone paving of about one foot six inches in thickness. The material next to be removed was gravel and sand mixed, about nine inches; then yellow clay, three inches; and lastly, four feet six inches gravel mixed with boulder-stones of moderate size, and evidently, by the seams of fine sand, shewing that the excavation was then in the natural ground; and as one of the workmen observed, when throwing it out, 'that has not been moved since the morning of the flood.' The total depth sunk below the sill of the doorway was eighteen feet seven inches, viz.:

1 ft.	7 in.		from sill to under side of the projection of floor,
8 ,,	0 ,,		to the level of bottom floor or commencement of dressed ashlar work,
3 ,,	6 ,,		rubble masonry—foundation of floor and loose stones with earth,
0 ,,	9 ,,		gravelly sand,
0 ,,	3 ,,		yellow clay,
4 ,,	6 ,,		gravel, sand, and boulder stones,
18 ,,	7 ,,		

being seven feet one inch below the ground-floor of the tower. Considering the nature of the materials, and the depth in which the men were working, it appeared conclusive that the ground beneath had never been disturbed, and consequently the object for which the work had been undertaken had been fully and satisfactorily executed, not leaving a doubt upon the minds of any present, that prosecuting the work any longer would be useless waste of time and labour. Mr. Molloy, a respectable farmer, who is seventy years of age, states, that for fifty-six years, that his memory serves him, no excavation similar to the present had been made within that period.

"Mr. Long, C. E., Mr. C. Mayne, and Mr. Molloy, farmer, were present during the entire operation. After having satisfied myself as to the result of the excavation, the material taken out was thrown back into the tower."

Colonel Jones also exhibited rubbings from a rock at Drumlish, of which he read the following account:

"This rock, which it appears has engaged the attention of the curious for many years, is situate in a small pasture-field, in the parish of Killoe, townland of Clernaugh, barony of Granard, county of Longford.

"The field is now in the possession of Thomas M'Cann, who lives on the spot. It is on the west side, and within fifty yards of the high road, and about three quarters of a mile from Drumlish, on the way to Longford. The spot is more particularly shewn in the sketch plan accompanying this paper.

"At the upper corner of the field there is a large irregular patch of the scalp of a rock bare; it is about eight yards in length, and its greatest breadth is about three. The rock is laminated perpendicularly, and appears to be of clay slate, but accidentally discoloured, nearly all of it being deep red (the country people call it the Red Flag), but there is a small portion of it grey.

"The whole of this surface is more or less occupied with indented marks, all apparently without connexion, arrangement, or method; and perhaps there are more of these marks to be found, if more of the rock was uncovered, for upon each of the three little separated patches (see the plan which accompanies this Number) I find there are similar marks; but yet, in the adjoining lane, where there are two more patches of the rock bare, I found more of these marks.

"The surface being so large, I made transcripts of portions only, which accompany this notice. These were made by laying the paper on the rock, and rubbing it with heel ball; and as this gave a confused appearance from the roughness of the rock, I used a tint afterwards to make the spaces more solid, and thus the indentions appear white; and as this tinting was not done upon the spot, perhaps there may be some slight deviations from the original.

"I have shewn upon one of the papers the character of these marks; they are all angular; there are none of them sunk or cut square; they are deeply indented in the middle, and decrease in depth to the surface; at the ends they appear, therefore, like marks or scratches made with a nail or some pointed instrument, when the rock was in a soft state; for instance, like a gash made in the dough of a loaf, and when baked, this gash would become an angular furrow.

"The people of the neighbourhood do not speak Irish, and that renders it difficult to obtain from them any thing like satisfactory evidence upon the subject, but M'Cann says that many people have come (even from England) to examine this rock: he also recollects a preacher, named M'Quig, who examined it between twenty and thirty years ago, and who said he could understand parts of it, and that an account of it would be found in O'Halloran's History. He also says that it is mentioned in a book published about seven years ago—a book about which there were many law-suits. (This must be Lewis's Topographical Dictionary.)*

"The people have a saying, that great troubles are to come, and that the finishing battle is to be fought in the adjoining valley, and the ratification or settlement will be signed upon this rock."

DONATIONS.

Two ancient Iron Swords and a Spear Head, found near Kilmainham. Presented by the Directors of the Great Southern and Western Railroad.

Rubbings from an inscribed Stone, with Characters, supposed to have been Ogham, at Drumlish, near Granard. [Engraved to accompany the foregoing description.] *Also a Lithograph of a ruined Temple in Malta.* Presented by Colonel H. D. Jones, C. E.

* It is not mentioned under *Drumlish*, in Lewis's Typogr. Dictionary.

December 8, 1845.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

The President made a communication on some new applications of Quaternions.—See *Appendix*, No. V.

DONATIONS.

Registrum Prioratus Omnium Sanctorum juxta Dublin. Edited, for the Irish Archæological Society, by the Rev. Richard Butler. Presented by the Society.

Transactions of the Institution of Civil Engineers of Ireland. Vol. I. Presented by the Secretary, Thomas Oldham, Esq.

Memoires de l'Academie Imperiale des Sciences de St. Petersbourg. 6me serie, viz.: *Science Naturelles.* Tome IVme, 6me Livraison. *Divers Savans.* Tome IVme, 6me Livraison. *Recueil des Actes de la Seance Publique de l'Academie Imperiale des Sciences de St. Petersbourg; tenue le* 29 *Dec.* 1844; *et Science Politique.* Tome V., Livr. 5. Presented by the Academy.

Memorie de la Reale Academia delle Scienze di Torino. Tome XXXIX. (1836). Presented by the Academy.

Proceedings of the Geological Society of London. Vol. IV. Part 3, No. 104. Presented by the Society.

Flora Batava. Nos. 137, 138. Presented by the King of Holland.

Several Iron Swords, and other Weapons, found near Kilmainham. Presented by the Directors of the Great Southern and Western Railroad.

A large Collection of Iron Swords, Bosses of Shields, Spears, and other Weapons, some Fibulæ made of Bronze and Silver, all found in the Cuttings of the Great Southern and Western Railroad, near Kilmainham. Presented by Colonel Napier, Deputy Adjutant-General.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846. No. 52.

January 12, 1846.

GEORGE PETRIE, Esq, Vice-President, in the Chair.

The Earl of Enniskillen, William Lloyd, M.D., The Ven. Henry Cotton, D.C.L., Archdeacon of Cashel, Rev. John Connell, John C. Deane, Rickard Deasy, Conyngham Ellis, Arthur Nugent, Edward King Tenison, John Tyrrell, and T. Jolliffe Tuffnell, Esquires, were elected Members of the Academy.

Resolved,—"That the recommendation of the Council, to place £50 11*s*. at their disposal, for the purchase of a convoluted Gold Bracelet, be adopted."

The Rev. Professor Graves read a paper on the discussion of Algebraic Curves and Surfaces, with special references to the theory of Curves of the third degree.

The general equation of an algebraic curve or surface of the n^{th} degree being written in the form

$$U_n + U_{n-1} + U_{n-2} + \ldots\ldots + U_2 + U_1 + U_0 = 0, \qquad (1)$$

where U_n stands for a homogeneous function of the n^{th} degree of the rectilinear coordinates, any equation such as

$$A_m U_m + A_{m-1} U_{m-1} + A_{m-2} U_{m-2} + \ldots + A_2 U_2 + A_1 U_1 + A_0 U_0 = 0 \; (2)$$

in which m is not greater than n, and $A_0, A_1, A_2, \ldots A_m$ are constants, represents a curve or surface derived from the former by means of the following geometrical construction.

Draw from the origin O any right line meeting the surface (1) in the n points $N_1, N_2, N_3, \ldots\ldots N_n$; and assume on it m points $M_1, M_2, M_3, \ldots\ldots M_m$, so as to satisfy the m conditions

$$A_0 \Sigma\left(\frac{1}{OM_1}\right) = A_1 \Sigma\left(\frac{1}{ON_1}\right)$$

$$A_0 \Sigma\left(\frac{1}{OM_1 . OM_2}\right) = A_2 \Sigma\left(\frac{1}{ON_1 . ON_2}\right)$$

$$A_0 \Sigma\left(\frac{1}{OM_1 . OM_2 . OM_3}\right) = A_3 \Sigma\left(\frac{1}{ON_1 . ON_2 . ON_3}\right)$$

$$- - - - - - - - - = - - - - - - - - - - -$$

$$A_0 \left(\frac{1}{OM_1 . OM_2 . OM_3 \ldots\ldots OM_m}\right) = A_m \Sigma\left(\frac{1}{ON_1 . ON_2 . ON_3 \ldots\ldots ON_m}\right):$$

Then the points $M_1, M_2, M_3, \ldots\ldots M_m$ will lie on the surface (2).

If we suppose now that the coefficients $A_0, A_1, A_2, \ldots A_m$ are formed according to any assumed law; for instance, if they are given functions of m and n, we shall have, as m takes different integer values from 1 up to n, a series of curves or surfaces, derived from the original one, and related in a particular manner to it and to each other.

By making the coefficients $A_0, A_1, A_2, \ldots A_m$, all equal to unity, we form a series of curves or surfaces most easily derived from any given one; and the consideration of them suggests some interesting results.

The problem of drawing a tangent geometrically at a given point on a curve of the third degree has been elegantly solved by M. Poncelet; but we are now in a condition to solve it generally for any algebraic curve.

Having drawn any right line from the given point O, meeting the curve in $n-1$ other points $N_1\, N_2\, N_3 \ldots\ldots N_{n-1}$, let us assume on it a point M such that

$$\frac{1}{OM} = \Sigma\left(\frac{1}{ON_1}\right), \tag{3}$$

the locus of the point M will be a conic having a three point osculation with the curve at O. The tangent and osculating circle of this conic belong therefore likewise to the given curve. So also, in the case of a point O given on any algebraic surface, the surface of the second order, which is the locus of a point M determined in the same way, will have its lines of greatest and least curvature coincident with those of the given surface at O. All this is obvious, since if

$$U_n + U_{n-1} + \ldots + U_2 + U_1 = 0 \qquad (4)$$

be the equation of the given curve or surface, referred to axes passing through the given point O,

$$U_2 + U_1 = 0 \qquad (5)$$

will be the equation of the curve or surface of the second order, constructed in the manner described above.

Let us suppose $n = 3$, and (4) to be the equation of a plane curve of the third degree; its intersections with the conic (5)* determine the three right lines represented by the equation

$$U_3 = 0$$

which are obviously parallel to the asymptots. The directions of the asymptots being thus ascertained, we may readily determine their actual position. For this purpose draw tangents to the curve of the third order at the extremities of any one of the chords common to it and the conic; they will meet the curve in two points; and the line joining these points will cut the curve in the point through which the asymptot parallel to that chord passes.

Of course the results here stated are subject to modifications when O is a singular point.

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* As we are accustomed to regard the curve as generated by the motion of the tangent, whose equation is $U_1 = 0$, it seems natural to extend this conception, and to call the conic $U_2 + U_1 = 0$ *the generating conic:* the curve $U_3 + U_2 + U_1 = 0$ *the generating curve of the third degree,* and so on.

If we draw from o two right lines parallel to the asymptots of the generating conic, in virtue of the equation (3), they will each meet the curve of the third degree in two points equidistant from o. It follows, therefore, that the two rectilinear diameters of the curve, conjugate to these right lines, must pass through o. Hence we know how to construct these diameters. Again, as there are plainly but two diameters passing through o, the curve enveloped by all the diameters of the curve must be of the *second class*, that is to say, it must be a conic section.

If o be a conjugate point, the conic and its asymptots are imaginary; consequently o must lie within the envelope.

If o be a cusp, the conic degenerates into two coincident right lines. The diameters of the curve conjugate to them must, therefore, be coincident likewise. Hence o lies on the envelope.

If o be a double point, the conic is replaced by two intersecting right lines; and it is easy to see that, as we can draw from o two diameters of the curve, conjugate to the two systems of respectively parallel chords, o must lie outside the envelope.

The preceding theorems, relative to the envelope of the rectilinear diameters of a curve of the third order, and to the positions of singular points of the curve with respect to it, are due to Professor Plücker of Bonn, who obtained them by analytical methods. They are here presented as consequences, derived by purely geometrical considerations from the properties of the generating conic; in order to illustrate the importance of the place which it holds in the theory of curves of the third order.

Perhaps the most interesting class of *derived* curves or surfaces is that of the *Successive Polars* of a given curve or surface of the n^{th} order. The geometric mode of generating them is as follows:

Draw from any point o a right line meeting the surface in n points, $N_1, N_2, N_3 \ldots\ldots N_n$, and assume on it m points,

$M_1, M_2, M_3 \ldots\ldots M_m$, such that

$$\Sigma\left(\frac{1}{OM_1}\right)=\frac{m}{n}\Sigma\left(\frac{1}{ON_1}\right)$$

$$\Sigma\left(\frac{1}{OM_1.OM_2}\right)=\frac{m(m-1)}{n(n-1)}\Sigma\left(\frac{1}{ON_1.ON_2}\right),$$

$$\text{- - - - - - - = - - - - - - - - - - -}$$

$$\frac{1}{OM_1.OM_2\ldots\ldots OM_m}=\frac{m(m-1)\ldots\ldots 2.1}{n(n-1)\ldots(n-m+1)}\Sigma\left(\frac{1}{ON_1.ON_2\ldots\ldots ON_m}\right);$$

the curve or surface which is the locus of the points M_1, M_2, $M_3 \ldots\ldots M_m$ will be the m^{th} *polar* of the point O with relation to the given curve or surface. It is plain that, so far as regards this method of geometrical generation, the polars are *successive:* that is to say, the $(m+1)^{th}$ polar is derived from the m^{th} in the same manner as the m^{th} from the $(m-1)^{th}$. In every case the products of the distances OM_1, $OM_2 \ldots OM_m$, taken r by r, have the same harmonic mean as the products of ON_1, $ON_2 \ldots ON_n$, also taken r by r: and this for all integer values of r from unity to m inclusive.

It follows from what has been already said, that if

$$U_n + U_{n-1} + \ldots + U_2 + U_1 + U_0 = 0$$

be the equation of the given curve or surface, referred to rectilinear axes passing through O,

$$\frac{m(m-1)\ldots 2.1}{n(n-1)\ldots(n-m+1)}U_m + \frac{m(m-1)\ldots 2}{n(n-1)\ldots(n-m+2)}U_{m-1} +$$

$$\ldots + \frac{m(m-1)}{n(n-1)}U_2 + \frac{m}{n}U + U_1 = 0$$

will be the equation of the m^{th} polar of the point O, with relation to the given curve or surface. By making m, in this formula, successively equal to the integer numbers from 1 up to $n-1$ inclusive, we obtain the equations of all the *successive polars* of the origin. For instance, the equation of the first polar is

$$U_1 + nU_0 = 0$$

which involves in it the celebrated theorem of Cotes, so successfully used by Maclaurin. And the equation of the $(n-1)^{th}$ polar is

$$U_{n-1} + 2U_{n-2} + 3U_{n-3} + \ldots + (n-1)U_1 + nU_0 = 0,$$

an equation which we recognize as belonging to the curve or surface of the $(n-1)^{th}$ order, which passes through the points of contact of all the tangents drawn from the origin to the given curve or surface. It is by this geometrical property that M. Bobillier, who first directed attention to these successive polars,* has chosen to characterise them.

By substituting $\frac{1}{x}, \frac{y}{x}, \frac{z}{x}$, for x, y, z, in the general equation of the surface, and in the equations of its polars, Professor Graves shews that, when the point O recedes to an infinite distance, the whole series of successive polars become diametral lines or surfaces of the different orders belonging to the given curve or surface. This had, in fact, been observed, in the case of the first polar, by M. Poncelet, who has shewn that the theorem of Cotes is an extension of Newton's proposition relative to the rectilinear diameters of plane curves.†

This theory of polars enables us to give a geometrical construction of the problem, "From a given point in its plane to draw all the possible tangents to a curve of the third order."

We have only to construct the second polar of the given point, which will be a conic section, and its intersection with the curve will give the points of contact. Here we see the advantage of adopting the geometrical definition of polars employed in this paper.

In connexion with the present subject, Professor Graves announced an extension of a theorem deduced by Maclaurin

* See a Series of Articles contained in the eighteenth and nineteenth volumes of Gergonne's *Annales de Mathématique.*

† See Chasles' *Histoire de Geometrie*, p. 147.

from the theorem of Cotes. Maclaurin's theorem is: "If any transversal be drawn through a fixed point O, in the plane of a curve of the n^{th} order, so as to meet the curve in n points, the tangents drawn at these points will cut off upon any fixed right line passing through O, segments OT_1, $OT_2 \ldots OT_n$, the sum of whose reciprocals is constant."

The following is a more general theorem for plane curves: "*If any two transversals be drawn through a fixed point* O, *in the plane of a curve of the n^{th} order, so that one meets the curve in* n *points* M_1, $M_2 \ldots M_n$, *and the other in* n *points* M'_1, $M'_2, \ldots M'_n$; *the right lines* $M_1M'_1$, $M_2M'_2, \ldots M_nM'_n$, *will cut off, upon any fixed right line passing through* O, *segments* OT_1, $OT_2, \ldots OT_n$, *the sum of whose reciprocals is constant.*"

And we have an analogous one for surfaces: "*If any three transversals be drawn through a fixed point* O, *the first meeting a surface of the n^{th} order in* n *points*, M_1, $M_2, \ldots M_n$; *the second meeting it in* M'_1, $M'_2, \ldots M'_n$; *the third in* M''_1, $M''_2, \ldots M_n''$; *the planes* $M_1M'_1M''_2$, $M_2M'_2M''_2, \ldots M_nM'_nM''_n$ *will cut off, upon any fixed right line passing through* O, *segments, the sum of whose reciprocals is constant.*

The Rev. Dr. Drummond read a paper on the authorship of the poem entitled "The Exile of Erin."

This subject was taken up in consequence of a provincial newspaper having been sent to Sir William Hamilton, accompanied by a letter setting forth the claims of George Nugent Reynolds, Esq., to the authorship of that poem, and requesting that the matter might be brought under the consideration of the Royal Irish Academy. This task was at first declined, as unworthy of serious attention; but the claims of Reynolds continuing to be urged in several publications, and Thomas Campbell, the reputed author, represented as a plagiarist, Dr. Drummond, lest the silence of the Academy should be construed into an admission of the validity of Reynolds' claims, thought it an act of justice to both parties, to subject those claims to a strict scrutiny.

It is admitted by the friends of Reynolds, that Campbell's account of the poem has been unvarying; that he wrote it in Altona in 1801; that it soon became universally known as his; that it was published in various editions of his poems, and his right to it never questioned till nearly thirty years after its first appearance. He was then accused of having "abstracted" it from the library of the Marquis of Buckingham, though there was no Marquis of that title. In a letter addressed to the Editor of *The Times*, he indignantly repelled the charge as a calumny; and affirmed, that never in his life had he access to any papers of either Marquis or Duke of Buckingham; that he wrote the song in Altona, and sent it off immediately from thence to London, where it was published by his friend Mr. Perry in the *Morning Chronicle.* This statement of Campbell's was in perfect accordance with the account of the origin of the poem communicated to Dr. Drummond in Edinburgh, in the winter of 1811, by Dr. Robert Anderson, who had been Campbell's particular friend, viz., that it was written in Altona, in consequence of Campbell's having met with some expatriated Irishmen in that city, for whose misfortunes he felt a deep sympathy. This has been still farther corroborated by George Petrie, Esq., who affirms that he heard the same statement from certain of those very exiles whom he named, and who were well known in Dublin prior to their banishment. Campbell had spent the evening in their company, and their conversation having naturally turned on their ruined hopes and unfortunate country, he was greatly moved, and on retiring gave vent to his feelings in the song of the Exile of Erin. The following morning he gave them a copy of it, and by them it was speedily transmitted to Ireland. It is possible that one of those copies, or a transcript of one of them, may have fallen into the hands of Reynolds, that he spoke of it to his friends, and, as he was known to have had some propensity to rhyming, that their partiality led them to imagine that he was the author, though it does not appear that Reynolds himself ever made any such assertion. Some of his particular friends and near relatives, in lack of more conclusive evidence, swore before a justice of the peace in Dublin, that it was their belief that Reynolds was the real author; that the song was well known in their circle in the year 1799; that one of them, a lady, had given away a hundred copies of it among her friends, and that it had been noticed as the composition of Reynolds in the newspapers of the day. None of these newspapers, however, had been produced or named; and as to the belief or conviction of any individual, however respectable and trustworthy, it availed nothing in the determination of a question of this kind. It was stated in favour of Reynolds, that a wandering harper, called M'Cluskey, had said that he had learned the song in the Irish Harp Society of Belfast in 1799, and that G. N. Reynolds was the author. This statement was treated as a fabrication, for the following reasons. The principal founders and supporters of the Irish Harp Society were Henry Joy, Esq., of Belfast, and Edward Bunting, the well-known collector of ancient Irish melodies, of which he published a volume or "General Collection," in 1809. Mr. Joy wrote the learned and elaborate History of Irish Music which is prefixed to that volume. English songs appropriate to the airs were collected from various sources. Among them were three from Thomas Campbell, one of which is "the Exile of Erin." Had that song been ascribed to any claimant but Campbell, who gave it to Bunting as his own, Messrs. Joy and Bunting must have known it; and it is not for a moment to be imagined that either of those honourable men would have published the song as Campbell's, had there been but a breath of suspicion that he was not *bonâ fide* the real author; and as little is it to be imagined that Campbell, with the consciousness of its not being his own, would have suffered it to be sent forth with his name, in a volume to which public attention had long been carefully solicited, and which was about to pass into the hands of all readers of poetry, and all admirers of Irish music.

What, it was asked, were the literary pretensions of Reynolds, that he should be considered as the author of a lyric poem, to which he had written nothing equal? As to Campbell, no one acquainted with his poetry could doubt his competency to the task. Moreover, the style, the sentiments, the versification, were all in perfect harmony with the unquestioned productions of Campbell's muse, and particularly with the "Lines written on a Visit to Ayrshire." The warm friendly feelings expressed by Campbell for Ireland, entitle him to the gratitude of Irishmen; and it would ill become any native of our country to pluck a single leaf from the chaplet of fair renown that encircles the brow of Campbell.

> "Non ego illi detrahere ausim
> Hærentem capiti multa cum laude coronam."
>
> *Hor. Sat.* x. 48.

W. R. Wilde, Esq., read the following Memoir of the Dublin Philosophical Society of 1683:

"The year 1683 is memorable in the annals of scientific literature in Ireland for the formation of the Dublin Philosophical Society, the great prototype of all our existing learned bodies, but in particular of the Royal Irish Academy. It was commenced in October in that year by William Molyneaux, 'the friend of Locke,' and the distinguished mathematician and astronomer, who was the first secretary of this society.

"As there is no detailed account of this body in print, and as the notices of it which have as yet appeared are always exceedingly brief, and frequently incorrect, I have for some years past endeavoured to collect as much of its history and proceedings as the scanty records scattered through works and libraries afford. With these materials—with the manuscripts and correspondence of both William and Thomas Molyneaux placed in my hands by Sir Henry Marsh—from a careful examination of the documents belonging to it in the Manuscript Library of our University—and from the Minute Book still preserved in the British Museum, which has been accurately noted for the purpose—I have made, through these and other sources, some more memoranda of the history of the Philosophical Society than the usual accounts afford, and these I beg leave to offer to the Academy.

"In the manuscript correspondence of the Molyneaux's just alluded to, we find in a letter from William to his brother Thomas, then in Leyden, and dated 30th October, 1683, N. S., the following:—'I have also here promoted the rudiments of a society, for which I have drawn up rules, and called it *Conventio Philosophica.* About half a score or a dozen of us have met about twelve or fifteen times, and we have very regular discourses concerning philosophical, medical, and mathematical matters. Our convention is regulated by one chief, who is chosen by the votes of the rest, and is called *Arbiter Conventionis*, at present Dr. Willoughby (the name 'President' being yet a little too great for us). What this may come to I know not; but we have hopes of bringing it to a more settled society. The event you shall know. Sir W. Petty and all the virtuosi of this place favour it much, and have at some times given us their company.'

"From this it would appear that Dr. Willoughby was virtually, though not in name, the first President, and W. Molyneaux the original Secretary, although the former honour has been generally conferred on Sir William Petty, who, however, was not elected till the 1st of November, 1684.*

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* "At this election Sir W. Petty and Dr. Willoughby had equal marks for President, but upon a second election Sir William carried it by four votes, so he stood. Afterwards we had a handsome dinner at a tavern, so finished the day."—*Molyneaux Correspondence, Dublin University Magazine*, vol. xviii., p. 489.

"The first meeting took place on the 15th of October, 1683, when papers were read by Mr. William Molyneaux, Dr. Narcissus Marsh, afterwards Archbishop of Dublin, Mr. Foley, and Mr. St. George Ashe. It is remarkable, that although Ware, Birch, and Whitelaw have agreed in dating the origin of this society in 1683, Mr. Halliwell has, in a "Collection of Notes on the early History of Science in Ireland," published in the Proceedings of this Academy for 1841, stated that its first meeting took place on the 28th of January, 1684. In the winter of 1683, writes Archbishop Marsh in his Diary, 'was set up the Philosophical Meeting in Dublin, that met and formed itself into a society, in the Provost's lodgings. There, at the first opening of it, as a prelude to what we were to do, I in three or four days' time, composed *An Introductory Discourse to the Doctrine of Sounds*, which was sent to the Society in Oxford, and then printed in the *Philosophical Transactions*.'*

"Not having facilities for publishing their proceedings in Ireland, it appears that they determined upon offering them to the Royal Society; accordingly, on the 18th of December, 1683, the Provost, Dr. Robert Huntingdon, wrote a letter to Dr. Plot, of the Royal Society, giving an account of 'a weekly meeting of several ingenious men about philosophical subjects in Dublin.' This notice, which is recorded in the letter book of the Royal Society (vol. ix, p. 103), informs us that W. Molyneaux, then residing near Ormond's Gate (now Wormwood Gate), and who was at that period engaged in writing an 'Atlas for this Country,' was Secretary:—'And since,' he writes, 'you so generously, as well as charitably, offer your assistance, I think this will be the best method of conveyance, to transmit our notices to the Secretary of the

* The Diary of Archbishop Marsh, from a transcript in Marsh's Library, Dublin, published by the Rev. Dr. Todd in the British Magazine for July and August, 1845.

Royal Society, who, after he has perused them, can send them to Oxford' (where a similar society, under the care of Mr. Musgrave, had just been established), 'as you likewise by him may send hither. After Christmas that we next meet, our Secretary will pursue that course; you smoothing our way at London once again, as it seems you have already done.—After awhile we may perchance ease ourselves of that expense, and have our intelligence for nothing.* However, you may be sure we shall never grudge to defray all manner of charges that shall be incident to our correspondences, and we have raised a fund out of which to do it. By Moses Pit,† if not before, you may expect one or two of their discourses at large: for the way is for particular subjects mentioned in the foregoing meeting to be treated on by particular persons the next, and when they have done, every one that has anything to add or object has his time to express it. I don't give you the names of our society, because you know few of them, except the Bishop of Ferns and Leighlin, Sir William Petty, and Dr. Willoughby, and besides you will receive it more authentic from the Secretary. Several of the number meet on Sunday nights, as the whole company does on Mondays, to discourse theologically, of God, suppose, and His attributes, and how to establish religion and confute atheism by reason, evidence, and demonstration.'

"Having complimented Dr. Plot, and conveyed to him the thanks and acknowledgments of this 'young society for the promotion of philosophy, on account of the advantageous correspondence offered to it by the Royal Society,' he encloses him an account of some previous meetings tending to its 'better regulation, settlement, and future transactions,' and also the Minutes for October 15, 1683.

* The Royal Society charged only half payment to the Members of the Dublin Society, (see Minutes for 4th July, 1685).

† Moses Pit, a celebrated London bookseller, and publisher of "The Atlas."

" When I first commenced this inquiry some years ago, I was under the impression that the Transactions of this society were still in existence, and would one day or another be discovered, and acknowledged by some of the public libraries or private collections in these kingdoms. I have since, however, convinced myself of the contrary being the fact, and feel assured that no manuscript volume of the Transactions of the Dublin Philosophical Society is, or perhaps ever was, in being.

" The Minute Book of this Society, from 1683 to November, 1686, with its revival in 1693, and again in 1707, is still preserved in the British Museum (Addit. MSS., 4811). In the Manuscript collection in the Library of Trinity College we find among some scattered papers lately collected by the Rev. Dr. Todd, rough drafts of the minutes of the Dublin Philosophical Society, in the handwriting of William Molyneaux, from January the 28th to June the 9th, 1684, all of which accord with the notices of this body still existing in the papers of the Royal Society. On the first of these dates we find the officers for that year were appointed, and the 'obligation subscribed.' At that time there was no President (as already stated in the Molyneaux correspondence); Dr. Willoughby was appointed Director, and William Molyneaux Secretary and Treasurer. The members present were Dr. Narcissus Marsh, Sir William Petty, and Messrs. Bulkeley, Cuff, Foley, Baynard, Ash, Mullen, Follet, Baggot, and Mr. Keogh, who was represented by proxy. At this meeting Sir William Petty read a paper on Concentric Circles.

" On the 18th of February the Minutes closed with this notice :—' Nicholas Hudson, our operator, attended on us.' (MS., T.C.D., Cl. I. Tab. 4, No. 18, p. 11).

" The unpublished Letter Books of the Royal Society, and Birch's History of that body, likewise contain the Minutes of the Dublin Philosophical Society from its first meeting on the

15th of October, 1683, to the end of 1687, after which we have not been able to discover any record of its proceedings from these sources.

"The principal papers read to this body, all of which are enumerated in the Minutes, were either printed in the Philosophical Transactions, or formed the material for distinct works or monographs, which were published by their respective authors, and many of the communications were delivered in form of *vivâ voce* discourses at one sitting, and debated at the next.

"There are two manuscript volumes of county histories in the library of the University of Dublin (from which the History of West Connaught is now about to be printed by the Irish Archæological Society), which have generally been supposed to have formed part of the Transactions of the Philosophical Society; but as some of the papers in these are dated in 1682, prior to the creation of that body, and as we have no notice or allusion made to any of them in the Minutes of the Society, which are in every other respect so full and explicit, we feel assured that they were written and intended for the general survey of Ireland under Sir William Petty.

"Dr. Plot was desired to acquaint the Provost of Trinity College that the Royal Society very willingly embraced the correspondence of the Society in Dublin, and had ordered their secretary to write to them in the manner proposed; accordingly, Mr. Aston wrote to Mr. Molyneaux to that effect, a letter, dated the 26th of February, 1684, which is inserted in the unpublished Letter Book of the Royal Society, (vol. ix. p. 111).

"This courtesy of the Royal Society is alluded to in one of the letters of William Molyneaux to his brother Thomas, then residing in Holland, a portion of which I extract from the interesting correspondence of those gentlemen, which I published some years ago in the University Magazine.—'I know,' says William Molyneaux, 'you would willingly hear what has become of our meeting here in Dublin, of which take this following account. Since my last to you concerning this particular, we have constantly every Monday had a meeting, at which one or other would produce discourses no ways contemptible, till about a week before Christmas, we received a letter from Dr. Plot, directed particularly to the Provost, Dr. Huntingdon, but designed in general for us all, in which he takes notice of our design here on foot, for Dr. Huntingdon had formally given him an account thereof, and encourages us to go on vigorously therewith, promising us all the assistance we can desire, as, likewise, the favourable countenance and encouragement of the Royal Society, as also of such another philosophical meeting as our own, begun within these three months at Oxford: assuring us also of the constant correspondence of them, and that we may at any time command whatever we may please to hear communicated from them. This encouragement from so great a man, as he is secretary both to the Royal and Oxford Societies, made us think upon modelling ourselves into better form; and, accordingly, the Bishop of Ferns, Sir Wm. Petty, Dr. Willoughby, and I, were pitched upon to draw up rules, to be presented to the consideration of the rest after the holidays; so that yesterday (Jan. 7, 1684) our rules were presented, and are ordered to be read at three several meetings before they pass. The rules are much the same as those of the Royal Society, and we have entrance money, and a weekly contribution, but what it will yet come to, God knows.'

" On the 10th of May, William Molyneaux wrote to his brother, then at Leyden, the following notice of the young Society :—' Our Society goes on; we have a fair room in Crow's Nest' (off Dame-street), 'which now belongs to one Wetherel, an apothecary, where we have a fair garden for plants,' where they first met in April of that year. And again, upon the 14th of June we read: 'Our society has built a laboratory by Dr. Mullen's directions, in the same house where we have taken a large room for our meeting, and a small repository.'

"Subsequent to the general meeting in November, 1684, a list of the members of the Philosophical Society was forwarded to Mr. Aston, to which I have added the names of some seven or eight others, who, either prior or subsequent to the publication of this list, were, I have positive assurance, connected with this society, prior to 1688.

President, Sir William Petty, Knt., M.D.
Director, Charles Willoughby, M.D.
Treasurer, William Pleydall, Esq.
Secretary, William Molyneaux, Esq.

MEMBERS.

Narcissus Marsh, Bishop of Leighlin and Ferns.
William Lord Viscount Mountjoy.
Robert Huntingdon, D.D., Provost of Trinity College.
John Worth, D.D., Dean of St. Patrick's.
John Baynard, A.M., Archdeacon of Connor.
Sir Robert Redding, Bart.
Sir Cyril Wyche, Knt., P.R.S.
Richard Bulkeley, F.T.C.D., afterwards Knt. and Bart.
Patk. Dun, M.D., afterwards Knt.
Henry Fenerly, Esq.
J. Finglass, M.A.
Samuel Foley, F.T.C.D., afterwards Bishop of Down and Connor.
Daniel Houlaghan, M.D.
John Keogh, M.A.
Dudley Loftus, afterwards Judge of the Prerogative Court.
George Tollet, Professor of Mathematics.
—— Patterson, Surgeon.
John Maden, M.D.*
Allen Mullen, M.D.

* The name of Madden (or Maden, as it is written in the Minutes of the Philosophical Society) is intimately connected with the rise of science, literature, and medicine in this country. The John Maden, M.D., here alluded to, was son of Thomas of Maddenton, and died in 1703. His family were connected with, and he himself was the intimate friend of the Molyneaux. His son, the Rev. Samuel Madden, commonly called "Premium Madden," was the founder of the Royal Dublin Society in 1731. See "The Tribes and Customs of Hy-Many," by John O'Donovan, printed for the Irish Archæological Society.

William King, Sch. T.C.D., afterwards Archbishop of Dublin.
Richard Acton, B.D., F.T.C.D.
St. George Ashe, F.T.C.D., afterwards Bishop of Cloyne.
Mark Baggot, Esq.
John Bulkeley, Esq.
Paul Chamberlain, M.D.
Robert Clements, Esq.
Francis Cuff, Esq.
Christopher Dominick, M.D.
William Palliser, F.T.C.D., Professor of Divinity, and afterwards Archbishop of Cashel.
Edward Smith, F.T.C.D., Professor of Mathematics, and afterwards Bishop of Down and Connor.
John Stanley, M.A.
Jacobus Silvius, M.D.
—— Walkington, Esq.*
Sir Paul Ricaut, ——

Corresponding Member—Doctor, afterwards Sir Thomas Molyneaux, Bart.

"These men formed the *stellæ majores* of Irish literature and science at this period; and nearly every one of those of whom we have any subsequent account attained to considerable eminence either here or in England.

"At this time Sir William Petty designed to remodel the society, and drew up a code of laws for its future regulation and government, which were deemed worthy of being referred to the Council of the Royal Society, to see how far they might be useful to that body. We here find from authentic documents that some of the principal men of learning and science at that time in Great Britain, and even on the Continent, looked with a favourable eye on our Philosophical Society, and addressed to it, through its Secretary, several letters and papers upon scientific subjects, some of which are still preserved in the original Minute Book in the British Museum, and abstracts of which are to be found in the records of the Royal Society. At the end of the first year we find its progress thus recorded by William Molyneaux. 'Our society

* I have not been able to discover the Christian name of this gentleman, as the name was common in the University at that period. It was probably Samuel Walkington, who was a Scholar in 1680, for Edward Walkington, who was a Fellow in 1676, Archdeacon of Ossory in 1683, and afterwards Bishop of Down and Connor, in 1695, was elected into the Philosophical Society on its revival in 1693.

has been complimented in the philosophical acts, as you will find by the paper Mr. Ashe will send you, wherein for curious subjects (invented by our learned and ingenious Provost) I think we may vie with any Oxford ever had, and truly most of the poems and speeches therein were excellent. Thus, Tom., you see that learning begins to peep out amongst us. The tidings, that our name is in the journals of Amsterdam, was very pleasing to me, and really, without vanity, I think our city and nation may be herein something beholding to us, for I believe the name Dublin has hardly ever before been printed or heard of amongst foreigners on a learned account.' The Minutes of the Oxford Society were likewise regularly transmitted and read at the meetings of the Philosophical Society of Dublin.

" On the 11th of May, 1685, 'Mr. Molyneaux going for England, Mr. Ashe was chosen Secretary; and Mr. Tollet was then nominated Treasurer in Mr. Pleydell's place.' These gentlemen were continued in office at the November meeting of that year, and Lord Mountjoy was elected President. In June, 1686, Mr. Edward Smith* was chosen Secretary, and the other officers of the Society were re-elected at the general meeting, together with the following council:—Sir R. Redding, Sir Paul Ricaut, the Provost, Dr. Willoughby, and Mr. W. Molyneaux. They then adjourned to the 5th of November. The last notice of the Society at this period which we have been able to discover, is in the minute-book of the Royal Society, in which, according to Birch, we read, that on the 13th of July, 1687, 'the minutes of the Dublin Society for several months past were read;' but there is no detail of their

* In the minutes for 21st July, 1684, we read as follows:—" Ordered, That the thanks of this society be returned to Mr. Smith, for the honour he did us in the public act in the College on this *lemma paradoxon vetus Ægyptiacum, quod sol nonnunquam oritur in occidente. Demonstratur de Societate ad promovendam scientiam naturalem Dublinii nuper instituta."*—*Birch's History of the Royal Society*, vol. iv. p. 324.

proceedings given. Whether the Society actually ceased to exist at that period is not precisely known, but Dr. Hutton and other authorities are of opinion that it did not till 1688.* The minute-book in the British Museum has no entry after the 6th November, 1686. For some time, both previously and subsequently to the last note in the minute-book, it would appear from the letters and other communications made by several of its members directly to the Royal Society, that its meetings were few and irregular: even so early as the 10th of August, 1685, we read thus in the Secretary's letter to the Royal Society enclosing the minutes :—' Our company of late has been very thin, and people's heads so much dulled with politics, that next meeting, I believe, we shall adjourn till the term.'

" The unsettled state of this country in 1687 and 1688 caused a complete rupture of all society, public as well as private, and several of the principal members of the Philosophical Society removed from Dublin.

" The subjects entertained by this Society, during the first four years after its establishment, may be considered under the following heads: Mathematics and Physics; Polite Literature; History and Antiquities; and Medical Science, including Anatomy, Zoology, Physiology, and Chemistry. And with some pains we have arranged, under their respective denominations, the following list of the *principal* subjects, together with the names of their authors, as recorded by the Dublin Philosophical Society during the early years of its existence :

" MATHEMATICS AND PHYSICS.

" Mr. W. MOLYNEAUX.—De apparente Magnitudine Solis.—Explanation of the Volution of Concentric Circles.—On Telescopic Sights.—On the viewing of Pictures in Miniature with the Telescope.—Calculations on the Solar Eclipse.—An Essay on Crysta-

* Hutton's Mathematical Dictionary, vol. ii. p. 61.

lography.—Experiments on Hydrostatics.—On the Hydroscope, and the variations of the Barometer.

Dr. T. MOLYNEAUX.—Account of the Astronomer Huygens.

Dr. MULLEN.—Magnetical Experiments (several papers).

Lord MOUNTJOY.—On the Air Gun.

Sir W. PETTY.—Magnetical Observations.—On Weather Registries.—Ship-building.—On the Construction of Carriages.—On Concentric Circles.

Mr. ST. G. ASHE.—Review of De Chasles' Book on Motion.—On the Evidence of Mathematical Demonstration.—On the Solar Eclipse.—On the Weather Registry, T. C. D.—Experiments on Freezing.

Mr. BULKELEY.—On Wind Gauges.—A new Pump for Ships.—The Mechanism of Carriages.

Dr. SMITH.—De Angulo Contactus.

Mr. STANLEY.—Discourse on the Motion of Water.

Mr. TOLLET.—On the Longitude.—On Gunnery.

Mr. WALKINGTON.—Observations on Archimedes.—Observations on Algebra.

Dr. FOLEY.—Objections against Algebraic Calculations.—Computatio Universalis.

Mr. KING.—On the Difference in Size between the horizontal and Meridional Sun.—On the Acceleration of Descending Weights, and the Force of Percussion.—On Hydraulics.—On the Trisection of an Angle.

Dr. NARCISSUS MARSH.—De Radiis Reflectis et Refractis.—Magnetical Observations.

Dr. WILLOUGHBY.—On the Mirage seen at Rhegium in Italy: and on Winds.—On the Lines of Latitude and Longitude.

POLITE LITERATURE, HISTORY, AND ANTIQUITIES.

Archdeacon BAYNARD.—Concerning the Instruction of Youth for the Universities.

Dr. LOFTUS.—Concerning Pere Simon's *Histoire Critique.*

Dr. HUNTINGDON.—On the Obelisks and Pillars of Egypt.

Dr. FOLEY.—On the contagious Communication of a strong Imagination.

Mr. KING—On the Bogs and Loughs of Ireland.

Dr. Mullen—On fifteen cinerary Urns, and Bones found together, at Dontrilegue, County Cork, three feet deep, each covered with a small Stone, and varying in size from a Pottle to a Pint,

Dr. Smith—On cinerary Urns, found in the Caves at Warringstown, and at Loughbrickland, in the County of Down.

MEDICAL SCIENCE,—INCLUDING ANATOMY, ZOOLOGY, PHYSIOLOGY, AND CHEMISTRY, ETC.

Dr. Allen Mullen [or Moulin]—On the human and comparative Anatomy, and the Structure of the Ear (several papers).—Experiments, consisting of injecting Fluids into the Thorax of Animals.—Experiments on the Blood.—On Digestion.—On the Mineral Waters of Chapelizod.—On Poisons.—On Runnet and Coagulum.—On the Organs of Respiration and Circulation, by removing a Portion of a Dog's Lung, &c.—Dissection of a Monstrous Kitten; and a Chicken with two Bills.—Dissection of a Man who died of Consumption.—Observations on the Serum.—On the Peculiarities of the Pulse.—Dissection of Hydatids attached to the Diaphragm.—De Alkali et Acido.—On Ligature of the Jugular Vein in a Dog.—On various Chemical Phenomena.—On Ovarian Disease.—On Ague.—Observations on Scurvy Grass.*

Mr. W. Molyneaux.—On the Phenomenon of Double Vision.—On the petrifying Qualities of Lough Neagh.—Report on the Sirones or Acari.—The Dissection and microscopic Investigation of a Water Newt.—On the Circulation.—On the Pulvis Fulminans.—On the Connaught Worm.

Dr. T. Molyneaux.—On the Anatomy of the Bat.

Lord Mountjoy.—On the Mode of Bleaching in Holland.

Sir W. Petty.—Observations on Consumption.—On the Mode of examining Mineral Waters.

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* Mr. Dalrymple, in his admirable "Anatomy of the Human Eye," in writing of the vascularity of the lens and its capsule, says, that "Haller, in his Description of the Eye, quotes *an Englishman* of the name of Allen Moulin, as the first observer, and in fact the discoverer of these long-denied vessels." Mullen, or Moulin, was, however, an Irishman, and the diseases referred to are published along with his Dissection of the Elephant burned in Dublin in 1681, and entitled "New Anatomical Discourses on the Eyes of Animals."

Mr. St. George Ashe.—On the Fossils and Petrifactions of Londonderry.—On a remarkable Case of Hæmorrhage.*—On Hermaphrodism.—Account of a Man in Galway who suckled his Child, and had Pendulous Mammæ.

Mr. R. Bulkeley.—Experiments on venous and arterial Blood.—Discourse on Mr. Boyle's Book on Human Blood.—On Divers Alkalies and Acids.—On the Dissection of a Bat.

Mr. Patterson.—Various Dissections of the Human Subject.†—On Stone in the Bladder.—On Menstrua for dissolving the human Calculus.—On Cohesion between the Liver and Diaphragm.

Sir R. Redding.—On the Lampreys of the River Barrow.

Dr. Smith.—On the Waters of Lough Neagh.

Dr. Willoughby.—On Hermaphrodism.

Dr. Foley.—Explanation of the Theory of Vision.—Experiments on Vegetation.—On Fossils.

Dr. Houlaghan.—On the Mode of Discovering the Acidity of Liquors.—Description of a Human Kidney weighing forty-two Ounces.—On the Tests for Acids.—On the Dissection of a Monstrous Child with two Heads and three Arms.

Mr. King.—On the Mineral Waters of Clontarf and Edenderry.

Dr. Dun.—On the Analysis of Mineral Waters.

Dr. Narcissus Marsh—On Sounds and Hearing.—On the History and Classification of Insects.

Dr. Silvius.—De Acido et Urinoso.

Mr. Acton.—On the Scoter Duck found at Ireland's Eye.

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* We are not quite certain with regard to the author of this paper. Birch merely says, "Mr. Ashe." The Minutes of the British Museum, however, state that this paper was contributed by Thomas Ashe, Esq. We know not who this gentleman was—if a member of the Philosophical Society he would increase the number to 40.

† Human dissections were very rare in Dublin at that period. Mr. Patterson's communications to the Philosophical Society were founded upon the examination of the body of a malefactor procured by Dr., afterwards Sir P. Dun, to make a skeleton of. Mr. W Molyneaux says he "was constant at the dissection, and nothing curious was done, but only the chirurgeons and physicians that were present spoke at random as the parts presented themselves." This is the first notice of a dissection in Ireland that we have seen recorded. See University Magazine, vol. xviii. p. 479.

"During the remaining years of the seventeenth century, the unsettled state of Ireland precluded the possibility of literary enterprise or scientific investigation. From the following paragraph in the Diary of Archbishop Marsh, it would appear that an attempt was made to revive the Society in 1693; on the 26th of April in which year he writes, 'This evening, at six of the clock, we met at the Provost's lodgings in Trinity College, Dublin, in order to the renewal of our philosophical meeting, where Sir Richard Cox (one of the Justices of the King's Bench), read a geographical description of the *City and County of Derry*, and of the County of *Antrim*, being part of an entire *geographical description* of the whole *kingdom* of *Ireland*, that is designed to be perfected by him; wherein also will be contained a *natural history* of Ireland, containing the most remarkable things therein to be found, that are the products of nature. Upon his reading this essay he was admitted a fellow of this Society, together with Dr. John Vesey, Lord Archbishop of Tuam; Francis Roberts, Esq., younger son to the Earl of Radnor, some time Lord Lieutenant of Ireland: O Lord, grant that in studying thy works, we may also study to promote thy glory, (which is the true end of all our studies), and prosper, oh Lord, our undertaking, for thy name's sake.'* The manuscript volume in the British Museum recommences at this date, and informs us that the members of the old Society who met on this evening, or, as they are styled, 'the members before the warre,' were the Archbishop of Cashel, the Provost, Dr. Willoughby, and Sir Cyril Wyche. At the meeting of the 3rd of May, Mr. Cuff and the Bishop of Cork rejoined the body, and papers were read by Sir R. Cox, describing Judland, and by Sir Cyril Wyche, on Varro's Book, '*De Lingua Latina.*'—Dr. Thomas Molyneaux, Mr. Edward Walkington, and Mr. Bartholomew Van Homrigh,

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* The Diary of Archbishop Marsh, already cited.—British Magazine, for August, 1845.

were proposed at this meeting, and admitted on that of the 10th, when Sir. R. Cox finished his History of Judland; read some papers on Ireland, and on the bringing of the Society into its ancient model, &c. On this evening the Hon. Francis Roberts was elected President; Dr. Charles Willoughby, Secretary, and Francis Cuff, Esq., Treasurer.

"Bound up with this minute book are several copies of a letter, which I judge to be of the same date, and of which the following is a copy:

"'Sir,—The Dublin Society is again revived, and they have ordered me to give you notice of it, and desire me to renew their correspondence with you. We are as yett but very young, and therefor cannot hope to make any suitable returne, but must have a little time to settle, after the disorder the warr has put every thing into here. Mr. Roberts is chosen President, and our Society increases by new elections, so that we may expect it may be considerable, and then there may be something fit to be communicated from,

"'Sir, your most humble Servant,

"'Owen Lloyd,'—F.T.C.D.

"A considerable hiatus occurs after this entry; but it appears that in the year 1707, an attempt was made to re-establish the Society; but its success was not of any long duration, and this MS. contains a register of the philosophical papers read before the Society, from August 5th, 1707, to March 11th, 1708. The Earl of Pembroke, then Lord Lieutenant of Ireland, presided over the Society at this revival. Addit. MSS. 4812.

"The Minutes do not inform us who the members were that attended this re-union, but the following is a list of the papers read during that period:—

The Bishop of Clogher.—A Letter from, to Dr. Molyneaux, concerning an odd Hare's Tooth; afterwards accounted for by Dr. Molyneaux.

Mr. Derham.—On the Spots on the Sun.

The ARCHBISHOP OF DUBLIN.—A Discourse on Measuring Land.—A Letter on a fiery Eruption from the Bowels of a dead Cow.—Thoughts for Improving the Harbour of Dublin.—Scenography of an Engine to force Water out of a Quarry, &c.

Dr. THOMAS MOLYNEAUX.—On Mines and Minerals within the Kingdom of Ireland.—Account of a petrified Honey Comb, and an Essay on Antiquities.

Mr. W. MOLYNEAUX.—On Mercurial Phosphorus.

Dr. ROBINSON.—Concerning the Density of the Atmosphere.

Mr. WARING.—Account of the Occurrences of a Storm.—Letter to Dr. Molyneaux concerning the Cross-Bills.

Mr. E. CROW.—Account of Lightning near Tuam.

Mr. (afterwards Bishop) BERKELEY.—A Discourse on Infinities—An Inquiry whether the Figure of the Earth be spheroid.

Mr. NORMAN.—A Letter on Barnacles.

"These Minutes appear to have been in the possession of Sir H. Sloane, and were by him presented to the British Museum.

"Subsequently the Philosophical Transactions continued to be the medium of communication between the medical profession in Ireland and the public."

Mr. George Yeates presented returns of observations, made by himself, with thermometer, barometer, and rain and wine guages, from January to December, 1845, inclusive.—(See *Appendix*, No. VI.)

DONATIONS.

An Astrolabe, made in Florence in the 16*th Century.* Presented by Sir William Betham.

A Small Bronze Ink-Bottle, found in the Excavation made by the Great Southern and Western Railway Company near Kilmainham. Presented by W. Ogilby, Esq.

Three Bracteate Coins, found in the Round Tower of Kildare, and figured in Mr. Petrie's Essay on the Round Towers, p. 209. Presented by Rev. Mr. Brown of Kildare.

A Beautiful Bronze Dirk, found near Tullamore. Presented by Anthony Molloy, Esq.

January 26, 1846.

GEORGE PETRIE, Esq., Vice-President, in the Chair.

The Rev. Samuel Butcher, F.T.C.D., read the first part of a paper by the Rev. Dr. Hincks, on Hieroglyphics.

This paper commences with a review of the progress made in Egyptian learning, from the first discoveries of Drs. Young and Champollion to the present day. It was alleged, that very little progress had been made since the death of Champollion, the only point established since that event being the principle of peculiar letters and their complements, discovered by Dr. Lepsius. The causes of the want of progress since this discovery were affirmed to be two: 1st, ignorance of another principle, in some measure antagonistic to this, which was extensively applied in Egyptian writing; and, 2nd, an erroneous mode of investigating the phonetic powers of the letters. This consideration was postponed till the second part of the paper: the third part to contain the results, as far as yet known, of an investigation into the powers of the letters, conducted in the manner that would be shewn, in the second part, to be most likely to lead to the truth.

The present part was devoted to the establishment of the new principle above referred to. The principle is this: "the phonoglyphs which compose the proper Egyptian alphabet had *names*, which consisted of *themselves with the addition of certain expletive characters:* and these names might be, and often were, used in place of the single phonoglyphs. If, then, a phonoglyph belonging to the alphabet be followed by the expletive character which appertains to it, that expletive *may be*, and, for the most part, *should be, altogether neglected.*" It was added, that the single characters were occasionally, though not frequently, used for their names, and the name "Ptulmius" occurring so frequently on monuments of the

Greek age, was given as an example; the *two feathers* in this word representing, not, as has been heretofore supposed, an I singly, but IU, the name of that letter; while, on the other hand, in the name "Philipus," the name IU is twice written for the single vowel I.

In order to establish this principle, it was first shewn that it was adopted in transcribing foreign words, when written in Egyptian characters, in the papyri published in fac-simile by the Trustees of the British Museum, and mostly dated in the reign of Rameses the Great, and his grandson. A number of such transcripts were produced. Some of them were shewn not to represent the words that corresponded to them, which were preserved in Hebrew characters in the Old Testament, unless a quantity of, apparently, superfluous characters were removed; such were Ma-ru-ka-bu-ta for both singular and plural of the name of a chariot, Mirkéveth; I-u-ma for Yam, a sea; and Pu-ha-ru-ta for Phrat, the river Euphrates. Others were shewn to be written at times with those, apparently, superfluous characters, and at other times without them, as Astaruta and Astart, the name of the Syrian goddess; K-sh, Kash and Kshi, varieties of the name of a country which we know was Kush, the supplied vowel being *u* and not *a*. It was observed, as an essential point in the proof, that the vowel which was introduced in this seemingly unnecessary manner, was always the same after each letter; some letters, however, take for their expletives ideagraphic signs, which determine their pronunciation, and are thus equivalent to vowels. It was remarked that the letter may, in such cases, have for its expletive either the ideagraphic character, or the letter which it suggests or implies. This is an apparent but not a real exception to the law proposed.

It was shewn, secondly, that this principle was not confined to foreign words, though applied to them more systematically; but that several pure Egyptian words were written with superfluous characters. In order to meet the cavils which it was anticipated would be raised against this position, it was necessary to bring forward words, in which the alleged expletive could not be pretended to be properly a part of the word, there being no room for it either in the place where it was found, nor in any other part of the word, to which, according to the pretended law of transposition of vowels, it might be removed. Such were the instances of Ru-u-i-ha, for Ruha, "evening," the Coptic Ruhe; and Aahu, for Aah, "the moon," which the Greeks have transcribed by the single vowel A. Instances were also adduced, in which an ideagraphic character, or a consonant, appeared as an expletive in a pure Egyptian word; and also, an instance of two homophonous letters, which took different expletives, being interchanged, namely Tu and Ta, as formatives of the past participle, both of which, it was affirmed, should be read without the final vowel.

The principle having been thus established in the age of the papyri, it was shewn, in the third place, that it was not confined to that age, but was recognised in the time of the twelfth dynasty, and even previously thereto. This was shewn by a collation of texts, which were repeated in different steles, or in different parts of the same slab. It was shewn, in a variety of instances, that the same word was written sometimes with, and sometimes without, a vowel; which vowel was, according to the practice of the age of the papyri, the known expletive of the preceding consonant. It was argued that, if a vowel so circumstanced should be rejected as an expletive in the age of the papyri, it should be so also in the early ages to which the monuments now under consideration belonged.

In order to explain the origin of this practice, it was affirmed that all the Egyptian phonoglyphs originally represented syllables; and that, when a limited number of them was selected to represent the initial sounds in the respective syllables, they still retained their old names, as the sounds now appropriated to them could not be uttered alone. The five different modes used for completing the syllabic characters, by the addition of letters, were briefly explained; and it was then stated, that the old syllabic powers or names of the letters of the alphabet, were completed by the addition of another alphabetic character, representing the final sound in the syllable. This additional character is the expletive of the letter, and for it, as has been already noticed, an ideagraph, determining the pronunciation of the syllable, and thus equivalent to the first letter, may be obtained.

The reason why the practice of using expletives was retained, especially in foreign words, was the readiness with which some letters were confounded in the Hieratic texts. These letters had always different expletives, and a distinction was thus established between them, which would not exist if the expletives were omitted. The hieroglyphic texts in which expletives are chiefly found, were stated to be those which were copied from Hieratic, or, as they are called here, hieroglyphic originals.

Mr. Huband Smith read a paper descriptive of an ancient Wayside Cross, situate in the townland of Nevinstown, on the northern bank of the river Blackwater, about two miles from the town of Navan, in the county of Meath. One side bears an inscription; the opposite has a shield, with armorial bearings, *party per pale*, nearly effaced. Beneath the dexter side are the initial letters M. C., and, under the sinister, M. D. The height of the shaft is at present three feet six inches above the slab, in which a socket is cut to receive the tenon upon the lower end of the shaft. This slab stands on a low grassy hillock, the remains, doubtless, of an ascent of three or four stone steps, which, when complete, the cross surmounted.

Mr. Smith exhibited to the Academy a "rubbing," taken from the shaft, which shewed the present state of the inscription on the front, the shield on the back, and an ornamental pattern on each of the sides. He also produced a restoration of the entire, which shewed that the upper part of the shaft had been broken off, and with it the first line of the inscription. Of what remains the first line is illegible, but the rest is tolerably distinct. It is in the black-letter character of the sixteenth century, the letters being beautifully formed; and (filling up the contractions) it runs thus:

" . . . . . Armigeri, et Margaretæ Dexter uxoris ejus ac heredum eorum qui hanc crucem fecerunt anno Domini 1588 quorum animabus propicietur Deus, Amen."

This inscription leaves little doubt that this memorial was one of the Wayside Crosses so generally erected by the piety of individuals about the sixteenth and the preceding centuries, but which the ill-directed zeal of a subsequent period so unsparingly mutilated, and often wholly destroyed. Upon inquiry it proved that a road, leading from Navan to Rathaldron Castle, long the residence of one of the principal branches of the ancient family of the Cusacks, once passed close in front of this cross.

The name of the husband of "Margaret Dexter" Mr. Smith soon after learned from a manuscript in the possession of Mr. Henry T. Cusack. This MS. is written in French, and entitled "An Historical Memoir and Genealogy of the ancient and illustrious House of Cusack, of the Kingdom of Ireland." It appears to have been compiled by the Chevalier O'Gorman in the year 1767. It states that "Michael de Cusack, lord of Portrane and Rathaldron, married Margaret Dexter, who brought him, as a marriage portion, the castle, town, and lands of Rathaldron. He was 'Greffier' [a term which Boyer translates 'Registrar,' or Keeper of the Rolls] of Westmeath and of Louth in 1553, one of the Barons of the Exchequer in 1580, and died in 1589." From this it may be safely concluded that the initials "M. C.," upon the cross, are those of "Michael Cusack," and that his was the name sculptured on the upper part of the cross, now lost.

In conclusion, Mr. Smith submitted that it would be desirable to have careful drawings, and, where practicable, rubbings also, made of all such existing monuments, in order that these most interesting memorials, which contain valuable confirmations of written documents, as well as curious illustrations of the manners and customs of bygone times, may be preserved from oblivion; and stated that he would be much gratified by receiving any communications on the subject, though they went no further than to state the existence of such crosses, in order to complete the materials for a general history of these Christian memorials, so deeply interesting, even in an historical point of view alone.

Rev. Charles Graves, F.T.C.D., read a Memoir, by Mr. George Boole, of Lincoln, on Discontinuous Functions.

The author deduces in succession three theorems for the expression of the discontinuous function, $f(x)$. The first theorem, which is free from signs of integration, implies that between the limits $x = a$, and $x = a + \Delta a$,

$$f(x) = \frac{1}{\pi}\left(\tan^{-1}\frac{a + \Delta a - x}{k} - \tan^{-1}\frac{a - x}{k}\right)f(x), \qquad (1)$$

provided that we suppose k a positive quantity, and take the limit to which the second member approaches, as k approximates to 0. When $x = a$, or $a + \Delta a$, the first member of the above equation must be divided by 2; and when x transcends those limits, the first member is to be replaced by 0. From this formula, the author deduces his second theorem, involving one sign of integration, viz.:

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{k da f(a)}{k^2 + (a - x)^2}, \qquad (2)$$

in the second member of which the limits $-\infty$ and ∞ may be replaced by any other real limits, p and q, when all the values of x, for which $f(x)$ does not vanish, lie between the limits p and q. This theorem is subject to the same conditions, with reference to the interpretation of the second member, and to the changes of value which it undergoes in passing the limits, as the preceding one.

By an integral transformation, Mr. Boole then deduces the third, or Fourier's theorem, involving two signs of integration, viz.:

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} da\, dv\, \varepsilon^{-kv} \cos(av - xv) f(a). \qquad (3)$$

He remarks, that when this theorem is written in the form,

$$f(x) = \frac{1}{\pi}\int_{-\infty}^{\infty}\int_{0}^{\infty} da\, dv \cos(av - xv) f(a),$$

we must attach to the symbol $\int_0^\infty$ a meaning different from its ordinary one, and understand by $\int_0^\infty dv\, \phi(v)$, the limit of $\int_0^\infty dv\, \varepsilon^{-kv} \phi(v)$, for decreasing positive value of k. Mr. Boole proposes to designate an integral of this kind as taken in a *limiting* sense, and he observes that some anomalous results have been obtained by writers who have neglected the distinction here implied. Thus, from the equations

$$\int_0^\infty dx \cos x = 0, \qquad \int_0^\infty dx \sin x = 1,$$

in which the sign $\int_0^\infty$ has been used in its limiting sense, have been deduced, by taking that symbol in its ordinary sense, and integrating without reference to the factor understood, ε^{-kx}, the incompatible conclusions,

$$\cos\infty = 0, \ \sin\infty = 0.$$

Mr. Boole remarks that, by a converse error, other writers have been led to infer the incorrectness of Fourier's theorem.

From Fourier's theorem the author finally deduces a theorem for the discontinuous expression of $\frac{f(x)}{t^n}$, viz.:

$$\frac{f(x)}{t^n}=\frac{1}{\pi\Gamma(n)}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} da\,dv\,dw\cos\left((a-x)v-tw+\frac{n\pi}{2}\right)w^{n-1}f(a), \quad (4)$$

in which the symbols $\int_0^{\infty}$ are both used in a limiting sense. This theorem, it is observed, is susceptible of important applications, in the theory of definite multiple integrals.

As respects the expression of discontinuity, the formulæ (1), (2), (3), are shewn to be equivalent. The advantage which the second possesses over the first is, that x enters into the second member in a rational form; and the advantage of the third, or Fourier's theorem, over them both, is, that x enters into the second member exponentially, which affords facilities for both the direct and the inverse processes of the differential calculus.

In the fourth theorem the author remarks that both x and t enter exponentially.

Sir William R. Hamilton made some observations on Mr. Boole's communication.

Professor Harrison made some remarks on the peculiarities of the anatomy of the Emu. [This paper, not having been received in proper time for insertion here, will be printed in the Appendix, No. VII.]

Professor Allman announced the addition of Deotis Maritima to the Irish Fauna. This plant he had observed during the latter part of the summer of 1845, in the sand-hills in the neighbourhood of Dungarvan, county of Wexford.

DONATIONS.

Memorie della Reale Academia della Scienza di Torenzo. Serie Secunda. Tome. VI. Presented by the Academy.

Proceedings of the Philosophical Society of Glasgow, 1842, 1843, 1844, 1845. Presented by the Society.

Astronomical Observations made at the Radcliffe Observatory, Oxford, in the Year 1842. Vol. III. Presented by the Radcliffe Trustees.

Astronomical Observations made at the Observatory of Cambridge. Presented by the Rev. James Chatto, M.A.

Journal of the Asiatic Society of Bengal. No. CLV. 1844. Presented by the Society.

Index and Appendix to the Minutes of Evidence taken before the Commissioners of Inquiry into the State of the Law and Practice in respect to the Occupation of Land in Ireland. Parts IV. and V. Presented by the Lord Lieutenant.

Facts from Gweedore, with Hints to Tourists in Donegal. By Lord George A. Hill, M.R.I.A. Presented by the Author.

On the right Ascensions of the principal fixed Stars, deduced from the Observations made at the Cape of Good Hope, in the Years 1832 *and* 1833. By Thomas Hudson, Esq. Presented by the Lords Commissioners of the Admiralty.

Proceedings of the Zoological Society of London. Part XII.—1844. Presented by the Society.

Letter on the Irish Colleges Bill. By John D'Alton, Esq. Presented by the Author.

Annual Report of the Royal Cornwall Polytechnic Society. 1844. Presented by the Author.

Facts connected with the social and sanitary Condition of the Working Classes of Ireland. By Thomas Willis, F.L.S. Presented by the Author.

Manual of writing and printing Characters. By B. P. Welme. Presented by the Author.

An ancient Bronze Bit, found at Ballinaminton, King's County. Presented by George Marsh, Esq.

February 9th.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Richard Cane, Robert Franks, Charles W. Levinge, James Corry Sherrard, Pierce Morton, and Stephen O'Meagher, Esqrs., were elected Members of the Academy.

Mr. Robert Mallet read part of his paper on the Mechanics of Earthquakes.

The author first notices the various instances recorded of an apparently vorticose motion having occurred in earthquakes, as evidenced by the twisting displacement of objects, such as the superimposed stones of obelisks, &c., and then proceeds to demonstrate that the conclusion adopted by Mr. Lyell and other authors, that such a vorticose motion actually takes place, does not follow from the premises, and is inconceivable and impossible in many respects.

He proves that the twisting displacement is due to a mere alternate, straight-line motion, given by the earthquake-shock to the base upon which the displaced body rests. The insistent body is moved by the adhesion of its base, and its inertia, acting through its centre of gravity, will cause the body to twist whenever the point in the base, at which all the adhesion may be supposed to act, and which the author calls "the centre of adherence," lies either to one side or the other of a vertical plane, passing through the centre of gravity of the body twisted, and being on the line of motion of the base.

The alternate, straight-line motion having such great velocity, yet within narrow limits, as thus to move heavy bodies by their inertia, and which constitutes the earthquake-shock, the author defines as the passage of a wave of elastic compression though the solid crust of the earth, produced at any distant points, by any original sufficient impulse, such as the sudden bending, by elevation or depression, or the rupture of a portion of the earth's crust.

From this single principle, viz., that the true earthquake-shock consists simply in the transit through the solid crust of the earth of a wave of elastic compression, which the author believes to be now, by him, for the first time, enunciated, he proceeds to develope and account for, in detail, all the more important recorded phenomena of earthquakes, as well as many of the more perplexing secondary phenomena.

The original impulse, or origin of an earthquake, may be either under the sea, or on land, at a distance from the sea. In the former case, at the moment of originating the impulse, whether by bending or fracture, several distinct sets of waves set out from the same points, and at the same moment of time, but they move with very different velocities.

The wave of elastic compression, or great earth-wave of the author, makes its transit through the solid crust, outwards, in all directions, from the point or points of impulse, and moving at a speed proportionate to the specific elasticity and density of the formations through which it passes. This, the author shews, may be as much as 11,000 feet per second. This wave constitutes a real undulation of the surface through which it is passing, and may be also (if there is fracture at the origin) heard as a sound-wave in the solid, moving at the same rate. A sound-wave also travels through the water of the sea, and, moving more slowly than in the solid, is heard upon land after the shock has passed. Lastly, a rolling wave of translation, or great sea-wave of the author, is formed by the movement of the bottom, directly above the originating disturbance. This sea-wave, though setting out at the same moment as the shock or earth-wave, is rapidly outstripped by the latter—because its motion is dependent upon its own form and magnitude, and upon the depth of the sea upon and through which it moves. The great sea-wave, therefore, comes to land long after the shock has passed, and may be followed by several in succession, into which the original great sea-wave has broken, where the form of the soundings in-shore are suitable.

The apparent recession of the sea just at the moment of the earthquake-shock reaching the land, the author shows, is to be accounted for by a small undulation of the sea, carried, as it were, upon the back of the earth-wave, and moved along at its speed, and which he has called the "forced sea-wave." Such are the usual train of circumstances when the centre of disturbance is under the sea, accompanied, in addition, by a sound-wave through the air, when rupture of the crust has occurred.

When the centre of disturbance is far inland, the "great earth-wave" and sound-wave through the solid, with the sound-wave through the air, and the "forced-wave" upon the shore, are the only ones that can occur.

Dr. Apjohn observed, that Mr. Mallet appeared to him not only to state very correctly how the onward motion of the earth's crust, produced by an earthquake, might cause the upper stones of a pillar of masonry to be deranged from their position in the line of such motion, but also to have suggested, for the first time, the true cause of their partial rotation, or displacement in azimuth. In endeavouring, however, to remove the obvious objection to this explanation, viz., that the *returning movement* should restore such disturbed masses to their original position, he (Dr. Apjohn) could not but think that Mr. Mallet had introduced speculations as to the mechanism of terrestrial wave-motion, which, though very ingenious, appeared somewhat far-fetched and obscure, and, unless he (Dr. Apjohn) was much mistaken, certainly not necessary to the solution of the difficulty in question. In fact, it is impossible that the displacement produced by the forward motion could be *undone* by the returning stroke, unless upon a hypothesis almost infinitely improbable, viz., that, after the displacement, what Mr. Mallet calls the centre of adhesion shall have, in relation to the centre of gravity, such a position, that the moment of the weight of the displaced masses, referred to the centre of adhesion, shall have its original value, and tend, at the same time, to produce a motion of rotation opposite to that which has already occurred. Now, in order to this, the centre of adhesion must continue at the same side of the line of direction of the earthquake-movement passing through thecentre of gravity of the displaced materials, and we must also have $d \times \sin \theta' = d' \times \sin \theta'$, d and d' being the distance between centre of gravity and centre of adhesion before and after the first displacement, and θ and θ' the angles made by the direction of earthquake-movement with the lines connecting centre of gravity with centre of adhesion. It is scarcely necessary to say, that the fulfilment of such conditions in any particular case must be in the highest degree improbable.

The Secretary of the Academy read the second part of the Rev. Dr. Hincks' paper on Phonetic Hieroglyphics:

The object of the second part is to shew what data are most to be relied on for determining the exact powers of the Egyptian letters; the existence of an approximate alphabet is assumed, and the knowledge of facts grounded on the general correctness of this is to be applied to determine the exact alphabet. It is remarked, in the first place, that, as the powers of the letters probably varied at different times and in different parts of Egypt, it is necessary to assume a particular place and time, the alphabet of which is to be investigated. The place chosen is Thebes, and the time the interval between the deaths of the first and third kings of the name of Rameses, during which the principal sculptures at Thebes were executed, and the papyri in the British Museum, of which facsimiles have been published, were written. The data which are considered the most valuable are transcriptions of foreign words occurring in the papyri and on the monuments of this period; while the words themselves, or transcriptions of them into Hebrew letters, are preserved in the Hebrew Scriptures, and in many cases transcriptions into Greek letters are also met with. As the alphabet, which, we previously found, was formed from transcriptions of Greek and Roman proper names into Egyptian characters made in the later ages, so it is by similar transcription, made in the time and at the place chosen for a standard, that this alphabet must be corrected and completed. Transcriptions of Egyptian words into Hebrew letters are a useful auxiliary to the other kind of transcriptions, especially when they contain the peculiar Hebrew letters which represent sounds unknown to the Greeks. Against these, however, the objection lies, that they probably represent the pronunciation of Lower Egypt, which may have differed from that of Thebes. The transcription of Egyptian words into Greek characters in Theban papyri of the Ptolemaic period, and in the names of kings, are also to be taken into consideration, chiefly, however, to supply the proper sounds of those letters, the Hebrew representative of which were ambiguous; the Maronetic points, by which a certain value was affixed to these letters, being shewn to be of no authority. In the case of S and SH, where the two sounds are expressed by the same letters in Greek as well as in Hebrew, we are compelled to seek a distinction in the Coptic equivalents of the ancient Egyptian words. It is maintained, however, that, owing to the Coptic representing the Egyptian language in its latest form, when many words had been corrupted, it should not be admitted as evidence in opposition to clear indications of the powers of the letters found in ancient transcriptions. Interchanges of letters, if habitually made in texts of the standard period, are admitted to be good evidence of the identity in power of the letters interchanged. But it is observed that the number of letters thus exchanged is very limited, and a caution is given against depending too much on manuscripts, the writers of which were often very careless, and committed gross mistakes. This is especially the case with funeral MSS., on which no dependence whatever should be placed in reference to the present object. An opinion is expressed that all these MSS. are of a late age, and that the famous one at Turin, of which Dr. Lepsius has published a copy, is not earlier than the second century before Christ. It is shewn, too, that it was transcribed from a hieratic original, or from a hieroglyphic one which had been copied from a hieratic one. At the end of this part suggestions are given as to the aid which may be derived from the Indo-Germanic languages in determining the powers of Egyptian characters in some particular instances. The whole of this part is preparatory to the third, in which the principles laid down here will be applied to the practical determination of the powers of the letters.

Mr. Mallet presented his Translation of the Report of the Institute of France upon M. Arnollet's System of Atmospheric Railways.

February 23, 1846.

GEORGE PETRIE, Esq., Vice-President, in the Chair.

Thomas Butler, Esq., John T. Evans, M.D., Richard R. Madden, M.D., Robert C. Williams, M.D., and Henry W. Massy, Esq., were elected Members of the Academy.

The Secretary of the Council having read to the Academy the following Resolution of the Council of the 16th of February,

"That the Council are of opinion, that it is not expedient that the same person should be elected to the office of President more than five times in succession:"

It was Resolved,—That the Academy do concur in the preceding opinion expressed by the Council.

Mr. Robert Mallet completed the reading of his paper on the "Mechanics of Earthquakes."

The author pointed out the correspondence of his theory with the actual velocity of earthquake-shocks, so far as these have been observed, and by numerous quotations shewed how completely his theory accounts for the complex phenomena often detailed, and many of which have been heretofore inexplicable.

He shewed that an exact knowledge of the velocity of earthquake-waves passing under the bed of the ocean, would enable us to ascertain, with considerable certainty, what the geological formations are, which, constituting this bed, form more than two-thirds the whole surface of our globe, which hitherto has been a geological blank. He also indicated the means of experimentally determining the velocity of waves of elastic compression in the crust of the earth, and proposed the establishment of geological observatories, both separate and in connexion with the magnetic observatories scattered over the face of the globe, for the purpose of registering and recording with suitable instruments, all the motions of the water of earthquake-waves which occur; and he has shewn reason to believe that these (though so small as to be inappreciable without the aid of proper instruments) are much more frequent than has been hitherto supposed; in fact, Arago has actually observed an earthquake-shock at his magnetic observatory at Paris, which was imperceptible there without the aid of instruments, and the origin of which lay in the south of France.

Dr. Lloyd took this occasion to mention that he had frequently observed certain abnormal movements of the magnets in the Dublin Observatory, which, like that noticed by M. Arago, and referred to by Mr. Mallet, he was inclined to ascribe to earth-tremors, propagated from remote centres of disturbance. These movements were *vertical* oscillations of the magnets, which came on suddenly, and by which all the instruments were, in general, simultaneously affected. That they were not due to any sudden change in the direction or intensity of the magnetic force, is evident from the fact that they were in general unaccompanied by any changes in the mean position of the magnetometers, such as would result from a sudden magnetic disturbance. It appeared equally evident that they were not the result of any ordinary extraneous disturbing causes, such as currents of air; for, besides that the instruments are well protected from such influences, they frequently occurred at times of perfect calm, and when there was no movement within the Observatory. Under these circumstances it was difficult to avoid the conclusion that they were the effects of mechanical movements of the earth's crust itself, which were too slight to affect the senses directly. Under this supposition, Dr. Lloyd stated that he had given instructions to his assistants to keep a record of these movements; and that he was now in possession of a registry of them for the last two years, the times of which he hoped soon to compare with those of recorded earthquake-shocks, and thus to establish or disprove the conjecture respecting their origin which he entertained.

The Chairman read some extracts from a letter he had received from Dr. Lappenburg, and presented to the Academy, on the part of Dr. Lappenburg, of Hamburg, the volume of the Encyclopædia in which his Essay on Ireland is printed.

DONATIONS.

On some Roman Vestigia, recently found at Kirkley Shore, in Westmoreland. By Captain W. H. Smith, R. N., &c. Presented by the Author.

Vestiges of the Natural History of Creation. 5th Edition. Presented by Anonymous.

Twenty-four Maps of the Geological Survey of Great

Britain. From No. 19 to 33, and from 35 to 43, inclusive. Presented by the Earl of Lincoln, Chief Commissioner of Woods, Works, and Land Revenues.

Natural History of New York. In 10 vols. 4to, with a geological Map accompanying each set. Presented by the Legislature of New York.

Transactions of the American Philosophical Society, held at Philadelphia for promoting useful Knowledge. Vol. IX. New Series. Part 2. Presented by the Society.

Transactions of the historical and literary Committee of the American Philosophical Society. Vol. III. Part 1. Presented by the Society.

Proceedings of the American Philosophical Society for January and April, May and August. Presented by the Society.

A Cast in Plaster of the ancient Harp, called Brien Boru's Harp, in the Museum of Trinity College, Dublin. Presented by the Provost and Senior Fellows.

Two heads of Fossil Elks, found at Castleknock. Presented by Algernon Preston, Esq.

March 16. (Stated Meeting.)

Sir WILLIAM R. HAMILTON, LL.D., President, in the Chair.

The following Report from the Council was read and received, and ordered to be entered in the Minutes:

During the past year the twentieth volume of the Transactions of the Academy, containing Mr. Petrie's long expected paper on the Round Towers of Ireland, was published, and delivered to such members as were entitled to it.

The great beauty of this volume, the laborious and extensive research which it displays, and the approbation with which it has been received by the public, will, it is hoped, prove ample compensation to the Academy for the delay that unavoidably took place in its publication.

Although it contains only a portion of Mr. Petrie's essay, the Council have recommended, and the recommendation has been adopted by the Academy, that in consideration of its bulk and value, and the numerous and beautiful wood-cuts which adorn it, the volume be received as acquitting Mr. Petrie of his engagement; but they have declined entering into any further agreement with him, as to the publication of the remainder of the essay.

In this resolution the Council have also had the concurrence of the Academy; and they may add, that the twentieth volume of the Transactions has been received with such favour by the members, that it was found necessary to order fifty additional copies, over and above the usual number which the Academy had originally agreed to take from Mr. Petrie, in order to meet the demand of those, not otherwise entitled to it, who desired to obtain it by purchase.

The twenty-first volume of the Transactions is in progress; four sheets have been printed off in the department of Science, and eleven in the department of Polite Literature.

In the publication of the Proceedings much delay and difficulty have been experienced. Two additional parts, however, are ready, and are now laid on the table; but they extend only to July of last year, leaving the proceedings of the present session still in arrear.

One great cause of this delay has been the difficulty of obtaining in proper time, from those who read papers before the Academy, the necessary abstracts of such papers. This throws an immensity of unnecessary labour and trouble upon the Editor of the Proceedings; but the Council have now made a rule, which will in a great measure remedy this evil; and they hope that in future there will be nothing to complain of on this head.

During the past year a great number of most important and interesting papers have been read before the Academy; and on one occasion his Excellency the Lord Lieutenant was pleased to honour us with his presence, when the Rev. Dr. Robinson gave an account of the first observations made with Lord Rosse's six-foot reflector.

The Museum has been fitted up in a manner which has given great satisfaction, and has attracted a great number of visiters to the Academy. An inventory of its contents has also been commenced, on a plan suggested by Mr. Clibborn, and is in progress.

Of the donations to the Museum during the year, may be noticed the singularly interesting specimen of an ancient Irish waxed wooden tablet presented by the Rev. James Spencer Knox; the valuable collection of ancient Swords, and other antiquities, found in the works of the Dublin and Cashel Railway, near Kilmainham, presented by Colonel J. E. Napier and the directors of that railway; and the donation of ancient weapons, by the Shannon Commissioners, which are also of very great interest and value. The importance of this class of donations, possessing a peculiar value from their authenticity, can scarcely be too highly estimated; and the Council therefore thought it right to address a circular to the Directors of the railways now in progress in Ireland, calling their attention to this subject, and requesting their co-operation with the Academy in the attempt that has been made to establish here a public museum of Irish Antiquities.

The increase of our collection of MSS. has made it necessary to provide for their security; the Council have therefore made rules prescribing the conditions upon which access to the MSS. is to be henceforth permitted, with a view to the exclusion of such persons as would be likely to injure them, or to make an improper or dishonest use of their contents. Under this head should be noticed the recovery of two of the missing leaves of the Leabhar Breac, which were presented to the Academy by Messrs. Hodges and Smith, having been purchased by them, along with some other volumes in MS. which they have also presented to our library.*

During the past year friendly communications have been opened with the Literary and Scientific Society of Toronto, the Irish Society of London, the Archæological Institute of Great Britain and Ireland, and the Belfast Library and Society for promoting knowledge.

The Council have now to notice, with regret, the retirement from the office of President, of the distinguished gentleman who has for the last eight years presided over the Academy. Sir William Hamilton's determination to resign a place whose duties he has so long discharged with honour to himself and to the Academy, is already well known to every member; it was communicated to the Council by a letter received by them on the 17th of November, which contained also a suggestion that the voluntary resignation of the President afforded a favourable opportunity for considering, without the infringement of delicacy

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* See their letter in the Proceedings, vol. iii. p. 73.

towards an existing President, the expediency of introducing any rules of practice with respect to the election of the President, which, in the judgment of the Council, might seem desirable.

The President being by our charter an annual officer, it is of course the undoubted right of the Academy to re-elect the same individual as often as they think fit; and with this right, as it is a chartered right, perhaps not even the Academy itself, could directly interfere. But as a majority of the Council were in favour of some limitation, the following resolution was communicated to the Academy, on the 23rd ult. :—

"RESOLVED,—That the Council are of opinion, that it is not expedient that the same person should be elected to the office of President more than five times in succession."

In this opinion the Academy at the same meeting expressed their concurrence.

During the past year the distinguished name of William Wordsworth was added to our list of Honorary Members; and forty ordinary members have been elected during the same period, whose names are as follows:

J. A. Galbraith, Esq.
J. Jameson, Esq.
John F. Waller, Esq.
C. Bournes, Esq.
W. C. Dobbs, Esq.
W. Henn, Esq.
Digby P. Starkey, Esq.
B. Wilme, Esq.
J. Clarridge, Esq.
Adolphus Cooke, Esq.
Wyndham Goold, Esq.
C. K. King, M.D.
C. W. Williams, Esq.
James S. Close, Esq.
D. Conolly, Esq.
David Moore, esq.
Rev. Classon Porter,
James Talbot, Esq.
The Earl of Enniskillen.
The Ven. Henry Cotton, D.C.L.
Rev. John Connell.
John C. Deane, Esq.
Richard Deasy, Esq.
Conyngham Ellis, Esq.
W. Lloyd, M.D.
A. R. Nugent, Esq.
E. K. Tenison, Esq.
F. J. Tyrrell, Esq.
J. Tuffnell, Esq.
Richard Cane, Esq.
R. Franks, Esq.
Charles W. Levinge, Esq.
Pierce Moreton, Esq.
James C. Sherrard, Esq.
Stephen O'Meagher, Esq.
Thomas Butler, Esq.
John T. Evans, M. D.
Richard R. Madden, Esq.
Robert C. Williams, M. D.
Henry W. Massey, Esq.

The painful task now remains of recording the names of those who have been lost to the Academy during the past yearby death; amongst them will be found two of the oldest Members of the Academy: Dr. Whitley Stokes, elected a Member in 1787; and the Hon. Justice Johnston, who was elected a Member in 1790.

Of Dr. Stokes, whose distinguished career is so well known, it will be unnecessary to say much. The philanthropic and practical objects to which his useful life was devoted, left him but little time for purely literary or scientific pursuits, and consequently his nme occurs but seldom in the Minutes of the Proceedings of this Academy. But the Council cannot permit that name to pass away from the list of Members of the Academy, without expressing their respect for the memory of one to whose zeal and energy is very mainly to be attributed the present flourishing and progressing state of the practical sciences of Botany, Mineralogy, Geology, and Zoology, in Dublin and in the University.*

We have also lost our late excellent and active Treasurer, Dr. Thomas Herbert Orpen, whose useful life was passed in the constant discharge of those benevolent virtues which leave little of incident to be recorded in a biographical notice. He commenced practice as a physician in Dublin, in 1803, about which period he took the regular medical degrees in the University, having previously studied at Edinburgh; and the charitable institutions of Dublin engaged a great portion of his time and attention. For a great part of his life he kept a diary of the weather, noting the heights of the barometer and thermometer, and the direction and intensity of the wind, twice every day. These observations, which extend through a period of thirty-five years, are now in the Academy's library.

The names of the remaining deceased Members, with the dates of their elections, are as follows:—

1816, George Kiernan, Esq.
1824, Hugh Ferguson, M. D.
1829, John Houston, M.B.
1836, Thomas Arthur, Esq.

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* A brief Memoir of Dr. Stokes has appeared in the Dublin University Magazine, for August, 1845.

1844, The Most Noble the Marquis of Downshire.
1845, Robert S. Bradshaw, Esq.
„ Thomas Davis, Esq.
„ Sir Richard Franklin,

There are therefore now on the books of the Academy, 162 life Members, and 203 annual Members.

The President delivered the following Address:

"My Lords and Gentlemen of the Royal Irish Academy,—Although it is, I believe, well known to most, perhaps to all, of you, that it has been for a considerable time my wish and intention to retire this evening from the Chair to which, in 1837, your kindness called me, on the still lamented event of the death of my distinguished predecessor, the late admirable Doctor Lloyd, and in which your continuing confidence has since replaced me on eight successive occasions, yet a few parting words from me may be allowed, perhaps expected; and I should wish to offer them, were it only to guard against the possibility of any one's supposing that I look upon my thus retiring from your Chair as a step unimportant to myself, or as one which might be taken by me with indifference, or without deliberation. It was under no hasty impulse that I resolved to retire from the office of your President into the ranks of your private members, nor was it lightly that I determined to lay down the highest honour of my life.

"My reasons have been stated in an Address delivered in another place, at a meeting of some members of your body. They are, briefly, these: that after the expiration of several years, I have found the duties of the office press too heavily upon my energies, indeed, of late, upon my health, when combined with other duties; and that I have felt the anxieties of a concentrated responsibility—exaggerated, perhaps, by an ardent or excitable temperament—tend more to distract my thoughts from the calm pursuits of study, than I can judge to be desirable or right in itself, or consistent with the full redeeming of those pledges which I may be considered to have long since given, as an early Contributor to your Transactions.

"When I look back on the aspirations with which first I entered on that office from which I am now about to retire, it humbles me to reflect how far short I have come of realizing my own ideal; but it cheers me to remember how greatly beyond what I could then have ventured to anticipate, the Academy itself has flourished. Of this result I may speak with little fear, because little is attributable to myself. Gladly do I acknowledge that it has been my good fortune, rather than my merit, to have presided over your body during a period in which, through the exertions of others much more than through my own (though mine, too, have not been withheld), the Academy is generally felt to have prospered in all its departments. The original papers which have been read; the volumes of Transactions which have been published; the closer communication which has been established with kindred societies of our own and of foreign countries; the enhanced value of our Library and Museum, which have been, at least, as much enriched in the quality as in the quantity of their contents; the improved state (as it is represented to me) of our finances, combined with an increased strength of our claims on public and parliamentary support; the heightened interest of members and visiters in our meetings, which have been honoured on four occasions, during my presidency, by the presence of representatives of Royalty; even the convenience and appropriate adornment of the rooms in which we assemble;—all these are things, and others might be named, in which, however small may have been the share of him who now addresses you, the progress of the Academy has not been small, and of which the recollection tends to console one who may, at least, be allowed to call himself an attached member of the body, under the sense, very deeply felt by him, of his own personal and official deficiencies.

"Whoever may be the member elected by your suffrages, this evening, to occupy that important and honourable post which I am now about to resign, it will, of course, become my duty to give to that future President my faithful and cordial support, by any means within the compass of my humble power. But if it be true, as I collect it to be, that your unanimous choice will fall upon the very member whom, out of all others, I should have myself selected, if it could have been mine to make the selection—with whom I have been long connected by the closest ties of college friendship, strengthened by the earnest sympathy which we have felt in our aspirations for the welfare of this Academy which has already benefited by his exertions in many and important ways—then will that course, which would have been in any event my *duty*, be in an eminent degree my *pleasure* also.

"And now, my Lords and Gentlemen, understanding that an old and respected member is prepared to propose for your votes, as my successor, the friend to whom I have ventured to allude—very inadequately, as regards my opinion of his merits, yet, perhaps, more pointedly than his modesty will entirely forgive or approve of,—I shall detain you no longer from that stage of the proceedings of the evening which must be the most interesting to all of us, but shall conclude these words of farewell from this Chair, by expressing a hope that my future exertions, though in a less conspicuous position, shall manifest, at least in some degree, that grateful and affectionate sense which I must ever retain of the constant confidence and favour which you have, at all times, shewn towards me."

Resolved,—That the thanks of the Academy be given to Sir William R. Hamilton, and that the Academy desire to express their entire sense of the value of his services as President, of his high and impartial bearing in the Chair, and of his untiring efforts to advance the interests of the body; and they also wish to record their satisfaction that he has determined to remain in the Council of the Academy.

The ballot for the Annual Election having closed, the scrutineers reported that the following gentlemen were elected Officers and Council for the ensuing year:

President—Rev. Humphrey Lloyd, D.D.
Treasurer—Robert Ball, Esq.
Librarian—Rev. William H. Drummond, D.D.
Clerk and Assistant Librarian—Edward Clibborn.

Committee of Science.

Rev. Franc Sadleir, D. D., Provost; James Apjohn, M.D.; James Mac Cullagh, LL.D.; Robert Ball, Esq.;

Sir Robert Kane, M.D.; G. J. Allman, M.B.; Sir William R. Hamilton, LL.D.

Committee of Polite Literature.

Samuel Litton, M.D.; Rev. William H. Drummond, D.D.; Rev. Charles W. Wall, D.D.; John Anster, LL.D.; Rev. Charles Graves, A.M.; Rev. Samuel Butcher, A.M., F.T.C.D.; Rev. James Wilson, D.D.

Committee of Antiquities.

George Petrie, Esq., R.H.A.; Rev. James H. Todd, D.D.; Henry J. Monck Mason, LL.D.; J. Huband Smith, A.M.; Captain Larcom, R.E.; William R. Wilde, Esq.; F. W. Burton, Esq., R.H.A.

DONATIONS.

Journal of the Asiatic Society, Nos. 69, 70, 72, 73, 74, 77. New Series. Presented by the Society.

Journal of the Royal Asiatic Society of Great Britain and Ireland. No. 26. Part II. Presented by the Society.

Observations des Phenomènes periodiques de l'Academie Royale des Bruxelles. Extrait du Tome XVIII. des Memoirs. *Resumè des Observations magnetiques et meteorologiques faites a des Epoques determinèes. Academie Royal de Bruxelles.* Extrait du Tome XVIII. des Memoirs. Presented by the Academy.

Annales de L'Observatoire Royal de Bruxelles. Tome IV. By A. Quetelet. Presented by the Academy.

Nouveau Memoires de l'Academie Royale des Sciences et Belles Letters du Bruxelles. Tome XVII., 1844, et Tome XVIII., 1845. *Memoires Couronnès et Memoires des Savants Etrangers par l'Academie Royale des Sciences et Belles Lettres de Bruxelles.* Tome XXII., 1844, et Tome XXIII., 1845. Presented by the Academy.

Bulletin de la Sceance du 5 October, 1844, No. 9, Tome XI.; 2nd Nov., 1844, No. 10, Tome XI.; 15th Dec. and 30th Dec., Nos. 11 and 12, Tome XI.; No. 1, 11th Jan., 1845; No. 2, 1st Feb., 1845; No. 3, 1st March, 1845; No. 6, 7th June, 1845, Tome XII. Presented by the Academy.

Annuaire de l'Academie Royale des Sciences et Belles Lettres de Bruxelles. Onzieme Annuare, 1845. Presented by the Academy.

Memoir of Simon Stevin. By A. Quetelet. Presented by the Author.

Catalogue of Stars of the British Association for the Advancement of Science. Presented by the Association.

Flora Batava. Nos. 139 and 140. Presented by the King of Holland.

Geological Map of the State of New York. By Legislative Authority. Presented by the State of New York.

13th April, 1846.

Rev. HUMPHREY LLOYD, D.D., President, in the Chair.

John Alcorn, Esq., Abraham W. Baker, Esq., Philip Bevan, M.D., John Oliver Curran, M.B., Matthew D'Arcy, Esq., James Birch Kennedy, Esq., Michael H. Stapleton, M.B., and the Hon. and Rev. William Wingfield, were elected Members of the Academy.

The President, on taking the Chair, delivered the following Inaugural Address:

"GENTLEMEN,—My first—my most urgent duty—on this, to me most solemn occasion, is to thank you for the high distinction you have conferred. It would be idle to attempt to express how highly I esteem the honour: the thought that I had been deemed worthy to occupy the chair which has been filled by Kirwan, by Brinkley, and by Hamilton, might indeed well nigh overwhelm me, did I not know that there were other merits, more humble than theirs, upon which you set a value—other qualities less dazzling, which may find here their employment and their use. An institution such as this has been compared to the House of Solomon, in Bacon's philosophical fiction, the *New Atlantis*, in which the investigation of Truth is carried on by labourers of various kinds, to each of which he has assigned a separate task. We have had, in this Academy, the representatives of each of these classes: we, too, have had our 'Miners,' our 'Lamps,' and our 'Interpreters of Nature.' I am content to enrol myself in the lowest class; or if, by reason of the high trust which you have now reposed in me, other tasks should fall to my lot, I am proud to accept a new station among the Intellectual Workmen, and to perform the part of one whose office it is to harmonize and give effect to the labours of all.

"There is another personal consideration, to which I cannot refrain from alluding; and yet it is one upon which I hardly trust myself to speak. Among my predecessors in this high office, was one whom I am still more proud to follow:—my nearest relative filled this chair. I know how he was valued here; and I cannot but feel that much of the indulgent estimation which you have formed of my fitness for the same station, has come to me reflected from his memory, and that you hope to find in the son some of these qualities for which the father was loved and honoured.

"But, Gentlemen, whatever qualifications I may want, there is one to which I lay claim: I mean that of deep interest in the welfare of this Body, and zeal for its service. Here I will yield to none; and I console myself with the hope that it may make some amends in your estimation for the many wants which you will hereafter have occasion to observe.

"Gentlemen,—My predecessor in this chair, upon an occasion similar to the present, laid before you some of his views respecting the constitution of the Academy, and the means by which its future interests might be promoted. I am sure that you will permit me to follow this precedent, and to offer a few remarks—*firstly*, upon the mixed nature of that constitution under which we are here united for the Pursuit of Truth,—and, *secondly*, upon the progress that has been made, or that may hereafter be made, in that high object of our incorporation. It is of the *future* that it is important to speak : the precept

"Τα μεν οπισω επιλανθανομενος, τοις δε εμπροσθεν επεκτεινομενος"

holds good in the pursuit of knowledge, no less than in the advance in piety. But still our hopes of the Future, if they are to be more than dreamy visions, must be based upon the history of the Past.

"The first thing that must strike every one, in considering the constitution of this Academy, is the comprehensiveness of its scheme, and the wide scope of its labours; and we are inclined to ask, whether a constitution so large and so varied,—so opposed to modern precedent,—can be sound and healthful? When we look into the recent history of Associations for the advancement of knowledge, we see that each division of the wide domain of truth, as it has arisen into prominent view, by the labours of those engaged in its cultivation, has claimed for itself the concentrated energy, and the undivided resources, of an exclusive Society. In this manner the Royal Society of London, which included originally, and still includes, representatives from every department of Philosophy, has seen Society after Society spring up, manned by its own Members, and claiming to perform, in a more complete and effective manner, the separated portions of its work.

"Such a state of things is the natural result of increased activity in any department, and of the consequent demand which it makes of a larger portion of time, and of the other appliances of labour, than can be devoted to it in a body of mixed constitution and more comprehensive plan. Nor can it be doubted that such a multiplication of the instruments, by which Intellectual Force is concentrated and applied, is attended with the many advantages which arise from the *division of labour*, or that it has actually tended, and in a very important degree, to push forward and to extend the boundary which divides the known from the unknown.

"But perhaps these advantages, great as they are, have not been wholly unbalanced. Have we not reason to apprehend that *Philosophy* has suffered, while the portions of her mighty empire have asserted their independence, and erected themselves into separate kingdoms?

So far as we insulate any portion of Truth from the rest, by an exclusive devotion to its pursuit (and there can be no doubt that such exclusiveness tends to insulation,) so far we mutilate the fair proportions of Truth itself, and injure and impair the Philosophic Spirit, whose vital power should animate and pervade the whole. And the injury, great as it is, does not end here. There is an evil partaking of a Moral nature obviously springing from this exclusiveness, and which unhappily we see too often realized, unless where some counteracting power is brought in to check it. I mean its effect in narrowing our views, in rendering us bigots in Philosophy, and in causing us to undervalue that which we do not understand.

"Now, the mixed constitution of our Society has a manifest tendency to overcome, or, at least, to mitigate, these evils. I do not mean to say that these evils, and these means of combatting them, were distinctly perceived by the first founders of this Body. It is an humbling lesson, that Human Institutions, in which we have learned to find wisdom, have often had their origin in circumstance, and their growth amid the adjustments of conflicting interests. The plan of this Academy took its rise, I believe, in the union of two small Societies, calling themselves the *Palæosophers* and the *Neosophers*, starting originally from opposite extremities of the field of Truth. But, whatever may have been its origin, we may now derive from it lessons not only of mutual forbearance, but of mutual instruction. The Mathematician may imbibe from the Antiquarian the taste which will lead him to explore, with reverence, the early history of the efforts of those master-minds in Science, whose very failures are fraught with philosophic interest, and to trace the progress of discovery up to the first dawn of thought; and he will return from the investigation with clearer views of the Human Mind itself, and of the means by which it attains Truth. The Antiquarian may learn from the man of Science those habits of precise thought, and exact reasoning, which, in the mysterious twilight that surrounds the fascinating objects of his pursuit, he is apt to think inapplicable; and both may learn from the cultivator of Literature to value and to acquire that magic power which Language confers upon Thought.

"Having said thus much in vindication of the constitution of the Academy, suffer me, in the next place, to consider how far it has been effective in attaining the ends proposed. For this purpose, it will be requisite to take a brief survey of the recent advancement of knowledge in this country, so far as it has been influenced by this Academy. And if, in the brevity with which the necessary limits of this Address compel me to glance over the subject, I should appear to have overlooked, or not to have assigned its due weight to any portion of our labours, you will, I trust, attribute this to its true cause.

"The prominent place which the Mathematical Sciences have occupied in our Transactions, may be dated from the time when Brinkley was enrolled amongst our Members. But it is to the labours of your late President, and your late Secretary, in this department, that the Academy, in a great measure, owes the high place which it holds among the Scientific Bodies of Europe. Of these labours, it might, perhaps, be rash to single out any portion as preeminent, had not the Academy itself, and the Royal Society of London, by the awards of their highest honours, marked out the researches of Sir William Hamilton, and Professor Mac Cullagh, in connexion with the wave-theory of light, as of especial value. The theoretical discovery of Conical Refraction, by Sir William Hamilton, the theory of Crystalline Reflexion and Refraction, by Professor Mac Cullagh, and the general Dynamical Theory of Light by the same author, mark an era in this branch of science not inferior to that of Fresnel.

"Time will not permit me to do more than allude to the new branch of Analysis, which has recently engaged so much of the attention of Mathematicians, and which originated in the Theory of *Quaternions* of Sir William Hamilton, and has received an important modification and development in the *Triplet* theory of Professor Graves. As a member of the University, I rejoice to be able to add, that worthy successors, even to such men as I have named, are arising there; and that the recent union of the mathematical strength of Cambridge and of Dublin, in the *Mathematical Journal* which was so long and so ably supported by the former University, is likely to give a new impulse to this branch of science amongst us. And long may these scienees continue to flourish in the University and in this Academy! Independently of the magnitude and sublimity of their own proper objects,—independently of their direct value in Physical Science, as instruments of research,—they confer a no less important, but indirect service, in disciplining the Mind, and correcting those tendencies of other portions of our mental constitution, which, when unbalanced, are sure to mislead.

"Turning from the Mathematical to the Physical Sciences,—and first of all to Astronomy, which stands upon the confines of both,—we cannot fail to be struck by the fact, that in this one Island, with all its disadvantages of climate, there are no fewer than *four* Astronomical Observatories, each claiming a high place in the history of European Science; and that while, in other countries, these costly institutions have been, with but few exceptions, founded and endowed by their respective Governments, in Ireland (a country not certainly among the foremost in pecuniary resources) they have been erected, equipped, and, with but one partial exception, maintained by the munificence and public spirit of Individuals. The names of Mr. Cooper, and of the Earl of Rosse, will henceforward be added to those of Provost Andrews and Primate Robinson, as benefactors of science in this country; and Markree and Birr be united to Armagh and Dublin in the future history of Astronomy.

"The Dublin Observatory is the eldest of this noble sisterhood. As respects its connexion with this Academy, I need not remind you that its chair has been filled by two of your Presidents. With the labours of Brinkley the Dublin Observatory will always stand connected in the history of Science. I am sure that it is unnecessary for me to remind you of his researches connected with the problem of the "Stellar Parallax," of which your Transactions contain the first results—that great problem, whose final solution has at length been placed beyond question by the observations of Bessel. Of the other and better known *inequalities*, which affect the apparent places of the stars, all have been illustrated by the observations made with the Meridional Circle of the Dublin Observatory. In this important class of astronomical investigations, the able Director of the Armagh Observatory has had a distinguished share; and the labours of Dr. Robinson have conferred, as might have been expected, increased accuracy upon the resulting values of the Constants.

"And here, Gentlemen, you will permit me to pause for a moment, and, having named the name of Bessel, to offer a passing tribute to his memory. He, who but a few months since occupied the foremost place in the ranks of living Astronomers, is now no more! He died on the day which followed the last meeting and Anniversary of this Body; and those among us who had the happiness to form his acquaintance, during his short visit to England, and to the British Association, four years ago, will be able to sympathize with his personal friends, no less than with the world of science, in deploring his loss.

"Of the Astronomical and Optical labours of the Earl of Rosse, and of his great reflector—the marvel of astronomical science—it is needless for me to speak. No one who was present when the account of its construction, and of its first achievements, was given in this room by Dr. Robinson, can readily forget it; and for others, the printed notice of that account, in the last Number of our Proceedings, will give the fullest information we yet possess respecting it. Even from this statement of its earliest trials, it is manifest that the astronomical history of the *nebulæ* will, ere long, be re-made; and it must be satisfactory to us to know, that the noble artist has arranged a plan of systematic observation, directed to these remote and mysterious portions of the universe, which promises to reveal all that can be known, until a still higher optical power (if such be practically possible) shall be applied to their examination. The imagination is bewildered when it seeks to grasp the possible future, which may be opened to this and other departments of Astronomical Science by the application of such means: I will mention but one amongst the many anticipations which press for utterance. The observations of Bessel have detected proper motions in the fixed stars, *Sirius* and *Procyon*, which appear to establish the existence of *invisible companions*, of vast magnitude, about which they revolve. Is the invisibility of these great bodies *relative* only? and if so, may it not be dispelled before the optical power which Lord Rosse has brought to bear upon the Heavens?

"'Astronomy, however,' to use the words of one whose philosophic mind, and varied and profound acquirements, well entitle him to legislate for science, 'is only one out of many sciences, which can be advanced by a combined system of observation and calculation, carried on uninterruptedly. * * * * * * * in a utilitarian point of view, the Globe which we inhabit is quite as important a subject of scientific inquiry as the Stars. We depend for our bread of life, and every comfort, on its climates and seasons, on the movements of its winds and waters. We guide ourselves over the ocean, when Astronomical observations fail, by our knowledge of the laws of its Magnetism; we learn the sublimest lessons from the records of its Geological history; and the great facts which its figure, magnitude, and attraction offer to mathematical inquiry, form the very basis of Astronomy itself. Terrestrial Physics, therefore, form a subject every way worthy to be associated with Astronomy as a matter of universal interest and public support, and one which cannot adequately be studied except in the way in which Astronomy itself has been—by permanent establishments keeping up an unbroken series of observation.'

"Two of the leading branches of Terrestrial Physics—the sciences of Meteorology and Magnetism—have now, as you know, for the last six years, been investigated after one uniform and comprehensive scheme, in more than thirty observing stations scattered over the entire globe; and the very bounds of civilization itself have been overleaped in order to give a wider development to the system. In order to realize the view which Sir John Herschel has so often and so ably advocated, it is only necessary to give *permanence* to the more important of these Observatories, and to enlarge somewhat their sphere of labour. All the phenomena of which our Earth, its Ocean, or its Atmosphere, is the seat; the Tides and the Oceanic Currents, no less than the Winds; the temperature of the Earth and of the Sea, as well as that of the Air; the movements of the earth's crust, whether calm or convulsive, no less than the changes of the mysterious power which animates and pervades its mass; all these, and more which might be easily added, are the proper subjects for continued and systematic observation. We have arrived at a period in the history of these branches of science, when the more obvious phenomena have revealed themselves to our desultory efforts, and when the precise laws, and the quantitative measurements, which must form the basis of exact theory, can be reached only by sustained and systematic exertion.

"In these researches, no less than in those of Astronomy, this country has taken its part. The Meteorological Observatory at the Ordnance

Survey office in the Phœnix Park, planned and directed by Captain Larcom, has now been upwards of ten years in active operation, and may be taken as a model for similar establishments. Of the Magnetical and Meteorological Observatory of Dublin, founded in the year 1838 by the University, I have already had frequent occasion to speak at these meetings, and I hope before long to communicate some of the ultimate results.

"Of the Geology of Ireland I have, perhaps, less right to speak, as the subject has been appropriated by another and a younger Society. Yet there are two facts in its recent history of such importance, that it is impossible not to refer to them in noticing the labours of the members of this Academy. I mean the completion of the Geological Map of Ireland by Mr. Griffith, which, as the work of one man, is certainly one of extraordinary merit; and the recent arrangements for the continuation of the Geological Survey of this country, the first fruits of which are before the world in Captain Portlock's able and elaborate Report on the Geology of Londonderry.

"Passing now from the sciences of *Observation* to those of *Experiment*, we here also meet with labourers of our own Body, and our Transactions are enriched with the results of their successful toil. Here are to be found the hygrometric researches of Dr. Apjohn, which have solved one of the most intricate problems in Meteorology; and the still more refined researches of the same author upon the Specific Heats of the Gases, to which you have awarded your medal. Here too are to be found most of the Chemical researches of Sir Robert Kane, upon the chief of which you have conferred a similar reward; and to this body were communicated the first investigations of Dr. Andrews, upon the Heat developed in Chemical Combination, which have recently been honoured with the Royal Medal.

"To remind you of the progress which Natural History has made, and is yet likely to make in this country, I have only to mention the names of Ball, of Thompson, of Mackay, and Harvey, and Allman, whose contributions to the history of the *Fauna* and the *Flora* of Ireland are too well known to need any comment here. The researches of Dr. Harvey, indeed, have embraced a wider range; and his latest work, the *Phycologia Brittanica*, now in course of publication, cannot fail to sustain his high character as a descriptive botanist. As a member of the University, I rejoice to be able to add that, of the distinguished Naturalists just mentioned, four are now connected with her teaching; and that a large portion of the plan contemplated by her late head, with reference to the advancement of these branches of science within her walls, has now been realized.

"The contributions to the department of Polite Literature, which in the early volumes of our Transactions occupied a large and conspicuous place, have, I regret to say, been of late years less numerous. To whatever cause this may be ascribed, we are the more indebted to such men as Dr. Wall, Dr. Hincks, and Dr. Kennedy Bailie, who have enriched our volumes with the results of their learning and their research. But it would not be difficult to name others, fellow-countrymen and fellow-members, who are qualified to share with them the honour and the toil. The latest communication that we have received in this department,—the paper by Dr. Hincks upon Egyptian Hieroglyphics, the first part of which was recently read,—promises to throw much light upon the deciphering of these ancient and mysterious records, and, if the author be right in his theory, to add considerably to the discoveries of Young and Champollion.

"The study of Antiquities, on the other hand,—and especially of the Antiquities of Ireland,—has never been, and, I hope, never will be, out of fashion here. From the time of Molyneux, and of the Dublin Philosophical Society, the earliest of the learned Societies in Ireland from which we can trace our descent, the pursuit of Irish Antiquities has been a favourite one. Of the researches of our living Antiquaries, the most conspicuous, undoubtedly, is the important work of Mr. Petrie, on the Ecclesiastical Architecture of Ireland, which has been referred to in the recent Report of your Council, and which forms, as you know, the last volume of your Transactions. Of the value of that work we should judge inadequately, were we to confine our view to the light which it has thrown upon the subject discussed; it is, perhaps, still more valuable as an example of the mode of dealing with Antiquarian questions, and of the evidences which may be brought to bear in their investigation.

"The study of Irish Antiquities will, there can be no doubt, receive both aid and impulse from the institution of your Museum, a collection well worthy of this Academy, and of this Country. It may be rash in one wholly unacquainted with the subject, as I am, to offer any suggestion respecting it; yet I cannot but think that much more may be done, in advancing our knowledge of Antiquities generally, and especially of that higher department of it which borders so closely upon History,—the distribution of the early races of mankind,—by the comparison of our own Monuments, and other relics of early civilization, with those of other countries. The information we gather from a Cairn, a Torque, or a Spear-head, will then no longer be limited to the light which they may throw upon the arts and manners of our Celtic ancestors. We may obtain from them a knowledge of the geographical distribution of their various tribes, much in the same manner as the geologist recognizes the fragments of one of the great formations which compose the earth's crust by the comparison of their imbedded fossils;—we may approach the history of their families, and trace them up to the parent stock. Studied with this reference, Antiquities may, perhaps in an important degree, tend to advance the science of Ethnology; and be combined with the study of Language, and of Physiological characters, as a new instrument in its research.

"GENTLEMEN,—I fear I have already trespassed too long upon your time. But I desire, before I conclude, to offer a few remarks upon the future advancement of the objects of the Academy, and upon some of the modes by which it may be accelerated.

"The first and chief of these, beyond all question, is *rapid publication*. It is not to be expected that men, who find a reward for their toils in the sympathy with which they are hailed by those engaged elsewhere in the same pursuits—it is not to be expected that they will communicate to us the fruits of those toils, if they should be long withheld from public view. Already there are indications that researches, which should naturally find their place in our Transactions, are about to reach the public through other channels. I trust that this evil may be stayed. The injury that it inflicts is not merely the loss of so much that should add to our credit and our character as a public body, but this very loss itself reacts upon, and augments the evil from which it has sprung. Nor is it necessary for me to urge, that publication is the first,—the main and essential duty of such a body as ours. No matter what may be the interest of our Meetings,—no matter how far the study of Science, Literature, or Antiquities, may be aided by our Library and our Museum,—it is by our published works that we shall be judged, and by which we must stand or fall. I have only to add, that your Council are duly impressed with this feeling; and that your Officers are at present engaged in the consideration of some measures, which promise to give not only a speedy, but also an increased publicity to our Proceedings.

"Another instrument of progress, to whose efficacy I will advert, and which this Academy may, I think, effectively wield, is the *directing power* which it may reasonably assume, in pointing out to its Members problems of local interest remaining to be solved, and encouraging them to the task by the proposal of Honorary Rewards. The practice of proposing subjects for investigation, and of honouring them by Prizes, has existed, you must be aware, from the very origin of the Academy; and it has tended to elicit researches of considerable interest and value. Some years since, indeed, it was generally felt that the system had failed; and that opinion (in which at the time I shared) led to an alteration in the system of honorary rewards, with which you are of course acquainted. It may be doubted, however, whether this failure was the necessary result of the system itself, and not rather of the nature of some of the topics selected and proposed. It must be manifest, I think, that no encouragement which such a Society as this can bestow, will be likely to stimulate a man of genius to the investigation of an abstruse question, to which he feels no predisposing movement,—that no Reward can usurp the place of Inspiration itself. But there are problems of a different stamp, whose solutions may be expected as the certain result of well-directed labour; to such problems as these, especially when their local character invests them with additional interest, and in some degree prepares men's minds for the research,—to such problems the recommendation of a learned Society may, with full assurance of the result, direct the attention of its Members. We know how much our knowledge of the Antiquities of this country has benefited by the proposal of such questions. Allow me to suggest one or two of a similar character connected with Physical science, as examples of what may be done in other departments.

"In an interesting paper recently printed in the *Philosophical Magazine*, Colonel Sabine has suggested that the almost unparalleled mildness of the late winter may possibly be explained by an unusual extension of the Gulf-stream, bathing the shores of these Islands, and carrying with it a portion of the high temperature of the tropical region from which it flows. And the probability of this explanation has been augmented by the fact, that in the winter of 1821–2, a winter in many respects resembling the last, this great oceanic current, whose force is usually spent when it reaches the Azores, was actually observed in the neighbourhood of our shores. I have long speculated upon the probable influence of the Gulf-stream upon the Irish winters generally, which appear to be much milder, in comparison with those of England, than can be well accounted for upon the principles of insular climate alone; and I was glad to see, from Colonel Sabine's paper, that my conjectures had some real foundation. Whether or not they will account for the fact, may, I think, be easily tested by a series of observations of the temperature of the sea on the Eastern and Western coasts of the Island, and under the same parallel; and I cannot but think that such a result, throwing so great a light upon the Climatology of this country, would, if established, well reward the labour bestowed in the investigation.

"The Climate of Ireland, indeed, engaged a large share of the attention of the Academy during the life-time of Kirwan; and several papers on the subject, by himself and others, are to be found in the early volumes of our Transactions. Should the Royal Irish Academy, as I think it ought, take that subject again under its peculiar care, the knowledge of it might be extended and improved, by the observation of the times of the leafing and flowering of certain plants, after the plan suggested and carried out by M. Quetelet of Brussells, and now extensively followed in many parts of Europe. Such observations furnish us with a simple but admirable measure of the *total effects* of all the influential causes in their combination and union.

"Another subject of special inquiry, which might be fitly urged by this Society, is the History of the Tides on the coasts of Ireland. On this subject much has been already done; but probably much yet remains to be accomplished. Of the observations made in the summer of 1842, by the non-commissioned officers of the Ordnance Survey, under the direction of Colonel Colby, Mr. Airy, by whom they have been ably discussed in a paper recently printed in the *Philosophical Transactions*, observes, that 'extent of time alone appears wanting to render them the most important series of tide-observations that has ever been made.' Among the results to which Mr. Airy has arrived, is the remarkable one, that in the harbour of Courtown, on the coast of Wexford—'the only place on the earth in which such a result has been distinctly obtained,'—the Solar Tide exceeds the Lunar. Such a result as this, affords not only encouragement to fresh exertion, but also direction as to its application.

"Another, and most interesting subject of research, which this Academy might direct, if not undertake, is that to which attention has been recently drawn by Mr. Mallet,—the movements of the Earth's crust, whether convulsive and paroxysmal, or gentle and regular. The phenomena of Earthquake shocks in Scotland have been systematically observed for the last five years, at the instance of the British Association, and yearly reports of the results have been made, and published in its Proceedings. Although there appears to be nothing in this country analogous to the local movements at Comrie, in Perthshire, still there is no doubt that Earthquake shocks have been *felt* here; and that more refined methods of observation would detect numberless others, which wholly escape the cognizance of the unaided senses.

"These, and many other investigations, connected with the Physical, the Physiological, and the Monumental history of Ireland, appear to be fitting subjects, if not for the direct labours of this Academy, at least for its encouragement. Science has a right to demand such histories of local phenomena from the representatives of Science in each portion of the civilized globe, and shall this Academy be deaf to the call?

"Gentlemen,—I have, at the outset of these remarks, noticed the *moral*, as well as the *intellectual* benefits, which result from the union of different mental powers, such as this Academy presents, combined in the investigation of different portions of Truth. But there is a yet higher principle, to which this union may lead us—a yet holier temper which it may inculcate; I mean the contemplation of Truth itself as essentially one, under its many and diversified forms, and the habit of tracing all its varied and refracted rays to its One and Eternal Source. Strengthened by this high thought,—our feelings raised and spiritualized by this habit,—there is no danger that we shall give place to the weak apprehension (which is but a subtle form of unbelief itself), that any portion of Truth can ever prove inconsistent with any other. And the same principle, while it saves us from slavish *fear,* will also guard us from *presumption.* Standing in the presence of confessed and established truth, we shall feel that we are treading upon holy ground; and we shall demean ourselves, not with the elation and pride of conquest, but with the devotion of worship and of love."

It was Resolved,—That the President be requested to permit his Address to be printed in the Proceedings of the Academy.

The Rev. Charles Graves read a paper by Mr. George Boole, of Lincoln, on a Certain Definite Multiple Integral.

It has for some time been known that the evolution of definite multiple integrals can, in many cases, be effected by the employment of discontinuous functions. In illustration of this fact, the author notices the researches of M. Lejeune Dirichlet, founded on the properties of the discontinuous integral $\int_0^\infty \frac{d\phi \sin\phi \cos r\phi}{\phi}$, and those of Mr. Ellis based on Fourier's theorem. In his own investigations he employs the formula of triple integration,

$$\frac{f(x)}{t^n} = \frac{1}{\pi\Gamma(n)}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} da\,dv\,dw \cos\left((a-x)v - tw + n\frac{\pi}{2}\right) w^{n-1} f(a),$$

by the aid of which he deduces the value of the multiple definite integral,

$$v = \iint\ldots \frac{dx_1\,dx_2\ldots dx_n f\left(\frac{x_1^2}{h_1^2}+\frac{x_2^2}{h_2^2}+\ldots+\frac{x_n^2}{h_n^2}\right)}{\{(a_1-x_1)^2+(a_2-x_2)^2\ldots+(a_n-x_n)^2\}^{i}},$$

the limits of the integrations being given by the condition

$$\frac{x_1^2}{h_1^2} + \frac{x_2^2}{h_2^2} \ldots + \frac{x_n^2}{h_n^2} = 1.$$

The result of his analysis is

$$\mathrm{V} = (-)^{i-\frac{n}{2}} \frac{h_1 h_2 \ldots h_n \pi^{\frac{n}{2}}}{\Gamma(i)} \int_0^\infty \frac{ds \left(\frac{d}{d\sigma}\right)^{i-\frac{n}{2}} f(\sigma)}{s^{i-\frac{n}{2}+1} \{(s+h_1^2)(s+h_2^2)\ldots(s+h_n^2)\}^{\frac{1}{2}}},$$

in which

$$\sigma = \frac{a_1^2}{s+h_1^2} + \frac{a_2^2}{s+h_2^2} \ldots + \frac{a_n^2}{s+h_n^2},$$

and $f(\sigma)$ is a discontinuous function, which is supposed to vanish when $\sigma \angle 1$.

As particular examples of this result, the author deduces the attraction of an ellipsoid on an external or internal point, when the force varies as the inverse square and as the inverse fourth power of the distance. In the latter case some remarkable consequences are seen to flow from the discontinuous character of the function $f(\sigma)$. When the density is uniform, and the point external, all the elements of the integral which precede or follow the break in that function vanish, while at the break a single finite element occurs. This gives a finite algebraic expression for this case of an ellipsoid's attraction. When the ellipsoid is of variable density, and the point external, the attraction is given partly by a finite algebraic expression, and partly by a definite single integral.

Similar remarks apply to all inverse even powers of the distance, except the square.

Dr. Allman read a paper on the larva state of Plumatella, and on the anatomy of Polycera quadrilineata.

In this paper the author described the occurrence in *Plumatella fruticosa*, Allm., of a larva state presenting a very different form from that assumed by the mature animal. This larva was discovered in a glass of water containing specimens of the adult Bryozoon, and when first observed was enclosed in a delicate, transparent, egg-shaped sac, through which the various parts of the contained embryo might be seen with ease.

Within this external sac the embryo is suspended in a transparent cell, the rudiment of the future polypedom, and presents distinct traces of tentacula, stomach, and intestines. The rudimentary muscles may also be plainly seen. The tentacula are as yet very imperfect, they are short and thick, when compared with the same organs in the adult animal, and strongly suggest the idea of having been formed by the longitudinal division of what had been, in a still earlier stage, a continuous infundibuliform membrane. The cell or rudimental polypedom is of an oval figure, densely ciliated posteriorly. Where the cilia terminate, the membranous walls of the cell present an invagination to a considerable extent, and are again reflected outwards to undergo a still further invagination, by which a sheath is formed for the rudimental tentacula, and the digestive organs suspended in the visceral cavity.

When the embryo Plumatella is released from the external egg-shaped envelope, its locomotive powers being now no longer restrained, it soon becomes evident how active a creature it is, for withdrawing the anterior portion of its cell, which, as we have already seen, is deprived of cilia, within the posterior ciliated portion, the latter is completely closed around the anterior end, and the little embryo thus becomes closely wrapped in a natatory mantle, through whose agency it is carried through the surrounding fluid in endless and elegant gyrations.

Beyond this point in the development of Plumatella, Dr. Allman was unable to adduce any observations; the little larva, however, now described, presenting as it does all the essential elements of bryozoal structure, belongs undoubtedly to a phase considerably in advance of the well-known locomotive gemmules of the marine bryozoa.

Dr. Allman also made some observations on the Anatomy of *Polycera quadrilineata.*

In this mollusk the buccal mass is furnished with two powerful corneous jaws, acted on by distinct muscles, and contains a spinous tongue of very complex structure. The œsophagus leads to a stomach, which is imbedded in the anterior extremity of the liver, and from which an intestine first passes forward and then curves backwards, to terminate at the anal outlet, which is situated between the two posterior leaflets of the bronchial tuft. Two pair of glands are connected with the buccal mass. The liver, though in its natural condition it is compact, like that of Doris and its allies, may yet be unravelled so as to display its minute structure; and it is then seen to consist essentially of a ramified tube, whose branches end in slightly dilated *culs de sac,* and are furnished along their sides with closely-set spheroidal divertacula, which would seem to be the essential secreting portion of the organ.

The heart consists of an auricle and ventricle, and occupies the back of the animal immediately in front of the branchial tuft; it is furnished with auriculo-ventricular and aortic valves.

The brain consists of six supra-œsophageal ganglia, and the nervous collar is completed below by a band of nervous matter, which passes from the most external ganglion of one side, round the ventral aspect of the œsophagus, to the corresponding ganglion of the other side; the nerves which supply the dorsal laminated tentacula are furnished at their base with large ganglionic dilations; and two small pharyngeal ganglia are placed upon the ventral aspect of the œsophagus just as this tube leaves the buccal mass.

The eyes are almost sessile upon the anterior ganglia of the brain, having the whole thickness of the integumentary structures intervening between them and the external world. They consist of a spherical lens, imbedded in the anterior extremity of a pigment cup, which is penetrated posteriorly by the very short optic nerve, and the whole surrounded by a delicate capsule.

The auditory capsules, with their otolites, are easily demonstrable.

The generative apparatus is complex. It is hermaphrodite; the male organs consist of testis vas deferens and penis, and the female of ovary, oveduct, uterus (?), and certain accessory glands.

The President appointed, under his hand and seal, the following Vice-Presidents:

George Petrie, Esq.
Captain Larcom, R. E.
Sir William R. Hamilton, LL.D.
Rev. Franc Sadleir, D.D., Provost.

DONATIONS.

Famine and Fever in Ireland. By J. Corrigan, M.D. Presented by the Author.

Annual Report of the Leeds Philosophical Society, for 1844-5. Presented by the Society.

London University Calendar, for 1846. Presented by the University.

Memoirs of the Astronomical Society. Vol. XV. *Proceedings of the Royal Astronomical Society, from November* 8*th*, 1844, *to January* 9*th*, 1846. Presented by the Society.

Journal of the Asiatic Society of Bengal. Nos. 75 and 76 (for 1845). Presented by the Society.

Transactions of the Linnean Society of London. Vol. XIX. P. 4. *Proceedings of the Linnean Society of*

London, from p. 237 to 268, for the year 1845. *List of the Members of the Linnean Society of London, for* 1845. Presented by the Society.

The Quarterly Journal of the Geological Society of London. Vol. I. Presented by the Society.

Reduction of the Greenwich Observations of Planets, from 1750 *to* 1830. *Greenwich astronomical Observations,* 1843. *Greenwich magnetical and meteorological Observations, for* 1843. Presented by the Royal Astronomical Society.

Geological Survey of the United Kingdom of Great Britain and Ireland. Horizontal Sheets, coloured, Nos. 1 to 17, inclusive. *Vertical Sections,* 1 to 8, and 10 to 15, inclusive. *Index Sheet of Colours.* Presented by the Earl of Lincoln, Chief Commissioner of Woods, Works, and Land Revenues.

Philosophical Transactions of the Royal Society of London, for 1845. Part II. *Proceedings of the Royal Society.* Nos. 61 and 62. *List of Members of the Royal Society.* 30 Nos. 1845. *Greenwich magnetical and meteorological Observations,* for 1843. Presented by the Royal Society.

Report of the Secretary of the Navy of the United States, communicating a Report of the Plan and Construction of the Depôt of Charts and Instruments, &c. February 18, 1845. *Third Bulletin of the Proceedings of the National Institute for the Promotion of Science at Washington.* Presented by the President of the United States.

Practice of the Court of Record of the Borough of Dublin. By William P. Pike, Esq. Presented by the Author.

Some Account of the Dominions of Furney. By Evelyn P. Shirley, Esq. Presented by the Author.

Etudes sur la Mortalitè dans les Bagnes, et dans les Maisons centrales de Force et de Correction, depuis 1822 *jusqu'a* 1837, *inclusivement.* Par M. Ravoul Chassinat. Presented by the Author.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846. No. 53.

April 27.

REV. HUMPHREY LLOYD, D.D., President, in the Chair.

Dr. Todd gave an account of an Irish Manuscript preserved in the Royal Library, Paris. Ancien Fond. No. 8175.

After refuting the conjecture of M. Champollion Figeac, that this is the identical volume which was sent from Bretagne by M. de Robien to Paris, for the use of the Benedictine authors of the *Nouveau Traité de Diplomatique*, Dr. Todd proceeded to give an account of the contents of the manuscript.

It is a parchment volume, containing now 117 leaves small folio, or what modern booksellers would perhaps call imperial octavo size. It was found by the Revolutionary Commissioners, during the French Republic, in some house in Paris, and by them deposited in the National Library, as is attested by a curious note inserted in the volume, in the handwriting of M. Villebrune, then Librarian or Conservator of that Institution. Of its previous history nothing is known, except what may be gathered from a few entries made by the original scribes and some of its possessors.

The volume may be divided into seven portions, which are, in fact, different works, having been written by different scribes, and at different times, although now bound together. These are:

I. A book written by William Mac an Legha, some sheets of which are misplaced by the binder. The book, however, when this error is corrected, will be found to be quite perfect, and contains ten tracts, viz.:

1. The History of the Children of Israel. A note at the end tells us, that this work was transcribed in the year 1473, in the space of two summer days; and that it was written by William Mac an Legha, at Cluan Lorg, in the house of Cormac O'Betnachan.

2. The History of King Solomon.

3. A Tract against defiling or profaning Churches.

4. A Legend of Hell and its Torments, entitled Tenga bith nua, "The Eternal new Tongue."

5. A Legend describing the Condition of Enoch and Elias in Heaven, entitled, Da bron flatha nimhe, "The two sorrowful ones of the Kingdom of Heaven."

6. A Legend of a holy Monk and a Woman who went to him for Confession.

7. A Legend of two Children, one Jewish, the other Christian.

8. A Legend of an Eastern Woman and her Child.

9. A Legend of St. Brendan.

10. The Reasons for making Friday a Day of Fasting.

II. The second book bound up in the volume is stated to have been written by Flathri (who calls himself in truag, "the wretch, or miserable") for Donogh, son of Brien Mac Conor O'Brien, who must have been the same as Brien Duff, son of Brien O'Brien of the battle of Nenagh, who was the first of that branch of the O'Brien family who settled in the Castle of Carrig O'Gonnell, County Limerick, about 1449; and, therefore, it follows that this portion of the MS. must have been transcribed after that year. The Book of Flathri contains the following tracts:

1. Charta humani Generis, seu Speculum Peccatoris.

2. A Tract entitled Brud grádha De, "Stimulus amoris Dei."

3. A Tract on Alms.

4. A Dialogue between the Body and the Soul. It is at the end of this that Flathri gives us the information above stated, from which we learn that he was the writer of the volume. And the following tracts seem to be also in his hand:

5. A Legend of the Virgin Mary. This is imperfect, some leaves being lost between what are now fols. 14 and 15 of the MS.

6. A Tract entitled Do ḟeir proceptra, "Of the Rule of Preaching."

7. A curious Tract on the institution of the Festival of All Saint's Day.

8. On the Miracles attending the birth of our Saviour.

9. A Sermon on the Text, "Intrate per angustam portam."

10. The history of the Right of spiritual Direction of the Men of Ireland.

11. On the Virtues of Faith, Chastity, Humility, Charity, Fortitude, and Temperance.

At the end of this are notes in Irish, in different hands, giving two different calculations of the number of leaves in the volume. One of these states that it contains six score (which is corrected by another hand to seven score) leaves and one. But a later entry makes the number seven score, and a still later note adds, "and three leaves over."

We gain but little information from this note: because it must always be uncertain whether it refers to the whole volume, or only to that part of it which was written by Flathri for Donogh O Brien. If it refers to the whole volume, the loss sustained since the seventeenth century, when the note appears to have been written, will amount to twenty-three leaves. If only to the book of Flathri, the loss will be 130 leaves. Let us hope, therefore, that the note related to the entire volume, which is, perhaps, the more probable supposition. There is also another uncertainty attaching itself to the Irish mode of counting by scores, for it was very common to count six score to the hundred; and it is curious that if we count the volume so, the number of leaves it now contains will be exactly seven score and one; so that, on this supposition, the volume has remained uninjured for the last two hundred years.

III. The next part is a collection of the Lives of Saints, not all in the same hand. The name of the scribe is not given, but the great mass of this part of the manuscript appears to be in the hand of William Mac an Legha. It contains the following tracts:

1. Life of St. Maighnen, Abbot and Founder of Kilmainham, near Dublin.

2. Life of St. Mochua, founder of the Church of Timohue, in the Queen's County.

3. Life of St. Senan, of Scattery Island, in the mouth of the Shannon.

4. Life of St. George.

5. Life of St. Gregory the Great.

6. Life of St. Longinus, who pierced our Saviour's side on the Cross, and became blind in consequence, but was converted to Christianity.

7. Life of St. Juliana.

8. Life of the four Donalds. A Legend, which begins by telling us of three students who came from the diocese of Connor to be educated by Maolsuthan O Carroll, of the Eoghanacht of Loch Lein, and abbot of Inisfallen in the Lake of Killarney. This Maolsuthan, the story tells us, was spiritual director to Brien, son of Kennedy, i. e. to Brien Boru.

9. A Legend of Nicomedes, or Joseph of Arimathea.

10. Life of St. Columba or Columbkille. Followed by the curious legend of the saint, whilst he resided in the island of Aran in the bay of Galway: a tract of which we know no other copy.

11. The Legend of the Seven Sleepers. This tract ends imperfect, some leaves being lost, between fol. 57 and 58 of the manuscript.

IV. The next portion of the volume contains three theological treatises:

1. A Sermon on the verse *Novet in principio vigoris mei.* The initial letter N (which M. Champollion has mistaken for F) is illuminated in red and ornamented, shewing that here began a distinct book, which afterwards came to be bound up with the rest, but has no other connexion with it. A facsimile of the beginning of this Sermon is given by MM. Champollion and Silvestre, in the *Palæographie Universelle.*

2. Some Letters (apparently of Pope Innocent III.), translated into Irish.

3. A Dialogue between the Body and the Soul. This is the same tract of which another copy occurs also in this volume, in the portion of it written by Flathri for Donogh O Brien. This is probably the original, for a note at the beginning of it tells us that it was translated into Irish by William Maguibhne [Mac Gawney], and that Daniel O'Connell induced him to do so in the year of our Lord 1443. The tract is imperfect, some leaves being lost between fol. 73 and fol. 74.

V. Then follows another collection of Lives of Saints, containing three lives: this is the oldest portion of the MS. and is unfortunately imperfect at the beginning. It appears from the handwriting (for no other means remain of determining its age), to have been written in the 14th, or beginning of the 15th century. It contains—

1. A fragment of the Life of St. Patrick, imperfect at the beginning.

2. The Life of St. Bridgit.

3. The Life of St. Brendan, imperfect in the middle; the defect is supplied, however, by a more recent hand: so that the tract is complete, although not in its original state.

VI. Next follow two tracts, written, as appears from a note at the end, by Mailechlain, son of Illan Mac an Legha, for Donogh, son of Brien Duff O'Brien, "the head of the hospitality and munificence of the English and Irish of Ireland (Gall 7 Gaidel nErend), the year in which the son of the Earl of Ormond was treacherously killed by the Butlers." The son of the Earl of Ormond here mentioned, was probably James, commonly called Black James, natural son of James, fifth Earl of Ormond, who was slain by Sir Pierce Butler, between Dunmore and Kilkenny, March 17, 1518. The tracts are—

1. An ancient Exposition of the Lord's Prayer.

2. An Account of the Destruction of Jerusalem, entitled "The Avenging of the Blood of Christ."

VII. A miscellaneous collection of Theological Tracts, containing

1. The Vision of St. Adamnan.

2. An Account of the King of the Medes and Persians.

3. Liber Sententiarum. A fac simile of part of this Treatise is given by M. Champollion in his *Palæographie Universelle.* It consists of nineteen chapters.

4. A Treatise on Repentance.

5. A Tract entitled, Gleo Michil ler in beirt, "Michael's Combat with the Monster."

6. A Tract entitled, "The Ambition of the Angel, and the banishment of Adam out of Paradise."

7. A very short Tract, without title, on the same subject. On the lower margin the transcriber has written in Irish, "I have not found any more of this narrative to write;" so that it is probably incomplete.

8. A Tract entitled, "Words on the Sacrament:" This is a sermon or theological discourse on the Lord's Supper. At the end is this note: "I, John, son of the Earl of Desmond, wrote this at Carrig o Gonnell (a Carraig o Coinnell) in order to assist my companion, and faithful tutor, Mailechloin Mac Illion." This was probably John, son of Thomas, Earl of Desmond, who died 1536.

9. A Tract entitled, "History of the Monks of Egypt."

The remaining pages of the volume contain only some scribbling of no importance or interest.

Mr. Huband Smith exhibited to the Academy a "rubbing" taken from the tombstone of William O'Byrne (A. D. 1569), in the cathedral of Old Leighlin, county of Carlow.

This tombstone has been noticed but slightly in the "History and Antiquities of the County of Carlow," by John Ryan, Esq., published in 1833, from which Mr. Smith read a passage (pp. 344 and 345), in which a few words of the inscription are given, so as to identify the stone, which is said to be "generally reputed, even by men of education, to be that of a Bishop Kavanagh," but the writer professes his "inability to decipher the entire," and adds, that he "could not discover the exact year inscribed on the tomb."

The rubbing, now exhibited by Mr. Smith, was made by Mr. Robert J. Gabbett, of Cahirmoyle, County of Limerick, and the inscription, as deciphered from this rubbing, is as follows:

Hic jacet Willelhmus obrin filius inominati filii Willelmi filii Dabid rufi Generosus de Corraloske et ballenebrenagh ac burgensis Veteris Leghleniensis——obit xbii. die mensis Junii Aº. dⁱ. M. ccccc°. lxix. et ejus Uxoris Winna Kewanagh filia Maurici filii donati (   ) monens qui obiit . . . . die mensis . . . . Aº. dⁱ. M. ccccc. . . . . Quorum animabus propicietur deus. Amen.

Several contractions occur in the inscription, which, however, are easily filled up; a few letters also are wanting on the edge of the stone, which Mr. Smith had little doubt he supplied correctly from the context. The only word he was unable to read was the title, or designation, as he supposed, following the name of Donatus, or Donogh Kavanagh, and ending in the dissyllable "monens." Blanks are left on the stone for the exact date of the decease of Winna Kavanagh, from which Mr. Smith stated his conjecture that she was still living when the stone was placed over her husband's grave. The name "inominatus," Mr. Smith suggested, might be a latinization of the Irish Christian name "Fearganonym" (literally "a man without a name"), which appears frequently in the Patent Rolls and other historical documents of this period, as one commonly in use among various septs and families of Irish descent. He read an extract from the Patent Roll of the 28th, 29th, and 30th Henry VIII. (dorso, lxxi. 3.) of the enrollment of an "Indenture between Lord Leonard Gray, Viscount Grane, the King's Deputy, and Fergynanym Roe O'Byrne, whereby it is agreed that the said Fergynanym shall be the king's faithful subject, and serve at hostings with his power, at his own expense; that he shall pay to the King's use four-pence Ir. yearly, for every horse, mare, cow, bull and ox, being in future in the town of Ballihorsy, Cowlyth, Dwly, Dromor, and Kilparke. And the Deputy shall maintain and defend said Fergynanym, and his tenants, &c., and the possessions in the towns aforesaid, against all men, as well English as Irish."—17 Sep. 1536.

The O'Byrne mentioned in this indenture, Mr. Smith supposes was the same whose tomb remains in the Cathedral of Old Leighlin.

In conclusion, he suggested that careful rubbings of the tombstones which yet remain in the various churches and abbeys, especially in Kilkenny and Cork, and other places in the south of Ireland, and which are every day fast disappearing under the hand of time, would preserve a vast deal of curious information, of no inconsiderable value to the topographer, the genealogist, and the historian.

Mr. Ball brought under the notice of the Academy, as an unobserved fact, a beautiful provision in the fœtus of the spined dog-fish (*Acanthias vulgaris*), by which the mother is protected from being lacerated by the spines of the young before birth. He exhibited two perfectly developed young, which he had taken from the mother on the 30th of November last; in these the spines were each covered at the point with a small knob of cartilage, fastened by straps of the same material, passing down one on each of the three sides of each spine, in such a manner as evidently to become easily detached at birth, thus allowing the little animal to commence life effectively armed. He mentioned that the female in question contained a large number of eggs, in various states of development, in addition to the two fully-formed young; and he took occasion to remark, that this fish is so destructive to herrings that fishermen look on it with abhorence: in this he thought they were wrong, for he considers that some of the success of fishing with driving nets is to be attributed to the headlong haste with which shoals of herrings go along when pursued by enormous packs of dog-fishes, and that thus they serve man rather than injure him. Fishermen, however, destroy the dog-fish whenever it falls into their power, as they did the specimen which gave occasion to this notice.

Dr. Allman mentioned an analogous fact in the ova of *Cristatella mucedo.*

Rev. N. J. Halpin commenced the reading of a paper on some passages in the life of Shakspere.

Rev. Dr. Drummond presented to the Museum an ancient Ogham inscription, on the part of Francis W. Jennings, Esq.

Mr. Robert Mallet presented a drawing of a silver antique ring found in Ireland, and presented to the British Museum by Lord Enniskillen, containing an inscription in characters resembling Chinese.

DONATIONS.

Proceedings connected with the magnetical and meteorological Conference, held at Cambridge, in June, 1845. Presented by the Association.

The Laws of the Tides on the Coast of Ireland, as inferred from observations made in connexion with the Ordnance Survey of Ireland. By G. B. Airy, Esq. F.R.S.&c. Presented by the Author.

Transactions of the Geological Society of London. Second Series, Vol. VII. Parts 1 and 2. Presented by the Society.

On the Heat of Vapours. By Sir J. W. Lubbock. Presented by the Author.

Philosophical Transactions for the Year 1845. Part 1. Presented by the Royal Society.

Reduction of the Greenwich Observations of Planets, from 1750 to 1830. *Greenwich Astronomical Observations,* for 1843. Presented by the Royal Society.

Journal of the Royal Asiatic Society. No. XVI. Part 1. 1845. Presented by the Society.

Dramatic Sketches, and other Poems. By the Rev. James Wills, M.R.I.A. Presented by the Author.

Eleventh Annual Report of the Poor Law Commissioners, with Appendices, for 1845. Presented by the Commissioners.

The Repeal Dictionary. Part 1. By John O'Connell, Esq. M.P. Presented by the Author.

Inaugural Address to the Members of the Temperance Institute in Cork. By Edward Kinnealy, Esq. Presented by the Author.

An old Dutch Tobacco Box, found near Newmarket, County of Clare. Presented by Sir Lucius O'Brien, Bart.

Vertical Sections of the Geological Survey of Great Britain. Sheet No. 9. Presented by the Chief Commissioners of Woods and Land Revenues, on the part of Her Majesty's Government.

Results of the Mackerstoun Observations. No. I. Presented by J. Allan Broun, Esq.

May 11.

REV. HUMPHREY LLOYD, D.D., President, in the Chair.

John Aldridge, M. D., and George Lefroy, Esq., were elected Members of the Academy.

The reading of the Rev. N. J. Halpin's Paper, on some passages in the life of Shakspere, was resumed and concluded.

The object of this paper is to vindicate the poet's memory from aspersions thrown upon his character, as a father and a husband, by Malone, Drake, De Quincey, Moore, &c.

Those aspersions—unfounded in either fact or tradition—are chiefly inferences, rashly drawn, from the poet's last will and testament, and consist of two charges, viz.: favouritism towards one of his daughters, and neglect of his wife: which Drake lays down as "the most striking features" of that document. To these are added, from other sources, calumnies respecting the education of his children, and jealousy of his wife; all of which it is the object of the paper to refute.

The inference of favouritism towards his eldest daughter, deduced from the unequal division of his property, is shown to be false, by proving that the inequality was the result of the undutiful conduct of the younger, who had married a person of inferior station, who was either unable or unwilling to make a settlement upon her or her issue; whereas the elder had made a match to her father's entire satisfaction. Malone, Drake, &c. assert that, at the time of making his will, the poet was ignorant of his second daughter's marriage, and still spoke of her as an unmarried woman; whereas the reverse is the fact. He was aware of the marriage, and thereupon made the final disposition of his property; and though his resentment prevented him from mentioning the husband's name, he still indirectly recognizes the marriage, by including him as the husband with whom, at the end of three years, his daughter may be found united in wedlock. The clauses of the will (from which these facts are elicited) are fully discussed, and the memory of Shakspere rescued from the charge of "favouritism" or unjust partiality.

The charge respecting his supposed neglect of his wife is next examined. It rests upon three points: the omission of her name in the first draft of his will; the neglect of any ostensible provision for her support; and the interlineation which conveys to her his "second best bed." Malone, followed by Drake and others, interprets these facts into proofs of the unhappiness of the marriage through life, and the unfriendly feeling with which it closed in death. The last point, in particular, Malone construes into "cutting her off," not with a shilling, but an old piece of furniture; and Moore translates it into "a bitter sarcasm."

With reference to the first, Mr. Halpin argues that the poet's omission of his wife's name in the first draft of the will, and the subsequent interlineation, no more imply the absence of conjugal love, than the similar omission and interlineation of his friends' and fellows' names (Burbage, Hemings, and Condell) intimate his want of friendship for them—an inference which, in their case, the biographers never thought of drawing.

With reference to the second, Mr. H. concurs, to a certain extent, with Mr. Charles Knight's solution of the point, namely, the provision already made by law,—the widow being entitled to dower, or the third of all her husband's freehold property during her life; and further suggests, from other provisions of the will, the extreme probability that she and her children had been, previous to her marriage with the poet, provided for by a marriage settlement, and that she consequently had brought him a fortune.

With respect to the third, the vindication which Stevens suggested is confirmed by reference to the testamentary habits of the times; and the bequest is proved, by parallel with a similar bequest of William Herbert, first Earl of Pembroke, to Queen Elizabeth, to have conveyed, not "a bitter sarcasm," but the tender memorial of a love and attachment surviving the grave.

The calumnies derived from the will being thus disposed of, Mr. Halpin next adverts to the poet's alleged neglect of the duty of a father in the education of his children. Drake asserts that neither of his daughters had been taught to write; and sustains his assertion on the evidence of a legal document still existing, attested, as he thinks, by the mark of his daughter Judith.

This calumny the author treats as a superfetation of the similar degrading ignorance ascribed to the father of the poet, but without the shadow of evidence in the document on which it is founded. A fac-simile of the signatures to the entry on the books of the corporation of Stratford-upon-Avon (published by Mr. Knight), amongst which John Shakspere is said to have figured as a marksman, exhibits the name of that worthy corporator without any mark; proves that the mark assigned to him belongs, in reality, to George Whately, the high bailiff for the current year; and leads to the juster inference that, so far from not being able to write at all, John Shakspere probably wrote the best hand of any man in the corporation.

With reference to the daughters, the assertion as to both is disproved by the production of a *fac-simile* of the signature of the elder (Susanna) affixed, with her seal, to a legal instrument still existing; and with reference to the younger, it is shewn to rest on an ignorant or wilful mistranslation of the word *signum* into mark, instead of seal, &c.

The paper goes on to argue, from the nearness of their births, that both daughters were educated together; that whatever instructions or accomplishments the one had received, the other had at least the same opportunities of acquiring; and that, as Susanna is recorded to have been a woman of high attainments, it may be justly inferred that Judith was not deficient.

The character of Susanna Shakspere is then discussed, and her moral, intellectual, and poetical faculties asserted on the evidence of a contemporary. That her education embraced the Latin language, at least, is proved by the production of one indisputable, and several probable, pieces of her composition in Latin, as well as English metres; and the whole is brought to a conclusion by a vindication of her mother's memory, on the testimony to her virtues furnished in the monumental inscription placed over her remains by the piety and love of this exemplary daughter.

Edward J. Cooper, Esq., made some remarks upon the four Comets which were lately visible, and which were all observed in one night at Markree—namely, Biela's double comet, the two comets of De Vico, and Brorson's comet. He likewise read a communication from Professor Schumacher respecting a fifth comet, which has just been discovered.

The President commenced the reading of a paper "On the Variations of the Magnetic Declination at Dublin, as deduced from four years' observations."

DONATIONS.

Abhandlungen der Königlichen Gessellschaft der Wissenschaften zu Gottingen. II ter *Band von den Jahren* 1842–4. Presented by the Society.

Bericht über die zur Bekanntmachung guigneiten Verhandlungen der Koniglichen Preuss. Akademie der Wissenschaften zu Berlin. From July, 1843, to June, 1845.

Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. For 1842 and 1843. Presented by the Academy.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846. No. 54.

May 25.

REV. HUMPHREY LLOYD, D.D., President, in the Chair.

It was resolved, on the recommendation of Council, that a subscription be opened for the purchase of the *Domnach Airgid*, and that the Academy do subscribe fifty pounds.

The President resumed and concluded the reading of his paper "On the Variations of the Magnetic Declination."

The observations at stated hours, in the Magnetical Observatory of Dublin, commenced November 1, 1838, the hours of observation being at first limited to the day. In the beginning of the year 1840, the observations were taken *every alternate hour*, night and day, according to the more comprehensive scheme which received the sanction of the Royal Society, and which, under the recommendation of that body, has been followed in more than thirty observing stations scattered over the whole globe. This plan has been in operation at the Dublin Observatory until January 1, 1844, when it was discontinued, four years' observations having been found fully sufficient for the determination of all the phenomena connected with the diurnal movements. The results now laid before the Academy are chiefly those of the four years just mentioned. The observations have since been continued upon a different and reduced scale, and with a view to other classes of phenomena.

The changes which the magnetic declination undergoes, at a given place, may be reduced to three classes, namely, 1. *periodical variations;* 2. *secular variations*, which are either continually progressive, or else return to their former values in long and unknown periods; and 3. *irregular variations*, which observe, apparently, no law. The periodical variations hitherto noticed are those which depend upon the position of the Sun with respect to the horizon, or with respect to the equator, and which, therefore, complete their course in a *day*, or in a *year*. The author commences with the first of these.

Diurnal Variation.

In order to determine the laws of the diurnal changes, the observations of each month are combined separately, and the means of the results corresponding to the same hour taken. The observations having been continued for four successive years, there are thus four groups of mean results for each month, the means of which are then taken. And, finally, the mean values for the separate months are grouped together, so as to obtain the mean *yearly*, as well as the mean *summer* and *winter* course of the variation. The total number of individual observations thus combined exceeds 14,000.

When the mean results at each hour of observation, for the *whole year*, are examined, it is found that the course of the diurnal variation is regulated by the following laws:

1. About 7h. 30m. A.M., the north pole of the magnet begins to move to the *westward*, and, therefore, the declination *increases*. This increase continues until about 1 P.M., when the declination attains its *maximum*.

2. The north pole of the magnet then moves to the *east*,

and the declination *diminishes*, but at a slower rate than it has increased. This diminution continues until about 10 P.M., when the declination is a *maximum*. The whole diurnal *range*, between 1 P.M. and 10 P.M., is about 9.4 minutes.

3. During this increase, and subsequent decrease, the declination twice reaches its *mean* value, as deduced from the results of the entire day. The epochs of the mean declination are 9h. 30m. A.M., and 6h. P.M. nearly.

4. There is a second, but much smaller oscillation of the magnet during the night and morning, the north pole moving slowly to the *west*, from 10 P.M. until 3 A.M., nearly, when there is a second *maximum*, and then returning to the *east* until 7 A.M., when there is a second *minimum*.

When the means corresponding to the several hours of observation, taken for the six summer months, and the six winter months, separately, are examined, it is found that the course of the diurnal variation is very different in the two seasons. The following are the distinguishing circumstances:

1. The range is much greater in summer than in winter. The mean summer range amounts to 12·2 minutes; the mean winter range to 9·0 minutes.

2. In summer, the *morning* minimum is greater than the *evening* minimum, which is not the case in the curve for the entire year; and, consequently, the greatest range is between 7 A.M. and 1 P.M. In winter, on the other hand, the morning minimum, and the small preceding maximum, disappear altogether; and the course of the diurnal change presents only a single oscillation.

If now the inquiry be extended from the *seasons* to the *separate months*, it is found that the course of the diurnal change, in *each* of the six months from April to September inclusive, has all the characters of the mean summer variation, both as respects the *law* of the change and its *amount*. In like manner the diurnal variation, in each of the six winter months, corresponds with the mean variation for the whole period; the nocturnal oscillation vanishing, and the course of the representative curve, from 10 P.M. to 9 A.M., being in all nearly a straight line. Thus the curves for the separate months appear to distribute themselves into two groups, depending upon the position of the sun to the north or south of the equator; and it is remarkable that the transition from one to the other system appears to take place abruptly, and almost *per saltum*.

The whole *range* is nearly the same in each of the six summer months. The *maximum range* occurs in April, and its amount is 13′.3; this maximum is followed by a secondary minimum in July, which is succeeded by a secondary maximum in August. There is a sudden change in the magnitude of the range from February to March, and again from October to November. The *minimum* range occurs in December, and its amount is 7′.0.

The physical dependence of the phenomena of the changes of the declination upon the sun is evident from the fact, that they observe a diurnal and an annual period. In addition to this fundamental fact, it has been long ago observed, with respect to the diurnal change, that the time of the maximum of westerly declination follows the sun's meridian passage at a nearly constant interval, and that the morning and evening minima are in like manner connected (although not so closely) with the hours of sunrise and sunset; and another point of connexion between the cause and effect has been established by the fact, long since observed, of the greater magnitude of the range in summer than in winter.

Dr. Lloyd proposes to show that the sun acts by means of its heating power (as, in fact, is assumed both in the hypothesis of Canton and that of Christie); and that the connexion between the changes of declination and those of temperature is more intimate than has been hitherto supposed.

The force which produces the deviation of the magnet from its mean position, at any moment of the day, is measured by the sine of that deviation,—or, since the deviation is small, by the angle of deviation itself, or by the ordinate of the diurnal curve; and the sum of all these forces throughout the day, or the integral of the diurnal action, is measured by the *area of the diurnal curve.* Dr. Lloyd has computed this area for the several months of the year; and, on comparing it with the corresponding area of the diurnal curve of temperature, he finds that there is a marked agreement in the course of the two functions. The slight dissimilarities which exist between them may be accounted for by the circumstance, that it is to the heating power of the sun, exerted upon the *earth's surface*, and not upon its *atmosphere*, that we must ascribe the changes of declination; and the author feels assured, that as soon as we are in possession of data, respecting the diurnal changes of temperature of the earth's surface, sufficient to institute a comparison similar to that now made with the temperature of the air, the agreement of the laws will be found to be still more complete.

Annual Variation.

The annual variation of the declination was discovered by Cassini, in 1786. It appeared from the observations of Cassini, that the north pole of the magnet moved to the *east* during three months, viz., from the vernal equinox to the summer solstice; and, consequently, the declination *diminished.* During the remaining nine months, viz., from the summer solstice to the vernal equinox, it moved to the *west*, and the declination *increased.* The increase, during the nine months, preponderated over the decrease, which took place during the remaining three; and thus the declination was greater at the close of the year than at the commencement. This excess is the yearly amount of the *secular* change, which was then *additive.*

Although the law of the annual variation may be traced in the subsequent observations of Gilpin and Bowditch, it has, nevertheless, escaped the attention of more recent observers. There is but a faint indication of its existence in the Gottingen observations, which were made at the hours of 8 A.M. and 1 P.M.; and Professor Gauss finds, in the mean results deduced from these hours, no "important fluctuation dependent on season." A similar negative result is deduced by Dr. Lamont from the Munich observations, which were made twelve times in the day.

It will be easily understood, that the determination of the annual variation is much more difficult than that of the diurnal change; both on account of the much smaller frequency of the period itself, and the difficulty of preserving the instrument in the same unchanged condition during the much longer time, or of determining and allowing for its changes when they do occur. The Dublin observations appear to possess a peculiar value for this determination. Since the spring of 1841, the magnet of the declinometer has remained absolutely *untouched;* and the suspension thread (which elsewhere has frequently broken) has continued perfect since the instrument was first mounted.

The course of the annual variation in Dublin, as deduced from the results of the years 1841–4, is as follows:

1. During the first three months of the year, the mean daily declination is *nearly constant.*

2. In the month of April the declination begins to *increase;* and it continues to increase until the beginning of August, when it attains its *maximum.*

3. From the beginning of August it *decreases;* and the decrease continues, with rapidity, until the end of the year, when the declination is about five minutes less than at the commencement.

4. The *increase,* from the beginning of April to the beginning of August, is four minutes. The *decrease,* from the beginning of August to the beginning of January, is nine mi-

nutes; thus leaving a total decrease, from year to year, of five minutes.

When we compare the course of these changes with those observed by Cassini, we find that the directions of the movements are precisely opposite,—as, indeed, we might have been led *a priori* to expect from the opposition in the directions of the secular changes; and we are led to generalize the law as follows:

"From the beginning of the year to the beginning or middle of April, the mean declination undergoes little or no change. Thence, to the beginning of August, its movement is *retrograde* (or opposite in direction to the secular change); and from the beginning of August to the end of the year, it is *direct.*"

The phenomena just described are, it is manifest, the resultants of two distinct changes, namely, the *annual variation* properly so called, and the *progressive* or *secular change.* If the latter be subtracted, at the rate of 5.′0 per annum, or 0.′4 per month, the remaining numbers give the true annual period. When thus considered, the annual variation exhibits an *increase* of the declination during the first *seven months* of the year, and a *decrease* during the remaining *five months;* the apparently stationary condition of the declination, during the first three months, arising from the mutual compensation of the *periodical* and the *progressive* changes.

It appears, then, that the annual variation (unlike the diurnal in this respect) is a *single* oscillation. The *minimum* occurs near the end of January, and the *maximum* in the beginning of August; and the whole *range* of the change is 6.6 minutes.

The laws of the *annual variations* of the declination and of the temperature, present the most complete accordance in the epochs of *maxima* and *minima,* as well as those of the *mean* values. The maximum of temperature occurs about August 1,—that of declination about August 8. The minimum of temperature takes place about January 15,—the minimum of declination about January 25. Finally, the annual curve of temperature crosses the axis of abscissæ in two points, which correspond to May 1, and October 10,—the corresponding epochs in the curve of declination are May 10, and October 15.

Secular Variation.

The westerly declination is at present *diminishing* from year to year in these countries, and has been so since the year 1818, which was the time of the maximum. The present rate of the secular decrease in Dublin, as deduced from four years' observations, is 5.0 minutes annually.

With respect to the physical cause of the secular change, Dr. Lloyd said that he had been led to form an opinion very different from any of those heretofore held. From the remarkable relation which had been shown to exist between the annual and the secular changes, he was driven to conclude that they depended (ultimately at least) upon a common cause; and that thus the sun was the cause of the *secular*, no less than of the *periodical* changes, although not only the magnitude, but even the direction of the effect were different in different times.

Disturbances.

Having examined the *periodical* and the *secular* variations of the declination, as deduced from the observations made at the Dublin Observatory, it now remains to consider those which, from our ignorance of their laws, we have been accustomed to call "*irregular.*"

Professor Kreil seems to have been the first to notify the remarkable fact, that magnetic *disturbances* occur more frequently at certain hours than at others; and, that the *direction*, as well as the *frequency*, of these movements, has a dependence upon the time of the day. Colonel Sabine has since made a more complete and elaborate examination of this question, in his able discussion of the Toronto observations, and has arrived at conclusions for the most part confirmatory of those obtained by Professor Kreil.

In these investigations, however, those disturbances only are taken into account which exceed a certain arbitrary limit; and, even of these, the *frequency* is considered without any reference to their *magnitude*. In examining the question of the periodicity of disturbances, Dr. Lloyd has thought it necessary to pursue a different course. His method consists in taking the differences between each individual result, and the monthly mean corresponding to the same hour, and combining these differences in the same manner as the errors of observation (to which they are analogous) are combined in the calculus of probabilities. The square root of the mean of the squares of these differences is, in fact, a quantity analogous to the *mean error*, and which he therefore proposes to call the *mean disturbance*; and it is evident that its values, at the several hours of the day, and at the several seasons of the year, are measures of the *probable* disturbance to be expected at the corresponding times.

The values of this function have been deduced for the several hours of observation, in each month of the year 1843; and those for the entire year are obtained from them by a repetition of the same process. These numbers show that the mean disturbance follows a law of remarkable regularity in dependence upon the hour. During the day, i. e. from 6 A.M. to 6 P.M., it is *nearly constant*; at 6 P.M. it begins to *increase*, and arrives at a *maximum* a little after 10 P.M.: it then decreases with the same regularity, and arrives at its constant day-value about 6 A.M.

The preceding results are independent of the direction of the disturbance. If, however, we take the sum of the squares of the easterly and westerly deviations separately, we find that the *easterly* disturbances preponderate during the night, and the *westerly* during the day; the former being, however, much more considerable than the latter, and the difference reaching a maximum about 10 P. M.

It thus appears that the mean daily disturbance observes a regular period, both in magnitude and direction; and this period, it is worthy of remark, is precisely the reverse of that of the regular diurnal movement,—the *mean position* of the magnet being nearly constant during the *night*, the *mean disturbance* during the *day*;—the principal oscillation of the magnet, in the *regular* movement, being to the *west* during the *day*, while that of the *irregular* movement is to the *east* during the *night*. From these remarkable relations it seems evident that the two classes of phenomena are physically connected; and Dr. Lloyd is led to regard the disturbance which prevails about 10 P.M., as an *irregular reaction* from the regular day movement, and dependent upon it both for its periodical character and for its amount.

If this hypothesis be a just one, it will, of course, follow that the magnitude of the mean disturbance will vary, in some direct proportion to the daily range, and should, therefore, be greater in summer than in winter. This (which is contrary to the results deduced by Professor Kreil and Colonel Sabine, with reference to the *frequency* of disturbances exceeding a certain limit) appears to be the fact. The mean disturbance, deduced from the observations of 1843, is, for the summer six months, 2′.9, and for the winter 2′.2; so that it observes an *annual* as well as a *diurnal* period.

It by no means necessarily follows, from the results now stated, that *all* disturbances have a periodical character. There probably are *two classes* of disturbances, the results of distinct physical causes, of which one observes a period, while the other is wholly irregular; and it is manifest that, in such a case, the period of the former will necessarily be impressed upon the *resultant* mean disturbance. Dr. Lloyd stated that he had instituted, during the last year, a series of observations at short intervals, which seem to afford the means of testing this hypothesis, and of distinguishing the two classes of disturbances, if they really co-exist. The inspection of these observations has nearly satisfied him of the truth of this view; and he believes that it will be found, upon a more minute examination, that there are *two classes of disturbances*, one *periodical and local*, the other *irregular and universal.* Of the former, the principal (if not the only one) is that which occurs about 10 P. M., and which causes the north pole of the magnet to deviate to the east. The magnitude of this disturbance is on the average $10'.0$; and its mean duration is an hour and a half. The epoch of the maximum of easterly deflection varies from $7\frac{1}{2}$ P. M., to $1\frac{1}{2}$ A. M., the mean epoch being a few minutes before 10 P.M.; and, hence, it is evident, that its effect on the monthly mean curve is to produce a general increase of the negative ordinate between these limits of time, as well as the minimum which occurs at 10 P. M.

It seems to follow also, from these facts, that the ordinary mode of grouping the observations, by taking the mean of all the results at the same hour,—although it truly gives the *mean* diurnal curve for the period embraced by the observations,—does not represent the actual course of the movement during any one day. In order to obtain the *representative*, or *type* curve, as it may be called, it seems necessary to combine the results in a different manner, of which the author hopes to speak more fully upon a future occasion.

Mr. Ball exhibited a specimen of Apteryx Australis, recently purchased for the University Museum, and made some observations on the species, referring to the elaborate papers of Yarrell and Owen in the Transactions of the Zoological Society of London. He noticed the adaptation of the position of the nostrils of the bird to its wants; being placed at the end of the bill, it is enabled, by its powerful olfactory apparatus, to detect the burrowing larvæ on which it feeds. This is accomplished by snipes, woodcocks, &c., by means of extraordinary development of nerves on the end of the bill. They obtain their food by the delicacy of their power of feeling, as the Apteryx does by its powers of smelling.

Professor Graves read the following memorandum in reference to the discovery of the remains of a cedar ship, near Tyrella, in the county of Down, by George A. Hamilton, Esq., M. P.:

"About the year 1796, the late Mr. Hamilton, in pulling down some old buildings at Tyrella, found, to his surprise, that the beams, lintels, &c., were composed of cedar. On inquiry, he was informed that there was a tradition (for even then it was but a tradition) that a large ship, from the coast of Guinea, laden with slaves, ivory, and gold dust, had been wrecked, a great many years previously, in the Bay of Dundrum, and a considerable portion of her swallowed up in the sands near Tyrella; and though the shore, in the lapse of so many years, had undergone considerable changes, there was still a mark known by the country people under the name of 'The Cedar Ship.' I remember this well when a child. There were two pieces of wood, covered with sea-weed, to be seen at very low tides, sticking up in the sand.

"About the year 1815, my father, having collected a number of men, and having made an excavation in the sands, discovered the upper works of the ship, and succeeded in obtaining six elephants' tusks, a considerable quantity of cedar, a silver goblet, and the remains of chains supposed to be those with which the slaves had been confined.

"The situation of the wreck being under the level of low water, and the soft oozy nature of the sand rendering the work extremely difficult, prevented his proceeding further with the excavation.

"On the 10th of November, 1829, it occurred to me to make a similar attempt. The marks of the ship had been long effaced, and I found some difficulty in discovering the place. I succeeded, however, and in one tide I obtained sixteen elephants' tusks, a large quantity of cedar, four cannons, the remains of a number of swords, muskets, and chains, a number of small shells, some coral, a piece of metal, nearly in the shape of a horse-shoe, which, at the time, we supposed to be the handle of a trunk, and several pieces of a heavy metallic substance.

"Sir Charles Giesecke stated this substance to be a kind of iron dross, probably of volcanic production, which is abundant on the coast of Guinea, and the shells have been classified as of that description which the inhabitants there use for money."

Professor Graves exhibited specimens of the shells, coral, &c.; and mentioned that the piece of metal, supposed to have been the handle of a trunk, was one of the *manillæ*, or bracelets, used to this day for the purpose of barter by merchants trading on the coast of Africa, and identical in shape with the massive gold ornaments frequently found in Ireland.

Professor Graves also read the following memorandum, by Mr. Hamilton, relative to the discovery of what is termed by the country people "a North House," in the demesne of Hampton, and the opening of a tumulus near Knockingen:

"In the month of September, 1840, my brother-in-law, Mr. Rowland Burdon, of Castle Eden, in the county of Durham, being on a visit at Hampton Hall, it occurred to me one morning to ask him to examine two hillocks near Barnageera, in this neighbourhood, in order to ascertain whether they were artificial mounds, or whether they were some of those natural heaps of gravel called Eskers, which are found so frequently across Ireland.

"Mr. Burdon had satisfied himself that the first which he examined was natural, when his attention was attracted by a large stone in the face of a ditch, which had been made recently, traversing the hillock; he found it to be a flag, and, when pulled down, it proved the head-stone of a rude stone coffin, with a skeleton encased. There was no weapon or coin, or anything to indicate the date or circumstances of the interment. On learning this, we proceeded to make an excavation in the second mound, and found there also some bones, and a broken pipe, of a very large size, but in shape resembling the common tobacco pipes of the country.

"While thus engaged, an old man, one of my tenants, came up to me, and inquired whether I had ever seen 'the North House,' that had been found on my property, not far from the place where we then were. I had never heard the term 'North House' before, and asked him what he meant. 'Oh,' he said, 'a kind of house under ground, made of large flags and stones, with a passage like a large sewer leading to it.' The North House, to which he referred, proved to have been completely destroyed; the stones had been carried away for building some years previously. But one of the fields in my demesne at Hampton, having been usually called 'The North House meadow,' although the origin of the name had never before suggested itself, it occurred to me as not unlikely that the name might have been given to it in consequence of one of these North Houses having been at some time discovered in it.

"With the view of ascertaining this, we proceeded to make excavations in different parts of the field, and at length we happened upon the top stone of just such a chamber as the old man had described.

"It was constructed with large stones, in the rudest manner; the one stone projecting beyond that immediately below it, till a kind of bee-hive arch was formed: its height might have been six feet, and its diameter perhaps the same. There was a winding passage, or sewer, about three feet in height, and the same in breadth, constructed also of large flags and stones, and probably twenty yards in length, leading into it, and a small funnel, not more than one foot in its dimensions, at the opposite side of the chamber: the passage and the funnel were probably much larger, but they had been broken into as they approached the surface of the hill. We traced the side walls for a considerable distance. There was no cement used in the construction of this North House. We found in it bones of oxen and swine, and some sea shells; also the bones of birds, brought there, probably, by foxes.

" There was a larger and better defined mound than those I have mentioned, on the edge of the sea-cliff, near Knockingen, or Knocknagen, just where the little river Delvin, dividing the counties of Dublin and Meath, falls into the sea, and forms a small sandy bay. A portion of this mound had been already washed away, and the remainder seemed destined soon to share the same fate. On the beach immediately below there were several immense stones, which apparently had fallen from the mound. There was also, perhaps a hundred yards to seaward, a considerable number of similar stones. I could not help thinking that they had formerly formed a part of two North Houses.

" The mound at the edge of the cliff afforded so favourable an opportunity for examination, that, having obtained permission from Lord Gormanstown, on whose estate it was situate, we proceeded to dig it away. It was composed of small round stones, or shingle, from the shore. Our work was soon interrupted by huge stones, similar to those on the shore, and which appeared placed in a circle, buried in sand and shingle, around, but at some distance from the centre of the mound. Within this outer circle of stones we found, on what appeared to have been a floor of beaten clay, a large quantity of burned human bones, apparently of persons of different ages: we found amongst them the bones of very young children. In the centre of this circle there was a chamber constructed of immense flags, some of them more than six feet in height; and within this a rude stone basin, or rather a large stone of sandstone grit, with a cavity or hollow formed in it. This stone bore evident marks of fire; and around it, on all sides, were remains of charcoal or burned wood, and a quantity of burned human bones. Amongst these bones we found some beads, made of polished stone, in shape conical, with a hole through each, near the apex of the cone.

"A portion of the mound may still be seen overhanging the cliff, and if the section of it next the cliff be examined, the bones and charcoal may be easily observed.

"I gave the particulars of this discovery to Mr. D'Alton when he was about to publish his Memoir of Drogheda, and it is referred to in the first volume of his History of Drogheda. I stated to Mr. D'Alton that there was no tradition of the origin of this vast funereal pile, but he quotes a passage from Dr. Hanmer's Chronicles of Ireland, from which it would appear that a battle was fought between an army of marauders and Dermott Lamhdearg, King of Leinster, about the commencement of the fifth century, at Knock-na-cean, *i. e.*, the Hill of Heads, the marauders having landed at the 'Follesse of Skerries.'

"Rude stone coffins, composed of the common flag-stones of the country placed together in the form of a coffin, with skeletons, are found very frequently in this neighbourhood.

"Although it is unconnected with the foregoing, I may as well state, as a matter of curiosity, that Mr. Burdon, about the same time, when visiting the Hill of Tara, discovered and brought home to me a regular joint of a basaltic column, brought, no doubt, in the days of Tara's greatness from the Giant's Causeway. He discovered it accidentally; it was covered by the sod, and was not far from the pillar supposed to be the Lia Fail."

Rev. Samuel Haughton, Fellow of Trinity College, read a paper on "The Equilibrium and Motion of elastic solid, and fluid Bodies."

The object of the paper is to deduce, by the method of the 'Mecanique Analytique' of Lagrange, the laws of solid and fluid bodies from the same physical principles, and to discover by the same method the conditions at the limits.

The principle from which Mr. Haughton deduces the equations is, that the molecules of solid and fluid bodies *act on each other in the direction of the line joining them*, with a force which is a function of the distance; and in the case of crystalline structure, also of the direction of the line joining the molecules.

The general equation of equilibrium of a system is,

$$\iiint(\mathrm{X}\delta\xi + \mathrm{Y}\delta\eta + \mathrm{Z}\delta\zeta)\, dm = \iiint\delta\mathrm{V}\, dxdydz. \qquad (1)$$

Mr. Haughton shows from the definition of the medium, that

$$\mathrm{V} = \mathrm{V}_0 + \mathrm{V}_1,$$

where V_0 is a homogeneous function of the first degree of the six quantities,

$$\frac{d\xi}{dx},\ \frac{d\eta}{dy},\ \frac{d\zeta}{dz},\ \frac{d\eta}{dz}+\frac{d\zeta}{dy},\ \frac{d\zeta}{dx}+\frac{d\xi}{dz},\ \frac{d\xi}{dy}+\frac{d\eta}{dx}$$

and V_1 a homogeneous function of the second order of the same quantities. The function V_0 is zero in a solid body, and not in a fluid; which is equivalent to saying that in a solid body the molecular forces equilibrate each other without the aid of external forces, but that in fluids this is not the case, and that consequently a fluid left to itself would be dissipated by the action of its own molecular forces; this Mr. Haughton conceives to be true of all fluids, whether gaseous or liquid. It should be understood that the mutual *gravitations* of the particles are not included among the molecular forces, but among the external forces.

The values of V_0 and V_1 are shewn to be finally

$$\mathrm{V}_0 = p\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right) \qquad (2)$$

$$2\mathrm{V}_1 = \mathrm{A}\left(\frac{d\xi}{dx}\right)^2 + \mathrm{B}\left(\frac{d\eta}{dy}\right)^2 + \mathrm{C}\left(\frac{d\zeta}{dz}\right)^2 + \mathrm{L}u^2 + \mathrm{M}v^2 + \mathrm{N}w^2$$

$$+2\left(\mathrm{L}\frac{d\eta}{dy}\cdot\frac{d\zeta}{dz}+\mathrm{M}\frac{d\xi}{dx}\cdot\frac{d\zeta}{dz}+\mathrm{N}\frac{d\xi}{dx}\cdot\frac{d\eta}{dy}\right) + 2(\alpha_1 vw + \beta_2 uw + \gamma_3 uv)$$

$$+2\left\{u\left(\alpha_1\frac{d\xi}{dx}+\beta_1\frac{d\eta}{dy}+\gamma_1\frac{d\zeta}{dz}\right)+v\left(\alpha_2\frac{d\xi}{dx}+\beta_2\frac{d\eta}{dy}+\gamma_2\frac{d\zeta}{dz}\right)\right.$$

$$\left.+\, w\left(\alpha_3\frac{d\xi}{dx}+\beta_3\frac{d\eta}{dy}+\gamma_3\frac{d\zeta}{dz}\right)\right\}$$

where,

$$u = \frac{d\eta}{dz} + \frac{d\zeta}{dy}, \qquad v = \frac{d\zeta}{dx} + \frac{d\xi}{dz}, \qquad w = \frac{d\xi}{dy} + \frac{d\eta}{dx}.$$

It may be observed that the function V cannot be the same as the function used by Professor MacCullagh for light, which is a function of the quantities,

$$\frac{d\eta}{dz} - \frac{d\zeta}{dy}, \quad \frac{d\zeta}{dx} - \frac{d\xi}{dz}, \quad \frac{d\xi}{dy} - \frac{d\eta}{dx},$$

and that, consequently, if the latter represent light, that the molecules of the luminous ether cannot act on each other in the line joining them; in fact, the two functions mutually exclude each other.

From (2) is deduced

$$\iiint (X\delta\xi + Y\delta\eta + Z\delta\zeta)\, dm = \tag{4}$$

$$\iint p\,\delta\xi\, dy\, dz + \iint p\,\delta\eta\, dx\, dz + \iint p\,\delta\zeta\, dx\, dy$$

$$- \iiint \left(\frac{dp}{dx}\,\delta\xi + \frac{dp}{dy}\,\delta\eta + \frac{dp}{dz}\,\delta\eta\right) dx\, dy\, dz,$$

which will give the well-known equations of hydrostatics, and the conditions at the limits of the fluid.

If the function V_1 be transformed by changing the directions of the axes of coordinates, it can be shown that the transformation is the same as the transformation of the surface

$$\begin{aligned} &A x^4 + B y^4 + C z^4 + 6(L y^2 z^2 + M x^2 z^2 + N x^2 y^2) \qquad (5) \\ &+ 4yz(3\alpha_1 x^2 + \beta_1 y^2 + \gamma_1 z^2) + 4xz(\alpha_2 x^2 + 3\beta_2 y^2 + \gamma_2 z^2) \\ &+ 4xy(\alpha_3 x^2 + \beta_3 y^2 + 3\gamma_3 z^2) = 1. \end{aligned}$$

This surface may be called the characteristic surface of the function V_1, as, if it possesses any geometrical properties which simplify its equation, a corresponding simplification will take place in the function V_1.

It is then shown, by the aid of the auxiliary ellipsoid, whose equation is

$$(\mathrm{A}-\mathrm{L})x^2+(\mathrm{B}-\mathrm{M})y^2+(\mathrm{C}-\mathrm{N})z^2+2(\alpha_1+\beta_1+\gamma_1)yz$$
$$+2(\alpha_2+\beta_2+\gamma_2)xz+2(\alpha_3+\beta_3+\gamma_3)xy=1, \qquad (6)$$

that the function v_1 and the surface (5), may be referred to a system of rectangular axes, for which the following relations exist:

$$\aleph_1+\beth_1+\gimel_1=0; \quad \aleph_2+\beth_2+\gimel_2=0: \quad \aleph_3+\beth_3+\gimel_3=0; \qquad (7)$$

the Hebrew letters denoting what the Greek become after transformation of coordinates. The possibility of these equations in every case amounts to a proof of the existence of three axes at each point of a body, which are intimately connected with the molecular constitution of the body round the point.

The equation of equilibrium of a solid body is then shown to be

$$\iiint(\mathrm{X}\delta\xi+\mathrm{Y}\delta\eta+\mathrm{Z}\delta\zeta)dm=\triangle-\iiint(\mathrm{P}_1\delta\xi+\mathrm{Q}_1\delta\eta+\mathrm{R}_1\delta\zeta)dxdydz, \quad (8)$$

where

$$\mathrm{P}_1=\mathrm{A}\frac{d^2\xi}{dx^2}+\mathrm{N}\frac{d^2\xi}{dy^2}+\mathrm{M}\frac{d^2\xi}{dz^2}+2\left(\alpha_1\frac{d^2\xi}{dydz}+\alpha_2\frac{d^2\xi}{dxdz}+\alpha_3\frac{d^2\xi}{dxdy}\right)$$
$$+\alpha_3\frac{d^2\eta}{dx^2}+\beta_3\frac{d^2\eta}{dy^2}+\gamma_3\frac{d^2\eta}{dz^2}+2\left(\alpha_1\frac{d^2\eta}{dxdz}+\mathrm{N}\frac{d^2\eta}{dxdy}+\beta_2\frac{d^2\eta}{dydz}\right)$$
$$+\alpha_2\frac{d^2\zeta}{dx^2}+\beta_2\frac{d^2\zeta}{dy^2}+\gamma_2\frac{d^2\zeta}{dz^2}+2\left(\alpha_1\frac{d^2\zeta}{dxdy}+\gamma_3\frac{d^2\zeta}{dydz}+\mathrm{M}\frac{d^2\zeta}{dxdz}\right)$$

$$\mathrm{Q}_1=\mathrm{B}\frac{d^2\eta}{dy^2}+\mathrm{L}\frac{d^2\eta}{dz^2}+\mathrm{N}\frac{d^2\eta}{dx^2}+2\left(\beta_2\frac{d^2\eta}{dxdz}+\beta_3\frac{d^2\eta}{dxdy}+\beta_1\frac{d^2\eta}{dydz}\right)$$
$$+\beta_1\frac{d^2\zeta}{dy^2}+\gamma_1\frac{d^2\zeta}{dz^2}+\alpha_1\frac{d^2\zeta}{dx^2}+2\left(\beta_2\frac{d^2\zeta}{dxdy}+\mathrm{L}\frac{d^2\zeta}{dydz}+\gamma_3\frac{d^2\zeta}{dxdz}\right)$$
$$+\beta_3\frac{d^2\xi}{dy^2}+\gamma_3\frac{d^2\xi}{dz^2}+\alpha_3\frac{d^2\xi}{dx^2}+2\left(\beta_2\frac{d^2\xi}{dydz}+\alpha_1\frac{d^2\xi}{dxdz}+\mathrm{N}\frac{d^2\xi}{dxdy}\right)$$

$$\mathrm{R}_1=\mathrm{C}\frac{d^2\zeta}{dz^2}+\mathrm{M}\frac{d^2\zeta}{dx^2}+\mathrm{L}\frac{d^2\zeta}{dy^2}+2\left(\gamma_3\frac{d^2\zeta}{dxdy}+\gamma_1\frac{d^2\zeta}{dydx}+\gamma_2\frac{d^2\zeta}{dxdz}\right)$$
$$+\gamma_2\frac{d^2\xi}{dz^2}+\alpha_2\frac{d^2\xi}{dx^2}+\beta_2\frac{d^2\xi}{dy^2}+2\left(\gamma_3\frac{d^2\xi}{dydz}+\mathrm{M}\frac{d^2\xi}{dxdz}+\alpha_1\frac{d^2\xi}{dxdy}\right)$$
$$+\gamma_1\frac{d^2\eta}{dz^2}+\alpha_1\frac{d^2\eta}{dx^2}+\beta_1\frac{d^2\eta}{dy^2}+2\left(\gamma_3\frac{d^2\eta}{dxdz}+\beta_2\frac{d^2\eta}{dxdy}+\mathrm{L}\frac{d^2\eta}{dydz}\right),$$

Δ consists of double integrals, and gives the conditions at the limits.

The differential equations of *motion* derived from (8) are (no external forces X, Y, Z acting):

$$\varepsilon\frac{d^2\xi}{dt^2} = P_1, \qquad \varepsilon\frac{d^2\eta}{dt^2} = Q_1, \qquad \varepsilon\frac{d^2\zeta}{dt^2} = R_1. \tag{9}$$

These equations will admit of the particular integral

$$\xi = \cos\alpha \,.f(\omega), \qquad \eta = \cos\beta\,.f(\omega), \qquad \zeta = \cos\gamma\,.f(\omega),$$

$$\omega = lx + my + nz - vt,$$

provided it is possible to satisfy with real values of $(\alpha, \beta, \gamma, v)$ the equations of condition resulting from the substitution of these values in the equations of motion.

These equations of condition lead to the following construction for the directions of the *possible* vibrations of molecules, and the corresponding velocities of wave-planes.

Construct the six *fixed* ellipsoids,

$$\left.\begin{aligned}
P &= Ax^2 + Ny^2 + Mz^2 + 2a_1yz + 2a_2xz + 2a_3xy = 1,\\
Q &= By^2 + Lz^2 + Nx^2 + 2\beta_1yz + 2\beta_2xz + 2\beta_3xy = 1,\\
R &= Cz^2 + Mx^2 + Ly^2 + 2\gamma_1yz + 2\gamma_2xz + 2\gamma_3xy = 1,\\
F &= a_1x^2 + \beta_1y^2 + \gamma_1z^2 + 2Lyz + 2\gamma_3xz + 2\beta_2xy = 1,\\
G &= a_2x^2 + \beta_2y^2 + \gamma_2z^2 + 2\gamma_3yz + 2Mxz + 2a_1xy = 1,\\
H &= a_3x^2 + \beta_3y^2 + \gamma_3z^2 + 2\beta_2yz + 2a_1xz + 2Nxy = 1,
\end{aligned}\right\} \tag{10}$$

and from their common centre draw the normal to the wave-plane, this will pierce the surfaces in six points; let the corresponding radii vectores be $\rho_{,}\, \rho_{,,}\, \rho_{,,,}$, $r_{,}\, r_{,,}\, r_{,,,}$; with these construct the ellipsoid

$$\frac{x^2}{\rho^2_{,}} + \frac{y^2}{\rho^2_{,,}} + \frac{z^2}{\rho^2_{,,,}} + \frac{2yz}{r^2_{,}} + \frac{2xz}{r^2_{,,}} + \frac{2xy}{r^2_{,,,}} = 1. \tag{11}$$

The axes of this ellipsoid will be the *three* possible directions of molecular vibration, and the corresponding velocities of waves will be inversely as the lengths of these axes.*

The six ellipsoids just mentioned perform a very important part in the problem of elastic solids, as they reappear in the conditions at the limits, and afford a geometrical meaning for many of the results.

Mr. Haughton then determines from simple considerations the equation of the Sphæro-Reciprocal-Polar of the Wave-surface, or the *Surface of Wave-slowness of elastic Solids*, which occupies a position in this subject, analogous to that held by the index-surface in light. This surface, and the important results it leads to, are, as far as Mr. Haughton is aware, given by him for the first time; it is of the sixth degree, and has three sheets, and by means of it, the direction of a vibration passing from one medium into another may be determined.

The paper then proceeds to the discussion of three particular cases of elastic solids: 1. The case where the molecules are arranged symmetrically round three rectangular planes. 2. Round one axis. 3. The case of a homogeneous uncrystalline body.

In the first case, the following results are deduced: The traces of the surface of wave-slowness on the planes of symmetry, consist of an ellipse and a curve of the fourth degree. The surface possesses four nodes in one of its principal planes, where the tangent plane becomes a cone of the second degree, and the existence of these points will give rise to a conical refraction in acoustics, similar to what has been established in physical optics.

In general, for a given direction of wave-plane, three

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* After Mr. Haughton had obtained this construction, he found that M. Cauchy has given analytically, and for a particular case, a solution which involves an analogous ellipsoid; but M. Cauchy has not followed out the consequences of his analysis in the right direction, and has been misled in his attempt to apply his equations to the problem of light.

waves will be possible, the corresponding vibrations of the molecules being in three directions, at right angles to each other, though not, in general, parallel or normal to the wave-plane. Mr. Haughton investigates the possibility of the vibrations being normal and transversal, and finds that for particular directions of wave (given in the paper), the vibrations are, two in the wave-plane, and the third perpendicular to it. He discusses also at length the other two cases, together with the equations of condition which hold in general at the limits, and the geometrical interpretation of these conditions by means of the fixed ellipsoids (10).

The President read a note from Edward Cooper, Esq., giving the place of the new Comet, as observed at Markree, May 18d. 12h. 6m. Greenwich mean time.

DONATIONS.

Report of the Commissioners appointed to take the Census of Ireland for the Year 1841. Presented by W. R. Wilde, Esq.

Transactions of the Botanical Society of Edinburgh. Vol. II. Parts 1 & 2. Presented by the Society.

Lettres a S. A. R. le Duc Regnant de Saxe Coburg et Gotha, sur la Theorie des Probabilites appliquée aux Sciences morales et politiques. Par A. Quetelet. Presented by the Author.

Annales des Sciences Physiques et Naturelles, d'Agriculture et d'Industrie, publieés par la Société Royale d'Agriculture, &c., de Lyon. Tome VII. Annee 1844. Presented by the Society.

Popery unmasked. By John Ryan, Esq. Presented by the Author.

The thirteenth Annual Report of the Royal Cornwall Polytechnic Society. 1845. Presented by the Society.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846. No. 55.

June 8th.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Thomas John Beasly, Esq., Thomas Percy Boyd, Esq., and Rev. Robert J. M'Ghee, were elected Members of the Academy.

The President announced to the Academy, that Arthur R. Nugent, Esq., M. R. I. A., on the part of the Rev. Francis Brownlow, had deposited the ancient MS. called the Book of Armagh, in the Museum of the Academy, and that he had given to Mr. Nugent the following receipt for the same, subject to the approval of the Academy:

"*Royal Irish Academy*,
"*Dublin*, 4*th June*, 1846.

"Arthur R. Nugent, Esq., M. R. I. A., has this day deposited in the Museum of the Royal Irish Academy, on the part of the Rev. Francis Brownlow, that ancient Irish MS. called the Book of Armagh, the property of the said Rev. Francis Brownlow, with the understanding that the Academy will take the same care of the said book that they do of the best article in their Museum; and that the Academy will at any time return the said book to the said Rev. Francis Brownlow, or his heirs or representatives, on his or their demand, and without any delay, charge, or hindrance whatever.

"Signed, subject to the approbation of the Academy,

"H. Lloyd, *President*."

It was Resolved,—That the deposit of the Book of Armagh be accepted by the Academy, on the conditions named; and that the thanks of the Academy be voted to the Rev. Francis Brownlow, and to Arthur R. Nugent, Esq.

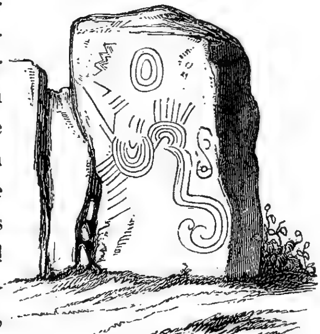
W. R. Wilde, Esq., exhibited and described the "Mias Tighearnain," an ancient Irish shrine, from the barony of Tyrawley, County of Mayo, which had been lent to him by Annesley Knox, Esq., for that purpose.

It was Resolved,—That the thanks of the Academy be presented to Mr. Knox, by whose permission the Mias Tighernain has been exhibited to the Academy.

Mr. Wilde presented a collection of Celtic antiquities, weapons, ornaments, domestic implements, sepulchral urns, and some animal remains, found in ancient tumuli, from Arthur R. Nugent, Esq., Portaferry, from whom Mr. Wilde made a communication in 1844. These interesting relics consisted of a very large stone celt, eight inches long; several flint arrowheads, among which is one of the most beautiful, both in form and execution, which the Academy has yet received; three flint knives, two very rude and apparently in the process of formation; a small sharpening stone; and four small circular stone discs, perforated in the centre, and probably used for the distaff;—all discovered in the County of Down, a locality remarkably rich in antiquities of this description. He also presented some silver pieces, among which was a shilling of Elizabeth and one of James I.

The sepulchral urns, two in number, one very perfect, the other in fragments, but capable of being restored, were discovered along with some incinerated bones, charred wood, and a quantity of the remains of some of the lower domestic animals, at Donaghanie, in the County of Donegal; and Mr. Wilde stated that he had received a communication on the subject from Mr. John Bell of Dungannon, informing him that they were found on opening a cairn contained "within a circle of large stones, measuring seventy yards in circumference. One of the Rev. John Davis's tenants requiring building materials, thoroughly laid open the tumulus, uncovering numerous sepulchral cells. These were replete with such rudely sculptured ornaments as are frequently found in cairn chambers. The vertical columns supporting large flags, shortening inwards, one over the other, are about six feet in height, and the roof-stones are kept in their places by the pressure of the heap or cairn on their extremities. This structure is similar to those near Drogheda and in Rosshire, and to that which once stood on the banks of the Carron in Stirlingshire."

From this description, the similarity of the cairn at Donaghanie to the great tumulus at New Grange will be at once recognised by the Academy; and the sculptured ornaments found in both these localities, and consisting of volutes, circles, and zig-zag characters, Mr. Wilde considered to be purely characteristic of the ancient Pagan burial places in Ireland, and perfectly distinct from that denominated Ogham writing. Of this sepulchral character a fine example is found upon the interior of one of the stones forming the upright pillars in the open kistvaen at Knockmany, in the County Tyrone, and of which the accompanying is a rude sketch. The animal remains found at Donaghanie consisted of the bones of several domestic animals, oxen, swine, cats, dogs, sheep, together with those



of geese, and other domestic fowl; and it is interesting to discover such traces of the distribution of these animals in the British Isles at a period so remote as the date of this tumulus points to.

Mr. Wilde recorded the discovery of a small tumulus on the western side of the great mound of New Grange, which had been opened by Lieutenant Newenham two years ago: it was about eight feet long, and consisted of a small stone passage leading into a little chamber, formed on the type of the great barrow in that vicinity. In this was discovered a vast collection of the remains of domestic animals, as well as several human bones, some perfect and others in a half-burned state. What gave particular interest to this excavation was the fact of the stones which lined the floor having been vitrified on the external face, which would lead to the conclusion that the cremation had taken place in the grave: and one of these vitrified stones Mr. Wilde presented to the Academy.

It is much to be lamented that ignorant persons, or those actuated by mere curiosity, should be allowed to open, and, as is very often the case, destroy those interesting monuments throughout the country, many of which possess an historical as well as an antiquarian and ethnological interest, and are alluded to in the ancient annals.*

* The following communication has been made to the Secretary by Mr. Wilde. "Some time ago, Arthur R. Nugent, Esq., opened a large sepulchral mound in the townland of Kintagh, in the neighbourhood of Portaferry, whence he writes to me: 'There was a circle of large stones containing an area of about a rood. Between each of these flat stones there was a facing of flat ones, similar to the building of our modern fences. The outer covering was coated with white pebbles, averaging the size of a goose-egg, of which there were several cart-loads,—although it would be difficult to collect even a small quantity at present along the beach. After this was taken away, we came to a confused heap of rubbish, stones, and clay, and then some large flag stones on their end, the tumulus still preserving a cone-shape. In the centre we came to a chamber about six feet long, formed by eight very large upright stones, with a large flag-stone at the bottom, on which lay, in one heap of equal thickness, a mixture of black mould and bones.' These bones, several of which are now in the Museum o lthe

Resolved,—That the thanks of the Academy be given to Mr. Nugent.

The Secretary announced to the meeting, that the Commissioners for the Improvement of the Shannon had forwarded a further donation of antiquities found in the bed of that river to the Museum; together with a section and plans of the small tower at Clonmacnoise.

Resolved,—That the thanks of the Academy be given to the Commissioners.

Dr. Allman exhibited a remarkable form of Saxifraga Leucanthemifolia, presenting the retrograde metamorphosis of flowers into bulbs, which were thickly scattered over the inflorescence, occupying the position of the leafy tufts described by Robert Brown in his Saxifraga Foliosa.

Rev. Samuel Butcher read a paper by the Rev. Edward Hincks, D. D., "On Persepolitan Writing."

In this paper various rectifications of the received mode of reading the first kind of Persepolitan writing were proposed; and an alphabet, or rather a combined alphabet and syllabary for the second was given, differing in some important respects from that of Westergaard.

Academy, are all human, and consist of the ribs, vertebræ, and the ends of the long bones, together with pieces of the skull, and some joints of the fingers of a full-grown person, and also several bones of a very young child, none of which had been subject to the action of fire. But among the parcel forwarded to me by Mr. Nugent, are several fragments of incinerated human bones. Either these latter were portions of the same bodies burned, or they belonged to an individual sacrificed to the manes of the person whose grave this was; and I am inclined to think the latter is the more probable, from the circumstances in which similar remains have been discovered in other localities. There were no urns, weapons, or ornaments of any description discovered in connexion with this tumulus; but Mr. Nugent states, that in the field where it was opened, small stone chambers, or kistvaens, have at various times been dug up, and in one of these a long, flat, and narrow skull was some time since discovered." W. R. W.

The changes proposed with respect to the first kind were these:

1. The vowel *a* might be inserted after any primary letter, before a vowel, either as a distinct syllable, or as a *guna* to the vowel, as well as before a consonant.

2. *W* after *u*, and *y* after *i*, are in general, both in the middle and at the end of words, absolutely mute. When not so, they are to be sounded as *a*, which they implicitly contain.

3. Secondary consonants, which are only used before particular vowels, are to be sounded in the same manner as the corresponding primary ones; and if a secondary consonant exist proper to any vowel, and the corresponding primary consonant appears to precede that vowel, an *a* is always to be supplied. If a secondary consonant be used without its proper vowel after it, that vowel must be supplied; *r* is here considered as a vowel. Thus the combination of the letters which Lassen calls *ṁi* would be *mi*; while his *mi* would be *me*, for *mai*. His *fr* would be *pr*; while his *pr* would be *par*.

4. Besides his mistake in giving values to the secondary consonants generally, different from those of their corresponding primary ones, Lassen has erroneously considered the secondary consonant corresponding to *d* before *i* to correspond to *k′*, i. e. *ch*; and he has given to three primary consonants the values *d*, *z*, and *z′*, i. e. *zh*; the true values of which Dr. Hincks maintains were *z*, *zh*, and *j*, or *dzh*.

The second Persepolitan alphabet, it is here maintained, consisted of characters representing nine elementary sounds: viz., four vowels, *a*, *i*, *u*, and *er*, and five consonants *p*, *k*, *t*, *s*, and *n*: and various combinations of these nine elements. In most cases, two or more characters, phonetically equivalent, represented the same element or combination.

Westergaard supposes a much larger proportion of the characters to represent elementary sounds than Dr. Hincks; and he supposes that an *a* might be inserted, as in the first kind of writing. Dr. Hincks maintains that every vowel is expressed at least once; but that both vowels and consonants might be expressed twice, at the end of one character and at the beginning of the next.

In addition to the correction made in Westergaard's alphabet by the addition of vowels to the consonants, which he supposed the complete representations of certain characters, and by the substitution of different vowels for those which he used, entirely new values are given by Dr. Hincks to five characters which Westergaard had improperly valued, and to five more which he had not valued at all.

Specimens of the inscriptions in this kind of writing, as read and translated, were added. The language was said to agree with the Indo-Germanic languages in having inflections; but to have inflections completely different from those of all these languages.

In a postscript to the paper it was stated, that the Babylonian and Assyrian alphabets were both of the same nature as this; so far as that some of the characters represented syllables and some elementary sounds; that the same sound was represented by two or more characters; that no vowel was omitted; and that vowels and consonants were habitually represented twice, when only to be sounded once. The number of elementary sounds in the Babylonian, or third kind of Persepolitan writing, was greater than in the second kind, as was the number of characters in use. Both the Babylonian and Assyrian had something in common with the second Persepolitan language; but they had also affinities with the Semitic languages.

Rev. T. R. Robinson made some observations on Dr. Hincks's paper, referring to researches on the same subject by Mr. Norris and Colonel Rawlinson.

Rev. S. Butcher read the third part of Dr. Hincks's paper on Egyptian hieroglyphics.

In it the principles established in the preceding parts were applied to ascertain the exact power of each of the letters of the Egyptian alphabet. Those contained in the alphabet lately published by Chev. Bunsen were first examined; and then the other characters alleged to be alphabetic: some of which were classed, in Chev. Bunsen's arrangement, among the syllabic signs, while others were altogether omitted. A new class of letters having the power of *ch*, and corresponding to the Hebrew צ, is established, to which the *long serpent* belongs, occurring in the word *chat*, signifying *ever*.

Robert Ball, Esq., exhibited various anatomical preparations of marine animals made by Mr. Goadby of London.

June 22.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Mr. Oldham, on the part of Mr. R. Mallet, who was unavoidably absent, described the objects, construction, and use of certain new instruments devised by the latter for self-registration of the passage of earthquake shocks.

Instruments previously intended for this purpose have not possessed the power of self-registration; they have consisted either in the trace left by the motion of a viscid fluid on the containing vessel, or they have been upon the principle of the inverted pendulum, or watchmaker's noddy. Instruments so constructed are objectionable, because having themselves times of vibration of their own, which may conflict with those of the earthquake shock, they are liable to fail in point of delicacy. They also possess several inconveniences of a mechanical kind in being adapted to self-registration.

The objects to be attained in the instruments which the author has had in view, are:

The self-registration—

1st. Of the time of transit, at a given point of the earth's surface, of an earthquake shock, or earth-wave, noting same to a small decimal of a second of time.

2nd. Of the vertical element, or altitude, of the earth-wave, at the moment of its transit, whether the wave be a positive or a negative one.

3rd. Of the horizontal element, or amplitude of the wave, at the same moment.

4th. Of the direction, as to azimuth, of the wave transit.

The principle adopted, as the means by which the wave, or shock, shall act upon the instrument, consists in availing ourselves of the oscillation of a column of mercury, in two vertical, and in four horizontal glass tubes, of peculiar construction. One end of the column of mercury in each tube is so adjusted in contact with one pole of a constant galvanic battery, that the oscillation produced in the mercurial column by the wave, in passing, breaks contact. The time during which the contact remains broken is proportionate to the amount of the vertical and horizontal elements of the wave. The breach of contact releases one or more of six pencils at the instant of its occurrence, and until contact is restored. Either of these continues to describe a trace upon a ruled sheet, placed upon a cylindrical barrel, carried round by the astronomical clock. The length of this trace is, therefore, a graphic representation of the amount of the respective element of the wave, and the pencil which marks it indicates the direction of the oscillation, whether vertically positive or negative, or horizontally from any point of the compass.

A somewhat similar arrangement marks, upon four dials, the hour, minute, second, and fraction of a second, at which the crest of the wave has passed the point of the observatory, or locus of the instrument. This is of peculiar importance for ascertaining the rate of progress of the wave between two distant observatories. The instrument cannot be understood in its details, without the aid of diagrams, as exhibited to the Academy.

The instrument is designed to register, by itself, for twelve hours at a time, and at such an interval its registrations require to be read off and noted.

Dr. Todd read a letter from C. T. Barnwell, Esq., containing some observations on two passages of Archimedes, *De Sphæra et Cylindro*, where commentators appear to have been strangely misled.

The first occurs in the Demonstration of Proposition I. of the first book.

In the demonstration of this proposition it is assumed that the triangles $AB\Delta$, $B\Gamma\Delta$ are together greater than the triangle $A\Delta\Gamma$ (fig. pag. 79, Oxf. ed. fol. 1792).

Dr. Barrow (in whose edition this is Prop. XII.) says, "liquet . . . . quia $AB+B\Gamma > A\Gamma$, et altitudo *communis est*," which is evidently not true, unless the triangle $AB\Gamma$ were equilateral.

In the German edition of J. C. Sturm (where this is Prop. IX.) the following most extraordinary inference is drawn from Euc. I. 24, viz., that, since (fig. in p. 80) $\Delta Z > \Delta E$, $\Gamma\Delta$ common, and the angle $\Gamma\Delta Z >$ the angle $\Gamma\Delta E$, the *triangle* $\Gamma\Delta Z >$ the triangle $\Gamma\Delta E$.

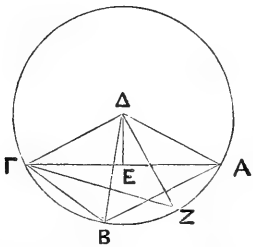
In the Oxford edition, the demonstration of Eutocius is condemned as invalid; but the editor, without stating the nature of his objection, contents himself with adding "sed res ipsa satis patet."

Flauti, of Naples (Corso, vol. I.) observes, and rightly, that the line ΔZ should have been directed to be drawn in the plane of the triangle $A\Delta\Gamma$, and states what he considers to be the objection of the Oxford editor, viz., that the triangle $\Gamma\Delta Z$ will not include the triangle $\Gamma\Delta E$ in the case of the angle $A\Delta B >$ the angle $A\Delta\Gamma$, and that it cannot, therefore, be inferred generally, that the first of these triangles $>$ the second. He then subjoins a different demonstration.

Hauber, in his excellent edition of this treatise (Tubingen, 1798), appears also to admit the objection; for he gives another demonstration of the assumption in question, which is, perhaps, preferable to Flauti's. Peyrard does not attempt any explanation or demonstration.

It is, however, very remarkable, that not one of the editors seems to have observed that, in the subsequent application of Prop. X. (see the Corollaries at the end of Prop. XIII. p. 86) the triangle $A\Delta\Gamma$ is composed with the *lesser* of the two conical surfaces intercepted between the lines $A\Delta$, $\Delta\Gamma$; and consequently, that the lesser of the two segments, into which the circle is divided by the line $A\Gamma$, is the one which should have been bisected in B.

The figure in p. 80, when corrected accordingly, will be this:



where, since the angle $\Gamma\Delta Z$ (=the angle $\Gamma\Delta B$) is $>\Gamma\Delta E$, and $<\Gamma\Delta A$ (since $\Gamma B < \Gamma A$), the triangle $\Gamma\Delta Z$ will evidently include the triangle $\Gamma\Delta E$, and the demonstration given by Eutocius will be valid. The only objection now to be made to it is, that it is unnecessary; for, since $\Gamma B + BA > \Gamma A$, and the perpendicular on ΓB or BA is also $>$ that on ΓA, it at once follows that the triangle $\Gamma\Delta B$ + the triangle $B\Delta A >$ triangle $\Gamma\Delta A$.

In every edition, the point Z appears to be in the circumference of the circle ΓBA, which seems to have misled Sturm and, perhaps, the Oxford editor also.

In a modern German edition of the whole of Archimedes, by Nirze (Stralsund, 1824), Hauber's demonstration is adopted, but in a less advantageous form, and with an erroneous figure.

The second occurs in the Demonstration of Proposition V. of the second Book.

At the foot of page 157, Oxford edition, it is said: "Since the ratio of ΔΛ to ΛX is given, as well as that of PΛ to ΛX, the ratio of PΛ to ΛΔ is also given." But this is untrue with respect to the first of these ratios, which is not given. Indeed, if it were, the analysis would here be at an end; for, since (p. 157, lin. penult.) ΔΛ:ΛX :: BZ:ZX, the ratio of BX:ZX would be given, as also ZX (since BZ is given) and the point X.

But it is remarkable that this corruption is as old as the time of Eutocius; for he has been led into an error so gross, that it is hardly possible to imagine how so able a commentator could have fallen into it. This error is no less than the assertion (page 160, line 25) that BX is given, *because its extremities are given;* whereas the whole object of the analysis is to find the point X.

This corruption of the text has been allowed to pass unnoticed into the Oxford edition, though it had been corrected in the old Latin translation, in the edition of 1544, as well as in the Greek text of that edition, except that in this latter there is an error of ΔΛ for PX.

Sturm, who does not give a regular translation of his author, has avoided the error in the text, but it is retained in Nizre and Peyrard.

Hauber has adopted the correction of the old translation, which is, undoubtedly, just; "since the ratio of PX to ΛX is given, that of PΛ to ΛX is also given."

The President described certain improvements in the construction of the Anemometer, and exhibited to the Academy the improved instrument.

The Anemometer of Osler, which is that employed in the Dublin Magnetical and Meteorological Observatory, registers, as is well known, both the *direction* and the *pressure* of the wind at every instant. The improvements which Mr. Osler has lately introduced into the construction of the instrument seem to leave nothing to be desired, so far as relates to the former part of its office. The place of the directing vane is now supplied by a vertical wheel with oblique vanes, which, by the intervention of wheel-work, is made to work round upon a fixed horizontal toothed wheel, and thus to maintain itself always in the vertical plane passing through the direction of the wind. This plan, which is that of the small directing wheel of the ordinary wind-mill, is found to answer its purpose admirably, and to avoid altogether the inconvenience which arose from the oscillations of the vane in the old construction. The only change which Dr. Lloyd has adopted in this part of the instrument, consists in placing this wheel *centrally*, instead of excentrically, with respect to the whole apparatus.

In the portion of the instrument destined to measure and register the *pressure* of the wind, which had always been found the least satisfactory, Dr. Lloyd employs a smaller vane-wheel, connected with the former by means of the framework of the instrument, and maintained by it always perpendicular to the direction of the wind. The pressure of the wind acts, through this wheel, upon a spiral spring coiled in a box, the vane-wheel and the spring being connected by an intermediate wheel and pinion. The axle of the vane-wheel carries an endless screw, which works upon a toothed wheel; and upon an arbor, attached to the axle of the latter, is coiled the chain which communicates with the registering apparatus below. The chain is kept tended, when the pressure of the wind is relaxed, by means of a small spring attached to the registering table.

The objects proposed to be attained by this arrangement are: 1. To augment the sensibility of the instrument, and to render it available for the registry of light gales, no less than high winds; and 2. To diminish, by means of the inertia of the wheel, the oscillatory movement of the registering pencil, occasioned by the unsteady action of the wind.

Mr. Petrie exhibited several ancient bells—the bell of St. Cuanna, of the county Clare, and the bell of St. Ruadhan, of Lorha, and some others. He also exhibited some bells supposed to be of pagan age.

The thanks of the Academy were given to Mr. Cooke for permitting the *Bearnan Cullain* to be exhibited to the Academy.

DONATIONS.

An ancient Bronze Bit and Flint Knife, part of the collection of the late John Echlin, Esq. Presented by Miss Echlin.

July 20. (Extraordinary Meeting.)

REV. HUMPHREY LLOYD, President, in the Chair.

Resolved, on the recommendation of the Council,—That a congratulatory Address be presented to His Excellency the Lord Lieutenant.

Resolved,—That the President and Secretaries (or officers) be a Sub-Committee to prepare a draft of an Address, which, having been agreed to by the Sub-Committee, was presented to the Meeting, whereupon

It was Resolved,—That the Address now presented be adopted, and that the President shall summon the Academy, as soon as His Excellency's pleasure is declared, as to the time when it would be convenient to him to receive the Address.

During the absence of the Sub-Committee, the Chair was taken, *pro tem.*, by George Petrie, Esq., V. P., when

Sir William R. Hamilton read a paper on the expression and proof of Pascal's theorem by means of quaternions; and on some other connected subjects.

This proof of the theorem of Pascal depends on the following form of the general equation of cones of the second degree:

$$\mathrm{S} \,.\, \beta \beta' \beta'' = 0; \tag{1}$$

in which

$$\left.\begin{aligned} \beta &= \mathrm{V}\,(\mathrm{V}\,.\,\alpha\alpha'.\,\mathrm{V}\,.\,\alpha'''\,\alpha^{IV}), \\ \beta' &= \mathrm{V}\,(\mathrm{V}\,.\,\alpha'\,\alpha''.\,\mathrm{V}\,.\,\alpha^{IV}\,\alpha^{V}), \\ \beta'' &= \mathrm{V}\,(\mathrm{V}\,.\,\alpha''\,\alpha'''.\,\mathrm{V}\,.\,\alpha^{V}\,\alpha), \end{aligned}\right\} \tag{2}$$

α, α', α'', α''', α^{IV}, α^{V}, being any six homoconic vectors, and S, V, being characteristics of the operations of taking separately the scalar and vector parts of a quaternion.

In all these geometrical applications of quaternions, it is to be remembered that *the product of two opposite vectors is a positive number*, namely, the product of the numbers expressing the lengths of the two factors; and that the *product of two rectangular vectors is a third vector rectangular to both*, and such that the rotation round it, from the multiplier to the multiplicand, is positive. These conceptions, or definitions, of geometrical multiplication, are essential in the theory of quaternions, and are hitherto (so far as Sir William Hamilton knows) peculiar to it. If they be adopted, they oblige us to regard the *product* (or the quotient) *of two inclined vectors* (neither parallel nor perpendicular to each other), as being *partly a number and partly a line;* on which account a quaternion, generally, as being always, in its geometrical aspect, a product (or quotient) of two lines, may perhaps not improperly be also called a GRAMMARITHM (by a combination of the two Greek words γραμμή and ἀριθμός, which signify respectively a *line* and a *number*). In this phraseology, the scalar part of a quaternion would be the arithmic part of a grammarithm; and the vector part of a quaternion would be the grammic part of a grammarithm. In the form given above, of the general equation of cones of the second degree, the six symbols, $\alpha, \ldots \alpha^{V}$, denote six edges of a hexahedral angle inscribed in such a cone; the six binary products $\alpha\alpha', \ldots \alpha^{V}\alpha$, of those lines taken in their order, are grammarithms, of which the symbols $\mathrm{V}.\alpha\alpha'$, &c., denote the grammic parts, namely, certain lines perpendicular respectively to the six plane faces of the angle; the three products

$$\mathrm{V}.\alpha\alpha'.\mathrm{V}.\alpha'''\alpha^{IV}, \text{ \&c.},$$

of normals to opposite faces, are again grammarithms, of which the grammic parts are the three lines β, β', β'', situated respectively in the intersections of the three pairs of opposite faces of the angle inscribed in the cone; and the equation (1) of that cone, which expresses that the arithmic part of the product of these three lines vanishes, shows also, by the principles of this theory, that these lines themselves are *coplanar:* which is a form of the theorem of Pascal.

The rules of this calculus of grammarithms, or of quaternions, give, generally, for the arithmic or scalar part of the product of the vector parts of the three products of any six lines or vectors $\alpha\alpha', \beta\beta', \gamma\gamma'$, taken two by two, the following transformed expression:

$$\mathrm{S}(\mathrm{V}.\alpha\alpha'.\mathrm{V}.\beta\beta'.\mathrm{V}.\gamma\gamma') = \mathrm{S}.\alpha\gamma\gamma'.\mathrm{S}.\alpha'\beta\beta' - \mathrm{S}.\alpha'\gamma\gamma'.\mathrm{S}.\alpha\beta\beta'; \quad (3)$$

and by applying this general transformation to the recent results, we find easily, that the equation (1), under the conditions (2), may be put under the form:

$$\frac{\mathrm{S}.\alpha\alpha'\alpha''}{\mathrm{S}.\alpha\alpha'''\alpha''} \cdot \frac{\mathrm{S}.\alpha''\alpha'''\alpha^{IV}}{\mathrm{S}.\alpha''\alpha'\alpha^{IV}} = \frac{\mathrm{S}.\alpha\alpha'\alpha^{V}}{\mathrm{S}.\alpha\alpha'''\alpha^{V}} \cdot \frac{\mathrm{S}.\alpha^{V}\alpha'''\alpha^{IV}}{\mathrm{S}.\alpha^{V}\alpha'\alpha^{IV}}; \quad (4)$$

which is another mode of expressing by quaternions the general condition required, in order that six vectors $\alpha, \ldots \alpha^{V}$, diverging from one common origin, may all be sides of one common cone of the second degree. The summit of this cone, or the common initial point of each of these six vectors, being called O, let the six final points be ABCDEC′: the transformed *equation of homoconicism* (4) expresses that the *ratio compounded of the two ratios of the two pyramids* OABC, OCDE, *to the two other pyramids* OADC, OCBE, *does not change when we pass from the point* C *to any other point* C′ *on the same cone of the second degree:* which is a form of the theorem of M. Chasles, respecting the constancy of the anharmonic ratio. An intimate connexion between this theorem and that of Pascal is thus exhibited, by this symbolical process of transformation.

As the equation (1) expresses that the *three* vectors $\beta\,\beta'\,\beta''$ are *coplanar*, or that they are contained on one common plane, if they diverge from one common origin, and as the equation (4) expresses that the *six* vectors $\alpha, \ldots \alpha^{V}$ are *homoconic*, so does this other equation,

$$\mathrm{S}\,.\,\rho\,(\rho-\gamma)\,(\gamma-\beta)\,(\beta-\alpha)\,\alpha=0, \qquad (5)$$

express that the *four* vectors $\alpha, \beta, \gamma, \rho$ are *homosphæric*, or that they may be regarded as representing, in length and in direction, *four diverging chords of one common sphere.* Thus, the *arithmic part of the continued product of the five successive sides of any rectilinear* (but not necessarily plane) *pentagon, inscribed in a sphere, is zero;* and conversely, if in any investigation respecting any rectilinear, but, generally, uneven, pentagon ABCDE in space, the product AB × BC × CD × DE × EA of five successive sides, when determined by the rules of the present calculus, is found to be a pure vector, or can be entirely constructed by a *line*, so that in a notation already submitted to the Academy (see account of the communication made in last December) the equation

$$\mathrm{S}\,.\,\mathrm{ABCDEA}=0, \qquad (6)$$

is found to be satisfied, we may then infer that the five corners A, B, C, D, E, of this pentagon, are situated on the surface of one common sphere. This *equation of homosphæricism* (5) or (6), appears to the present author to be very fertile in its consequences. To leave no doubt respecting its *meaning*, and to present it under a form under which it may be easily understood by those who have not yet made themselves masters of the whole of the theory, it may be stated thus: if we write for abridgment,

$$\left.\begin{aligned}
a_1 &= i\,(x_1 - x_2) + j\,(y_1 - y_2) + k\,(z_1 - z_2),\\
a_2 &= i\,(x_2 - x_3) + j\,(y_2 - y_3) + k\,(z_2 - z_3),\\
a_3 &= i\,(x_3 - x_4) + j\,(y_3 - y_4) + k\,(z_3 - z_4),\\
a_4 &= i\,(x_4 - x_5) + j\,(y_4 - y_5) + k\,(z_4 - z_5),\\
a_5 &= i\,(x_5 - x_1) + j\,(y_5 - y_1) + k\,(z_5 - z_1),
\end{aligned}\right\} \quad (7)$$

and then develope the continued product of these five expressions, *using the distributive, but not* (so far as relates to $i\,j\,k$) *the commutative property of multiplication*, and reducing the result to the form of a quaternion,

$$a_1\, a_2\, a_3\, a_4\, a_5 = w + ix + jy + kz, \quad (8)$$

by the fundamental symbolical relations between the *three coordinate characteristics* $i\,j\,k$, which were communicated to the Academy by Sir William Hamilton in November, 1843, and which may be thus concisely stated:

$$i^2 = j^2 = k^2 = ijk = -1; \quad \text{(A)*}$$

and if we find, as the result of this calculation, that the term

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* These fundamental equations between the author's symbols i, j, k, appeared, under a slightly more developed form, in the number of the *London, Edinburgh, and Dublin Philosophical Magazine* for July, 1844; in which Magazine the author has continued to publish, from time to time, some articles of a Paper on Quaternions; reserving, however, for the Transactions of the Royal Irish Academy, a more complete and systematic account of his researches on this extensive subject.

w, or the part of the quaternion (8) which is independent of the characteristics ijk, vanishes, so that we have the following equation, which is entirely freed from those symbolic factors,

$$w = 0, \tag{9}$$

we shall then know that the points, of which the rectangular coordinates are respectively $(x_1 y_1 z_1)$ $(x_2 y_2 z_2)$ $(x_3 y_3 z_3)$ $(x_4 y_4 z_4)$ $(x_5 y_5 z_5)$, are *five homosphæric points*, or that one common spheric surface will contain them all.

The actual process of this multiplication and reduction would be tedious, nor is it offered as the easiest, but only as *one* way of forming the equation in rectangular coordinates, which is here denoted by (9). A much easier way would be to prepare the equation (5) by a previous development, so as to put it under the following form :

$$\rho^2 \,\mathrm{S} \,.\, \alpha\beta\gamma = \alpha^2 \,\mathrm{S} \,.\, \beta\gamma\rho + \beta^2 \,\mathrm{S} .\, \gamma\alpha\rho + \gamma^2 \,\mathrm{S} \,.\, \alpha\beta\rho \,; \tag{10}$$

which also admits of a simple geometrical interpretation. For, by comparing it with the following equation, which is in this calculus an *identical* one, or is satisfied for *any* four vectors, $\alpha, \beta, \gamma, \rho$:

$$\rho \,\mathrm{S} \,.\, \alpha\beta\gamma = \alpha \,\mathrm{S} \,.\, \beta\gamma\rho + \beta \,\mathrm{S} \,.\, \gamma\alpha\rho + \gamma \,\mathrm{S} \,.\, \alpha\beta\rho, \tag{11}$$

we find that the form (10) gives

$$\rho^2 = \alpha\alpha' + \beta\beta' + \gamma\gamma', \tag{12}$$

if α', β', γ' denote three diverging edges of a parallelepiped, of which the intermediate diagonal (or their symbolic sum) is the chord ρ of a sphere, while $\alpha\beta\gamma$ are three other chords of the same sphere, in the directions of the three edges, and coinitial with them and with ρ; so that *the square upon the diagonal ρ is equal to the sum of the three rectangles under the three edges $\alpha'\beta'\gamma'$ and the three chords $\alpha\beta\gamma$*, with which, in direction, those edges respectively coincide. This theorem is only mentioned here, as a simple example of the *interpretation* of the formulæ to which the present method conducts ; since the same result may be obtained very simply from a more ordinary form of the equation of the sphere, referred to the edges $\alpha' \beta' \gamma'$ as oblique coordinates; and, doubtless, has been already obtained in that or in some other way. An analogous theorem for the ellipsoid may be obtained with little difficulty.

If we suppose in the formula (6), that the point E of the pentagon approaches to the point A, the side EA tends to become an infinitely small tangent to the sphere; and thus we find that V . ABCDA, or that *the vector part of the continued product* AB × BC × CD × DA, *of the four sides of an uneven* (or *gauche*) *quadrilateral* ABCD, if determined by the rules of multiplication proper to this calculus, *is normal to the circumscribed sphere* at the point A, where the first and fourth sides are supposed to meet. By the non-commutative character of quaternion multiplication, we should get a different product, if we took the factors in the order BC × CD × DA × AB; and accordingly the vector or grammic part V . BCDAB of this new quaternion product would represent a new line in space, namely, a normal to the same sphere at B: and similarly may the normals be found at the two other corners of the quadrilateral, by two other arrangements of the four sides as factors. To determine the lengths of the normal lines thus assigned, we may observe that if A′, B′, C′, D′ be the four points on the same sphere, which are diametrically opposite to the four given points A, B, C, D, then the four diameters A′A, B′B, C′C, D′D are given by four expressions, of which it may be sufficient to write one, namely:

$$\mathrm{A'A} = \frac{\mathrm{V \,.\, ABCDA}}{\mathrm{S \,.\, ABCD}}. \qquad (13)$$

The denominator of this expression denotes (as was remarked in a former communication) the sextuple volume of the pyramid, or tetrahedron, ABCD; it vanishes, therefore, when the four points A, B, C, D are in one plane: so that we have for *any plane quadrilateral* the equation,

$$\mathrm{S \,.\, ABCD} = 0. \qquad (14)$$

If the sphere is then to become only *indeterminate*, and not necessarily infinite, we must suppose that the numerator of the same expression (13) also vanishes; that is, we must have in this case the condition

$$\text{V . ABCDA} = 0. \tag{15}$$

In words, as the product of the five successive sides of an uneven but rectilinear pentagon inscribed in a sphere, has been seen to be *purely a line*, so we now see that *the product of the four successive sides of a quadrilateral inscribed in a circle is* (in this system) *purely a number:* whereas, for *every other* rectilinear quadrilateral, *whether plane or gauche*, the grammarithm obtained as the product of four successive sides *involves a grammic part*, which does not vanish. This condition (15), for a quadrilateral inscribable in a circle, could not be always satisfied, when D approached to A, and tended to coincide with it, unless the following theorem were also true, which can accordingly be otherwise proved: *the product* ABCA, or AB × BC × CA, *of three successive sides of any triangle* ABC, *is a pure vector, in the direction of the tangent to the circumscribed circle*, at the point A, where the sides which are assumed as first and third factors of the product meet each other. If $\text{A}_{\prime}$ be the point upon this circumscribed circle which is diametrically opposite to A, we find for the length and direction of the diameter $\text{AA}_{\prime}$ in this notation, that is, for the straight line *to* A *from* $\text{A}_{\prime}$, the expression:*

$$\text{AA}_{\prime} = \frac{\text{ABCA}}{\text{V . ABC}}; \tag{16}$$

the denominator denoting a line which is in direction perpen-

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* With respect to the *notation of division*, in this theory, the author proposes to distinguish between the two symbols

$$\text{Q}^{-1}\,\text{Q}' \quad \text{and} \quad \frac{\text{Q}'}{\text{Q}},$$

dicular to the plane of the triangle, and in magnitude represents the double of its area; while the numerator is, as we have just seen, in direction tangential to the circle at A, and its length represents the product of the lengths of the three sides, or the volume of the solid constructed with those sides as rectangular edges. We may add, that this tangential line ABCA is distinguished from the equally long but *opposite* tangent ACBA to the same circle ABC at the same point A, by the condition that the former is intermediate in direction between AB (prolonged through A) and CA, while the latter in like manner lies between AC (prolonged) and BA: or we may say that the line ABCA touches, at A, the segment *alternate* to that segment of the circle ABC which has AC for base, and contains the point B; while the opposite line ACBA touches, at the same point, the last mentioned segment itself. The condition for the diameter $AA_{\prime}$ becoming infinite, or for the three points ABC being situated on one common straight line, is

$$\mathrm{V} \,.\, \mathrm{ABC} = 0. \qquad (17)$$

This formula (17) is therefore, in this notation, the *general equation of a straight line* in space; (15) is the general *equation of a circle*; (14) of a plane; and (6) of a sphere.*

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which he inadvertently used as interchangeable in his first communication to the Academy: and to make them satisfy the two separate equations,

$$\mathrm{Q} \times \mathrm{Q}^{-1}\, \mathrm{Q}' = \mathrm{Q}';$$

$$\frac{\mathrm{Q}'}{\mathrm{Q}} \times \mathrm{Q} = \mathrm{Q}'.$$

He proposes to confine the symbol $\mathrm{Q}' \div \mathrm{Q}$ to the signification thus assigned for the latter of the two symbols which have been thus defined, and which, on account of the non-commutative property of multiplication of quaternions, ought not to be confounded with each other.

* The simpler equation of scalar form, $\mathrm{S} \,.\, \mathrm{ABC} = 0$, also represents a spheric surface, if B be regarded as the variable point; but a plane, if B be fixed, and either A or C alone variable.

It may seem strange that the line and circle should here be represented each by only *one* equation; but these equations are of *vector forms*, and decompose themselves each into three equations, equivalent, however, only to two distinct ones, when we pass to rectangular coordinates, for the sake of comparison with known results.

In the same notation of capitals, whatever five distinct points may be denoted by A, B, C, D, E, we have the general transformation,

$$\text{ABCDEA} = \text{ABCA} \times \text{ACDA} \times \text{ADEA} \div \text{ACADA}, \qquad (18)$$

in which the divisor ACADA, or ACA $\times$ ADA, is the product of two positive scalars; if then we had otherwise established the interpretation lately assigned to the symbol ABCA, as denoting a line which touches at A the circle ABC, we might have in that way deduced the equation (6) of a sphere, as the *condition of the coplanarity of the three tangents* at A, *to the three circles*, ABC, ACD, ADE. And we see that when this condition is satisfied, so that the points A, B, C, D, E are homosphæric, and that, therefore, the symbol ABCDEA represents a vector, we can *construct the direction of this vector* by drawing in the plane which touches the sphere at A, a line $A_1 A_2$ parallel to the line ACDA which touches the circle ACD at A, and cutting, in the points A_1 and A_2, the two lines ABCA and ADEA, which are drawn at A to touch the circles ABC, ADE; for then the vector ABCDEA, which is thus seen to be a tangent to the sphere, will touch, at the same point A, the circle A $A_1 A_2$, described on the tangent plane. In the more general case, when the condition (6) is *not* satisfied, and when, therefore, the rectilinear pentagon ABCDE, which we shall suppose to be uneven, cannot be inscribed in a sphere, the scalar symbol S. ABCDEA which has been seen to vanish when the pentagon can be so inscribed, represents the continued *product of the lengths of the five sides* AB, BC, CD, DE, EA, *multiplied by the sextuple volume of that triangular pyramid which is con-*

structed with three conterminous edges, each equal to the unit of length, and touching at the vertex A *the three circles* ABC, ACD, ADE, *which have respectively for chords the three remote sides of the pentagon,* and are not now homosphæric circles. And because, in general, in this notation, the equation

$$\mathrm{S}\,.\,\mathrm{ABCDEA} = \mathrm{S}\,.\,\mathrm{BCDEAB} \tag{19}$$

holds good, it follows that *for any rectilinear pentagon* (in space) *the five triangular pyramids constructed on the foregoing plan,* with the five corners of the pentagon for their respective vertices, *have equal volumes.*

Besides the characteristics S and V, which serve to decompose a quaternion Q into *two parts,* of distinct and determined kinds, the author frequently finds it to be convenient to use two other characteristics of operation, T and U, which serve to decompose the same quaternion into *two factors,* of kinds equally distinct and equally determinate; in such a manner that we may write generally, with these characteristics, for any quaternion Q,

$$\mathrm{Q} = \mathrm{SQ} + \mathrm{VQ} = \mathrm{TQ} \times \mathrm{UQ}. \tag{20}$$

The factor TQ is always a positive, or rather an absolute (or *signless*) number; it is what was called by the author, in his first communication on this subject to the Academy, the *modulus,* but he has since come to prefer to call it the TENSOR of the quaternion Q: and he calls the other factor UQ the VERSOR of the same quaternion. As the *scalar of a sum is the sum of the scalars,* and the *vector of a sum is the sum of the vectors,* so the *tensor of a product is the product of the tensors,* and the *versor of a product is the product of the versors;* relations or properties which may be concisely expressed by the formulæ:

$$\mathrm{S}\Sigma = \Sigma\mathrm{S}; \qquad \mathrm{V}\Sigma = \Sigma\mathrm{V}; \tag{21}$$

$$\mathrm{T}\Pi = \Pi\mathrm{T}; \qquad \mathrm{U}\Pi = \Pi\mathrm{U}. \tag{22}$$

When we operate by the characteristics T and U on a straight line, regarded as a vector, we obtain as the *tensor* of this line a *signless number* expressing its *length;* and, as the *versor* of the same line, an *imaginary unit*, determining its *direction*. When we operate on the product $\mathrm{ABC} = \mathrm{AB} \times \mathrm{BC}$ of two successive lines, regarded as a quaternion, we obtain for the tensor, $\mathrm{T.ABC}$, the product of the lengths of the two lines, or the area of the rectangle under them; and for the versor of the same product of two successive sides of a triangle (or polygon), we obtain an expression of the form

$$\mathrm{U.ABC} = \cos \mathrm{B} + \sqrt{-1} \sin \mathrm{B}; \tag{23}$$

the symbol B in the second member denoting the *internal angle* of the figure at the point denoted by the same letter, which angle is thus the *amplitude of the versor*, and at the same time (in the sense of the author's first communication) the *amplitude of the quaternion* itself, which quaternion is here denoted by the symbol ABC. In this theory (as was shown by the author to the Academy in that first communication), there are *infinitely many* different square roots of negative unity, constructed by lines equal to each other, and to the unit of length, but distinguishable by their *directional* (or *polar*) coordinates: the *particular* $\sqrt{-1}$ which enters into the expression (23) is perpendicular to the plane of the triangle ABC. It is the *versor of the vector* of that quaternion which is denoted by the same symbol ABC; and it may, therefore, be replaced by the symbol $\mathrm{UV.ABC}$, which we may agree to abridge to $\mathrm{W.ABC}$, so that we may establish the symbolic equation:

$$\mathrm{UVQ} = \mathrm{WQ}, \quad \text{or simply,} \quad \mathrm{UV} = \mathrm{W}; \tag{24}$$

we may also call WQ the *vector unit* of the quaternion Q. The expression (23) suggests also the denoting the *amplitude* of *any* quaternion by the geometrical mark for an *angle*, which notation will also agree with the original conception of such an amplitude; and thus we are led to write, generally, as a transformed expression for a versor,

$$\mathrm{UQ} = \cos \angle \mathrm{Q} + \mathrm{WQ} \,.\, \sin \angle \mathrm{Q}. \qquad (25)$$

The amplitude of a vector is in this theory a quadrant; that of a positive number being, as usual, zero, and that of a negative number two right angles. Applying the same principles and notation to the case of the continued product ABCDA of the four successive sides of an uneven quadrilateral ABCD, we find that the amplitude $\angle$ ABCDA of this quaternion product is equal to the *angle of the lunule* ABCDA, if we employ this term "lunule" to denote a portion of a spherical surface bounded by two arcs (which may be greater than halves) of *small* circles, namely, here, the portion of the surface of the sphere circumscribed about the quadrilateral ABCD, which portion is bounded by the two arcs that go from the corner A of that quadrilateral to the opposite corner C, and which pass respectively through the two other corners B and D. The tensor and scalar of the continued product of the four sides of the quadrilateral do not change when the sides are taken in the order, second, third, fourth, first; and generally,

$$\cos \angle \mathrm{Q} = \mathrm{SQ} \div \mathrm{TQ}; \qquad (26)$$

so that we have the equation,

$$\angle \mathrm{ABCDA} = \angle \mathrm{BCDAB}; \qquad (27)$$

hence the *two lunules* ABCDA and BCDAB, which have for their *diagonals* AC and BD the two diagonals of the quadrilateral, and with which the lunules CDABC and DABCD respectively coincide, are *mutually equiangular* at A and B. Thus, generally, for *any four points*, A B C D, the two circles ABC, ADC cross each other at A and C (in space, or on one plane), under the same angles as the two other circles, BCD, BAD, at B and D.

Again, it may be remarked, that the condition for a fifth point E being contained on the plane which touches, at A, the sphere circumscribed about the tetrahedron ABCD, is expressed by the equation

$$\mathrm{S} \,.\, \mathrm{ABCDAE} = 0\,; \tag{28}$$

this equation, therefore, ought not to be compatible with the equation (6), which expressed that the point E was on the sphere itself, except by supposing that the point E coincides with the point of contact A; and accordingly the principles and rules of this notation give, generally,

$$\mathrm{S} \,.\, \mathrm{ABCDEA} + \mathrm{S} \,.\, \mathrm{ABCDAE} = \mathrm{S} \,.\, \mathrm{ABCD} \,.\, \mathrm{AEA}, \tag{29}$$

in which by (14) the first factor S . ABCD of the second member does not vanish if the sphere be finite, that is, if the volume of the tetrahedron do not vanish, while the second factor may be thus transformed,

$$\mathrm{AEA} = -\,(\mathrm{EA})^2, \tag{30}$$

so that the coexistence of the two equations (6) and (28) of a sphere and its tangent plane, is thus seen to require that we shall have

$$\mathrm{EA} = 0\,; \tag{31}$$

which is, relatively to the sought position of E, the *equation of the point of contact.* These examples, though not the most important that might be selected, may suffice to show that there already exists a *calculus,* which may deserve to be further developed, for combining and transforming geometrical expressions of this sort. Several of the elements of such a calculus, especially as regards geometrical *addition* and *subtraction,* have been contributed by other, and (as the author willingly believes) by better geometers; what Sir William Hamilton considers to be peculiarly his own contribution to this department of mathematical and symbolical science consists in the introduction and development of those conceptions of GEOMETRICAL MULTIPLICATION (and division), which were embodied by him (in 1843) in his fundamental formulæ for the *symbolic squares and products of the three coordinate characteristics* (or algebraically imaginary units) i, j, k, which entered into his original expression of a QUATERNION $(w + ix + jy + kz)$, and by which he succeeded in representing, symmetrically, that is, *without any selection of one direction as eminent*, the three dimensions of space.

It is, however, convenient, in many researches, to retain the notation in which Greek letters denote *vectors*, instead of employing that other notation, in which capital letters (a few *characteristics* excepted), denote *points*. In the former notation it was shown to the Academy in last December (see formula (21) of the abstract of the author's communication of that date), that the equation of an ellipsoid, with three unequal axes, referred to its centre as the origin of vectors, may be put under the form:

$$(\alpha\rho + \rho\alpha)^2 - (\beta\rho - \rho\beta)^2 = 1;*$$

ρ being the variable vector of the ellipsoid, and β and α being two constant vectors, in the directions respectively of the axes of one of the two circumscribed cylinders of revolution, and of a normal to the plane of the corresponding ellipse of contact. Decomposing the first member of that equation of an ellipsoid into two factors of the first degree, or writing the equation as follows:

$$(\alpha\rho + \rho\alpha + \beta\rho - \rho\beta)(\alpha\rho + \rho\alpha - \beta\rho + \rho\beta) = 1, \quad (32)$$

we may observe that these two factors, which are thus *separately linear* with respect to the variable vector ρ, are at the same time *conjugate quaternions*; if we call two quaternions, Q and KQ, CONJUGATE, when they have *equal scalars* but have *opposite vectors*, so that generally,

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* Appendix, No. V., page lviii.

$$\text{K}Q = \text{S}Q - \text{V}Q, \quad \text{or, more concisely,} \quad \text{K} = \text{S} - \text{V}. \tag{33}$$

And if we further observe, that in general the *product of two conjugate quaternions* is equal to *the square of their common tensor*,

$$Q \times \text{K}Q = (\text{S}Q)^2 - (\text{V}Q)^2 = (\text{T}Q)^2, \tag{34}$$

we shall perceive that the equation (32) of an ellipsoid may be put, by extraction of a square root, under this simpler, but not less general form:

$$\text{T}(\alpha\rho + \rho\alpha + \beta\rho - \rho\beta) = 1. \tag{35}$$

Again, by employing the principle, that $\text{T}\Pi = \Pi\text{T}$, we may again decompose the first member of (35) into two factors, and may write the *equation of an ellipsoid* thus:

$$\text{T}(\alpha + \beta + \sigma) \,.\, \text{T}\rho = 1, \tag{36}$$

if we introduce an auxiliary vector, σ, connected with the vector ρ by the relation

$$\sigma = \rho(\alpha - \beta)\rho^{-1}, \tag{37}$$

which gives, by the same principle respecting the tensor of a product,

$$\text{T}\sigma = \text{T}(\alpha - \beta); \tag{38}$$

so that the auxiliary vector σ has a *constant length*, although it has by (37) a *variable direction*, depending on, and in its turn assisting to determine or construct the direction of the vector ρ of the ellipsoid; for the same equation (37) gives for the *versor* of that vector the expression

$$\text{U}\rho = \pm\,\text{U}(\alpha - \beta + \sigma). \tag{39}$$

Hence, by the second general decomposition (20), and by the equation (36), the last mentioned vector ρ itself may be expressed as follows:

$$\rho = \frac{\text{U}(\alpha - \beta + \sigma)}{\text{T}(\alpha + \beta + \sigma)}; \tag{40}$$

making then, in the notation of capital letters for points,

$$\alpha + \beta = \text{CB}, \quad \alpha - \beta = \text{CA}, \quad \sigma = \text{DC}, \quad \rho = \text{EA}, \qquad (41)$$

so that A is the centre of the ellipsoid, E a variable point on its surface, C the fixed centre of an auxiliary sphere, of which the surface passes through the fixed point A, and also through the auxiliary and variable point D, while B is another fixed point, we obtain the equation:

$$\text{EA} = \pm \text{U}\,.\,\text{DA} \div \text{T}\,.\,\text{DB}\,; \qquad (42)$$

which gives

$$(\text{EA})^{-1} = \mp \text{U}\,.\,\text{DA}\,.\,\text{T}\,.\,\text{DB}, \qquad (43)$$

and shows, therefore, that the *proximity* $(\text{EA})^{-1}$ *of a variable point* E, *on the surface of an ellipsoid, to the centre* A *of that ellipsoid, is represented in direction by a variable chord* DA *of a fixed sphere*, of which one extremity A is fixed, while the *magnitude* of the same proximity, or the *degree of nearness* (increasing as E approaches to the centre A, and diminishing as it recedes), is *represented by the distance* DB *of the other extremity* D *of the same chord* DA *from another fixed point* B, which may be supposed to be external to the sphere. This use of the *word* "proximity," which appears to be a very convenient one, is borrowed from Sir John Herschel: the *construction for the ellipsoid* is perhaps new, and may be also thus enunciated:—From a fixed point A on the surface of a sphere, draw a variable chord DA; let D′ be the second point of intersection of the spheric surface with the secant DB, drawn to the variable extremity D of this chord from a fixed external point B; take the radius vector EA equal in length to D′B, and in direction either coincident with, or opposite to, the chord DA; *the locus of the point* E, *thus constructed, will be an ellipsoid*, which will pass through the point B. This fixed point B (one of four known points upon the principal ellipse) may, perhaps, be fitly called a POLE, and the line BE a *polar chord*, of the ellipsoid; and in the construction just stated, the two variable points D, D′ may be said to be *conjugate guide-points*, at the extremities of coinitial and *conju-*

gate guide-chords DA, D′A of a fixed *guide-sphere*, which passes through the centre A of the ellipsoid.

We may also say, that *if of a quadrilateral* (ABED′) *of which one side* (AB) *is given in length and in position, the two diagonals* (AE, BD′) *be equal to each other in length, and intersect* (in D) *on the surface of a given sphere* (with centre C), *of which a chord* (AD′) *is a side of the quadrilateral adjacent to the given side* (AB), *then the other side* (BE), *adjacent to the same given side, is a* (*polar*) *chord of a given ellipsoid:* of which last surface, the form, position, and magnitude, are thus seen to depend on the form, position, and magnitude, of what may, therefore, be called the *generating triangle* ABC. Two sides of this triangle, namely, BC and CA, are perpendicular to the two planes of *circular section;* and the third side AB is perpendicular to one of the two planes of *circular projection* of the ellipsoid, being the axis of revolution of a circumscribed circular cylinder. Many fundamental properties of the ellipsoid may be deduced with extreme facility, as geometrical* consequences of this mode of generation; for example, the well-known proportionality of the difference of the squares of the reciprocals of the semi-axes of a diametral section to the product of the sines of the inclinations of its plane to the two planes of circular section, presents itself under the form of a proportionality of the same difference of squares to the rectangle under the projections of the two sides BC and CA of the generating triangle on the plane of the elliptic section.

If we put the equation (35) of an ellipsoid under the form

$$\mathrm{T}\,(\iota\rho + \rho\kappa) = \kappa^2 - \iota^2, \tag{44}$$

the constant vectors ι and κ will be in the directions of the normals to the planes of circular section, and may represent

* For the following geometrical corollary, from the construction assigned above, the author is indebted to the Rev. J. W. Stubbs, Fellow of Trinity College. If the auxiliary point D describe, on the sphere, a circle of which the plane is perpendicular to BC, the point E on the ellipsoid will describe a spherical conic.

the two sides BC and AC of the triangle, while $\iota - \kappa$ will be one value of the variable vector ρ or EA, namely, the remaining side of the same triangle, or the semi-diameter BA in the last mentioned construction of the surface; and by applying to this equation (44) the general methods which the author has established for investigating by quaternions the tangent planes and curvatures of surfaces, it is found that the *vector of proximity* ν of the tangent plane to the centre of the ellipsoid (that is, the reciprocal of the perpendicular let fall on this plane from this centre), is determined in length and in direction by the equation,

$$(\kappa^2 - \iota^2)^2 \nu = (\kappa^2 + \iota^2) \rho + \iota\rho\kappa + \kappa\rho\iota ; \tag{45}$$

while the two rectangular directions of a vector τ, tangential to a line of curvature, at the extremity of the vector ρ, are determined by the system of equations:

$$\nu\tau + \tau\nu = 0 ; \quad \nu\tau\iota\tau\kappa - \kappa\tau\iota\tau\nu = 0 ; \tag{46}$$

which may also be thus written :

$$\text{S} \,.\, \nu\tau = 0 ; \quad \text{S} \,.\, \nu\tau\iota\tau\kappa = 0. \tag{47}$$

Of these two equations (46) or (47), the former expresses merely that the tangential vector τ is perpendicular to the normal vector ν; while the latter is found to express that the tangent to either line of curvature of an ellipsoid is equally inclined to the two traces of the planes of circular section on the tangent plane, and therefore bisects one pair of the angles formed by the two circular sections themselves, which pass through the given point of contact. Indeed, it is easy to prove this relation of bisection otherwise, not only for the ellipsoid, but for the hyperboloids, by considering the common sphere which contains the circular sections last mentioned; the author believes that the result has been given in one of the excellent geometrical works of M. Chasles; it may also be derived without difficulty from principles stated in the masterly Memoir on Surfaces of the Second Order, which has been published by Professor Mac Cullagh in the Proceedings of this Academy. (See Part VIII., page 484.)

The length to which the present abstract has already extended, prevents Sir William Hamilton from offering on the present occasion any details respecting the processes (analogous in some respects to the calculi of variations and partial differentials) by which he applies the principles of his own method to investigations respecting surfaces and curves in space, or to physical problems connected therewith; he desires, however, to mention here that, in investigations respecting normals to surfaces, he finds it convenient to employ a new characteristic of operation of the form

$$(\mathrm{s} \,.\, \mathrm{d}\rho)^{-1}.\, \mathrm{d} = \alpha\,, \tag{48}$$

in order to obtain from a scalar function of a variable vector ρ, a new variable vector ν which shall be normal to the locus for which that scalar function is constant; and that the following more general characteristic of operation,

$$i\frac{\mathrm{d}}{\mathrm{d}x} + j\frac{\mathrm{d}}{\mathrm{d}y} + k\frac{\mathrm{d}}{\mathrm{d}z} = \triangleleft\,, \tag{49}$$

in which x, y, z are ordinary rectangular coordinates, while i, j, k are his own coordinate imaginary units, appears to him to be one of great importance in many researches. This will be felt (he thinks) as soon as it is perceived that with this meaning of $\triangleleft$ the equation

$$\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}y}\right)^2 + \left(\frac{\mathrm{d}}{\mathrm{d}z}\right)^2 = -\triangleleft^2, \tag{50}$$

is satisfied in virtue of the fundamental relations between his symbols i, j, k; which relations give also, as another result of operating with the same characteristic, this other important symbolic expression, which presents itself under the form of a quaternion:

$$\lhd (it + ju + kv) = -\left(\frac{\mathrm{d}t}{\mathrm{d}x} + \frac{\mathrm{d}u}{\mathrm{d}y} + \frac{\mathrm{d}v}{\mathrm{d}z}\right)$$

$$+ i\left(\frac{\mathrm{d}v}{\mathrm{d}y} - \frac{\mathrm{d}u}{\mathrm{d}z}\right)$$

$$+ j\left(\frac{\mathrm{d}t}{\mathrm{d}z} - \frac{\mathrm{d}v}{\mathrm{d}x}\right)$$

$$+ k\left(\frac{\mathrm{d}u}{\mathrm{d}x} - \frac{\mathrm{d}t}{\mathrm{d}y}\right). \qquad (51)$$

The President having taken the Chair,

The Rev. Charles Graves read a paper by Mr. George Boole, of Lincoln, containing investigations supplementary to his former papers on Discontinuous Functions and Definite Multiple Integrals.

The author commences his observations by pointing out a distinction among integrals which constitute the limits of more general forms, according as they are supposed to be obtained by the vanishing of one or of more constants. The integral $\int_0^\infty dx \cos(qx)\, x^{n-1}$, n being positive, considered as the limit of $\int_0^\infty dx\varepsilon^{-kx} \cos(qx)\, x^{n-1}$, he designates a limiting integral of the first class, because it involves the consideration of one vanishing constant. The same integral, when n is negative, he regards as the limit of the more general form.

$$\int_0^\infty \frac{dx\varepsilon^{-k'x} \cos qx \cos(n \tan^{-1} x)}{(k^2 + x^2)^{\frac{n}{2}}}$$

two constants k and k' vanishing; and designates it a limiting integral of the second class. Under this assumption he assigns as its value

$$\int_0^\infty \frac{dx \cos qx}{x^n} = \frac{\pi\,(\pm q)^{n-1}}{2\Gamma(n)}$$

the upper or lower sign being taken, according as q is positive or negative. Assuming as the definition of $\Gamma(n)$, the equation

$$\Gamma(n) = \int_0^\infty dx \cos(x)\, x^{n-1},$$

whether n is positive or negative, and regarding the integral in the second member as a limiting integral of the first or second class, according as n is positive or negative, the author shews that, universally,

$$\Gamma(n)\,\Gamma(1-n) = \frac{\pi}{\sin n\pi},$$

a theorem which is known to be true of Γ in its ordinary definition when n lies between 0 and 1, but not otherwise. This theory is further applied to explain the discontinuity of form which is apparent in integrals, the subjects of which become infinite within the limits of integration, with some other connected points.

The paper concludes with an application of Fourier's theorem to the solution of equations. It is proved that the value v of the definite integral

$$\mathrm{v} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} da\, dv\, \varepsilon^{(a-x)v\sqrt{-1}} f(a, v\sqrt{-1})$$

is symbolically expressed by the equation

$$\mathrm{v} = \left(-\frac{d}{dx}\right)^{\frac{d}{d\theta}} f(x, \varepsilon^{\theta}),$$

provided that $\theta = 0$, from which the following theorem is deduced:

If $f(u) = x$ and $\phi(x)$ be any function of x which makes $f[\phi(x)]$ real, then

$$\mathrm{F}(u) = -\left(-\frac{d}{dx}\right)^{\frac{d}{d\theta}-1} \varepsilon^{(f[\phi(x)]-x)\varepsilon^{\theta}}\, \mathrm{F}'[\phi(x)]\,\phi'(x),$$

which may be expanded in the form

$$\mathrm{F}(u) = \mathrm{F}[\phi(x)] + \mathrm{X}\mathrm{F}'[\phi(x)]\,\phi'(x) + \frac{1}{1.2}\frac{d}{dx}\mathrm{X}^2\mathrm{F}'[\phi(x)]\,\phi'(x) + \&c$$

wherein

$$\mathrm{X} = x - f[\phi(x)].$$

From this result the author deduces the theorems of Laplace and Lagrange for the expansion of implicit functions; and he shews that, through the arbitrary character of $\phi(x)$, they are particular cases of a class of theorems, infinite in number. Applied to the solution of the equation $f(u) = x$, by making $\mathrm{F}(u) = u$, this method gives that root which will be represented by $\phi(v)$, in which v is the least root of the equation

$$f[\phi(v)] = x.$$

The President read the following letter from Mr. Cooper:—

"*Markree Castle, July* 3, 1846.

"My dear Sir,—I now beg to transmit to you, for the favourable consideration of the Academy, the observations we have been able to make on comets at this Observatory, during the first six months of this year. They are preceded by the places of some stars with which we compared the comets, and which we were forced to determine, as they were not included in any catalogue we possess. The results of observations, made by Mr. Graham principally, for polar point on circle, are also added. The dates without places, signify that we have the observation, but not yet the stars of comparison.

"Believe me, my dear Sir,

"Your's very sincerely,

"Edward Cooper.

"*The Rev. H. Lloyd, D. D.*

&c. &c. &c."

DONATIONS.

Report of the Fifteenth Meeting of the British Association for the Advancement of Science, held at Cambridge in June. Presented by the Association.

Recueil de l'Academie des Jeux Floraux. Presented by Viscount De Mac Carthy, M. R. I. A.

Memoire sur deux Balances a Reflexion. Presented by Professor Wartman.

Journal of the Royal Asiatic Society. No. XXII. Part I. Presented by the Society.

Journal of the Asiatic Society of Bengal. No. CLXIV. Part I. Presented by the Society.

De la Methode dans l'Electricitie et le Magnetisme, &c. Presented by Professor Wartman.

Akademischer Almanach auf das Jahr, 1845. Presented by the Society.

Bulletins des Séances de la Societe Vaudoise, des Sciences Naturelles. Presented by the Society.

Proceedings of the Royal Asiatic Society of Great Britain and Ireland. Presented by the Society.

Magnetische und Geographische Ortsbestimmungen in Böhmen in den Jahren, 1843–1845. Von Karl Kreil. Presented by the Author.

Magnetische und Meteorologische Beobachtungen zu Prag, vom 1. *Jänner bis* 31 *Decr.* 1845. Von Karl Kreil. Presented by the Author.

Bulletins des Séances de la Societe Vaudoise des Sciences Naturelles. Tom. I. Années 1842–1845. Presented by the Society.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–47. No. 56.

November 9th, 1846.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The following Address, which was presented to the Lord Lieutenant on the 2nd September last, together with His Excellency's Answer, was ordered to be entered on the Minutes:

"*To His Excellency the Right Honourable John William Earl of Bessborough, Lord Lieutenant General and General Governor of Ireland, &c.*

"May it please your Excellency,

"We, the President and Members of the Royal Irish Academy, beg leave to offer to your Excellency our respectful congratulations on your acceptance of the high office of Representative of our Most Gracious Sovereign in this country.

"In virtue of the Charter under which we have been incorporated by our Royal Founder, for the Promotion of Science, Polite Literature, and Antiquities, the duties of Visitor of this Academy have devolved upon your Excellency, as Lord Lieutenant of Ireland.

"We cannot but rejoice that we are thus placed, in the present instance, in official dependence upon one who is both a native and a resident of this country, and to whose sympathies a Society such as our's, founded for the advancement of knowledge in Ireland, may confidently appeal.

"Composed, as our body is, of men conscientiously differing upon questions of the first importance, and united by the sole bond of kindred intellectual pursuits, we have had the satisfaction to know that, while the immediate objects of our incorporation have been successfully forwarded, the advance has been hallowed by those feelings of mutual good-will which are not less valuable than knowledge itself.

"That such feelings may take root and spread in this country, under your Excellency's Government, is our earnest hope and prayer."

ANSWER.

"Mr. President and Gentlemen,

"It is most gratifying to me to receive your congratulations on my acceptance of the high office to which Her Majesty has been pleased to appoint me; and I learn with great pleasure that, by virtue of the Charter under which you were incorporated, I am associated with your distinguished body as Visitor of the Royal Irish Academy.

"As a resident of Ireland I feel peculiar interest in a Society founded more particularly for the advancement of Science in this country; and it is most satisfactory to me to receive an Address from persons who, differing on questions of great importance, have done me the honour of concurring in their approval of my appointment."

Sir W. R. Hamilton having taken the Chair *pro tempore*, the President described a new instrument for observing the Magnetic Dip.

This instrument is similar to one furnished to the late Arctic Expedition under Sir John Franklin, and which was constructed by Mr. Barrow under the direction of Dr. Lloyd.

It is formed on the principle suggested by Gauss, of separating the needle altogether from the circle by which its position is determined; the moveable arms of the divided circle being furnished with compound microscopes, by means of which the extremities of the needle are observed. The circle is divided to 10′; and the readings are made to single minutes by the help of verniers.

By a simple modification in the construction of the instrument, it is capable of being used with needles of various lengths, not exceeding the diameter of the measuring circle; and it is hoped that it will thus serve to determine the question, not yet solved, as to the most advantageous dimensions of the dip needle.

Another, and more important, peculiarity in this instrument, is its adaptation to the determination of the intensity in absolute measure. In the received method, the *horizontal component* of the intensity is determined by a double observation, and the *total intensity* thence inferred by multiplying it by the secant of the inclination. Hence, in the high magnetic latitudes, where the inclination is considerable, any error in it will induce a large error in the inferred intensity; and in the neighbourhood of the magnetic pole the method fails altogether. The present instrument is adapted to measure the total intensity *directly;* and it is easily shewn that the result thus obtained is, in the high magnetic latitudes, much more accurate than that deduced in the ordinary way. The instrument may be employed, also, for the measurement of the horizontal and of the vertical components of the intensity.

Mr. Clibborn read a notice of certain Bronze Antiquities in the Museum of the Academy.

Mr. Clibborn stated that he had lately detected iron cores in the centre of the bronze composing the mouthpieces of several ancient horse-bits in the Museum. The fact was interesting, as it appeared to prove that, at the time these bits were manufactured, iron was a cheaper metal than bronze, it being used to save so much of that metal. It might, however, have been used for the purpose of chilling the bronze which composed the lateral rings, when their material in the molten state flowed through the openings in the mouth-pieces and formed these rings, which would otherwise have adhered to the mouth-pieces. In the Museum are many specimens of bronze castings, united by a process of this kind, yet there is no example of hard soldering, which appears to be a more modern invention.

The principle of covering iron with bronze, in the fluid state, is exhibited in the fabrication of the folded or lapped iron bells in the Museum, which are, in several instances, covered with bronze or brass, perfectly adhering to the iron surface. The bronze fills up the folds and joints, thus preventing any false vibration, and the bell sounds as if it were composed of one piece of metal.

He also directed the attention of the Meeting to the resemblance between the patterns on some curious antique plates in the Museum, and the details of a certain ornament very common in the initial letters in the manuscript Books of Kells, in the College Library, and the Book of Armagh, now deposited in the Museum of the Academy. He was disposed to infer, that these plates were of the same time as the MSS., or even earlier; for Mr. Westwood, who was the first to notice the resemblance, considered the pattern on them to be the type or original of the designs in the illuminations. This would support the conjecture that these plates were intended for Christian purposes, as pattens, or communion plates, probably; though the designs differ so very much from those of a later period.

The size and materials of these plates are the same as those of the *Mias Tighernain*, which may have been used as a patten also. This is rendered probable by the fact, communicated by the Rev. Dr. Kelly of Maynooth, that he had seen pattens of the same size, also made of copper, in use in the south of France. One of the plates in the Museum forms a cover for another, and is pierced with an opening, which is over a hollowed part of the under plate. The arrangements and proportions of the parts are such as to lead us to suspect that these plates, when placed together, may have formed a sort of poor-box, the alms having been dropped through the opening into the cupped part of the lower plate. It may be also observed that, on the most perfectly finished portion of one of these plates, there is the evident wear or impression of the thumb of a person who had, for a considerable time, handled it in the way a mendicant would, who might present it for alms. The back of this plate, and one of the others, has marks of fire on it. Mr. Clibborn also stated that a visitor to the Academy lately recognized these plates, as being very similar to two others, also composed of copper, which had been recently found in Armagh, and which he hoped would be soon deposited in the Museum, as they had been freely offered to our collection. Some light may thus be thrown on the use of these curious articles, which have hitherto been called shields; there being no evidence as to their original use, nor until lately was there any suspicion entertained of their belonging to the Christian period. From the perfection of the workmanship, and from some analogies in it and in the designs, it was also inferred that the large trumpets in the Museum might have been fabricated by artists of the same school as those who constructed the pattens.

The Secretary of Council read a translation of a letter from Professor Encke to M. Schumacher relative to Le Verrier's Planet. He also laid before the Academy the following note of observations made by E. J. Cooper, Esq., of Markree Castle, with his transit circle:

STARS OF COMPARISON.

No. 1.	*i* Capricorni.	No. 6.	*i* Aquarii.
2.	7451 Brit. Assoc. Cat.	7.	39 do.
3.	7487 do. do.	8.	45 do.
4.	42 Capricorni.	9.	50 do.
5.	δ do.	10.	σ do.

The following observations must hereafter undergo some modification, particularly those involving a comparison with 39 Aquarii, the declination of that star being lower than that assigned to it in the British Association Catalogue by about four seconds of space.

Apparent Places deduced from Mean Places, as given in the British Association Catalogue.

Mean Time, Greenwich.	Apparent Right Ascens.	Apparent Declination.	Compared Stars, &c.
1846.	h. m. s.	° ′ ″	
Oct. 3.401067	21 52 32.54	— 13 28 10.40	No. 7.
6.392739	20.74	29 13.93	No. 7. Bisection satisfactory, though blowing hard.
12.376121	0.27	30 53.67	No. 7. Hurried observation of Planet. Cannot depend on it. An opening only momentary in the clouds, which were general at the time.
14.370345	51 54.05	31 24.57	Nos. 1, 2, 3, 4, 5, 6. Got only the three first wires of the Planet, which was faint even at these.
16.365063	48.28	53.54	Nos. 1, 3, 4, 6, 7, 8, 9, 10. Good observation.
17.362062	45.73	32 2.03	Nos. 1, 2, 4, 6, 7, 8, 9, 10. Got only the three first wires of the Planet.
19.356785	40.94	36.65	Nos. 4, 5, 6, 7, 8. Good observation.
24.343014	30.60	33 19.78	Nos. 1, 2, 3, 5, 6, 7, 8, 9. Blowing hard, so that the clock was heard with difficulty; at some wires not at all.
26.337522	21 51 27.85	13 33 38.88	Nos. 5, 6, 7, 8, 9, 10. Satisfactory observation. Night had been very unpromising.

Sir W. R. Hamilton gave an account of the first observations of the new Planet made by him at the Observatory of Trinity College, Dublin.

DONATIONS.

Archæologia. Vol. XXXI. Presented by the Society of Antiquaries of London.

A Geographical Description of West or Hiar-Connaught, by Roderick O'Flaherty. Edited by J. Hardiman, M. R. I. A. Presented by the Irish Archæological Society.

Transactions of the Royal Society of Edinburgh. Vol. XVI. Part 2.

Proceedings of the Royal Society of Edinburgh. Vol. II. 1845–46, Nos. 27 and 28. Presented by the Society.

Address to the British Association for the Advancement of Science. Southampton, 1846. Presented by the Association.

Det Kongelige Danske Videnskabernes Selskabs Historiske og Philosophiske Afhandlinger. 7 Deel.

Det Kongelige Danske Videnskabernes Selskabs Naturvidenskabelige og Mathematiske Afhandlinger. 11 Deel.

Oversigt over det Kongelige Danske Videnskabernes Selskabs Ferhandlinger og dets Midlemmers Arbeider. I Aaret, 1844 and 1845. Presented by the Royal Society of Copenhagen.

Reports of the Council and Auditors of the Zoological Society of London. Read April 29th, 1844. *Proceedings of the Zoological Society of London.* Part 13. 1845. Presented by the Society.

Memoirs of the Geological Survey of Great Britain and the Museum of Economic Geology in London. Vol. I. Presented by the Commissioners of Woods and Forests.

On the Supply of printed Books from the Library to the Reading-Room of the British Museum. By A. Pannizzi, Esq. Presented by the Author.

Animadversions on the Library and Catalogues of the

British Museum. By Sir Harris Nicolas. Presented by the Author.

History of the Mace given to the Royal Society by King Charles II. By Richard Weld, Esq. Presented by the Author.

Two curious Blocks of Wood, with Mortices, found in a small Land-lake at Montagh, County Fermanagh. Presented to the Museum by the Earl of Enniskillen.

Proceedings of the Royal Society. Nos. 62 to 64.

List of Members of the Royal Society. Nov. 30, 1845. Presented by the Society.

The Quarterly Journal of the Geological Society of London. No. 7. Presented by the Society.

Memoires de la Societe de Physique et d'Histoire Naturelle de Geneve. Tome XI. 1re Partie. 1846. Presented by the Society.

Journal of the Statistical Society of London. Vol. IX. Part 3. Oct. 1846. Presented by the Society.

Seals of Archbishop Broderick and Archbishop Lawrence. Presented to the Museum by the Rev. Mr. Mayne.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 57.

November 30th, 1846. (Stated Meeting.)

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Thomas Moore, Esq., having been specially recommended by the Council, was elected an Honorary Member.

The Rev. William Roberts, F. T. C. D., read a paper on the definite integral

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 + n\sin^2\phi)}{\sqrt{(1 - k^2\sin^2\phi)}}\, d\phi.$$

It is clear that the only admissible values of n (real) are those of the parameter of an elliptic function of the third kind, to the modulus k, namely, $\cot^2\theta$, $-1 + k'^2\sin^2\theta$, and $-k^2\sin^2\theta$, where k' is the complement of k. The value of the definite integral may, in each of these cases, be expressed by elliptic functions of the first and second kinds, and by the remarkable transcendant $\Upsilon\left(\int\frac{\mathrm{E}(\phi)\,d\phi}{\sqrt{(1 - k^2\sin^2\phi)}}\right)$ by the aid of which functions of the third species, with a logarithmic parameter, can be calculated by tables of double entry. In fact, we have the following formulæ,

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 + \cot^2\theta \sin^2\phi)}{\sqrt{(1 - k^2 \sin^2\phi)}}\, d\phi = \tag{1}$$

$$\pi F(k', \theta) - 2F(k)\Upsilon(k', \theta) - \{E(k) - F(k)\}\{F(k', \theta)\}^2$$
$$- \tfrac{1}{2}\pi F(k') - F(k)\log(k \sin^2\theta)$$

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 - (1 - k'^2 \sin^2\theta)\sin^2\phi)}{\sqrt{(1 - k^2 \sin^2\phi)}}\, d\phi = \tag{2}$$

$$\pi F(k', \theta) - 2F(k)\Upsilon(k', \theta) - \{E(k) - F(k)\}\{F(k', \theta)\}^2$$

$$- \tfrac{1}{2}\pi F(k') + \log\left(\frac{k'}{k}\right) F(k).$$

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 - k^2 \sin^2\theta \sin^2\phi)}{\sqrt{(1 - k^2 \sin^2\phi)}}\, d\phi =$$

$$E(k)\{F(k, \theta)\}^2 - 2F(k)\Upsilon(k, \theta). \tag{3}$$

In equation (3) if we put $\theta = \frac{1}{2}\pi$, we will have, recollecting that

$$\Upsilon(\tfrac{1}{2}\pi) = \tfrac{1}{2}F(k)\,E(k) - \tfrac{1}{2}\log k',$$

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 - k^2 \sin^2\phi)}{\sqrt{(1 - k^2 \sin^2\phi)}}\, d\phi = \log(k')\,F(k). \tag{4}$$

Again, $\theta_{\prime}$ being the amplitude of the semi-complete function, we have

$$\sin^2\theta_{\prime} = \frac{1}{1 + k'}$$

and,

$$\Upsilon(\theta_{\prime}) = \tfrac{1}{8}F(k)\,E(k) - \tfrac{1}{4}\log\left(\frac{2k'\sqrt{k'}}{1 + k'}\right);$$

so that

$$\int_0^{\frac{1}{2}\pi} \frac{\log(\cos^2\phi + k'\sin^2\phi)}{\sqrt{(1 - k^2 \sin^2\phi)}}\, d\phi = \tfrac{1}{2}\log\left(\frac{2k'\sqrt{k'}}{1 + k}\right) F(k). \tag{5}$$

The values of the definite integrals (4) and (5) have been already deduced by Mr. Roberts from entirely different considerations, and published in Liouville's *Journal de Mathematiques*, May, 1846.

Some other interesting results may be obtained from our general formulæ. Thus, if in (2) we put $\theta = 0$, we will have

$$\int_0^{\frac{1}{2}\pi} \frac{\log(\cos\phi)\, d\phi}{\sqrt{(1-k^2\sin^2\phi)}} = \tfrac{1}{2}\log\left(\frac{k'}{k}\right) \mathrm{F}(k') - \tfrac{1}{4}\pi\mathrm{F}(k'), \qquad (6)$$

from which we may deduce, by an easy transformation,

$$\int_0^{\frac{1}{2}\pi} \frac{\log(\sin\phi)\, d\phi}{\sqrt{(1-k^2\sin^2\phi)}} = \tfrac{1}{2}\log\left(\frac{1}{k}\right) \mathrm{F}(k) - \tfrac{1}{4}\pi\mathrm{F}(k'); \qquad (7)$$

and, consequently,

$$\int_0^{\frac{1}{2}\pi} \frac{\log(\tan\phi)\, d\phi}{\sqrt{(1-k^2\sin^2\phi)}} = \tfrac{1}{2}\log\left(\frac{1}{k'}\right) \mathrm{F}(k). \qquad (8)$$

If we suppose k to vanish in formulæ (6) and (7), we obtain the well-known results, originally given by Euler,

$$\int_0^{\frac{1}{2}\pi} \log(\cos\phi)\, d\phi = \int_0^{\frac{1}{2}\pi} \log(\sin\phi)\, d\phi = \tfrac{1}{2}\pi \log \tfrac{1}{2}.$$

Denoting $\sqrt{(1-k^2\sin^2\phi)}$ by Δ, we can also derive from the above the value of the definite integral

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1 \pm \Delta\sin\theta)}{\Delta}\, d\phi. \qquad (9)$$

For, the sum of the integrals

$$\int_0^{\frac{1}{2}\pi} \frac{\log(1+\Delta\sin\theta)\, d\phi}{\Delta} \quad \text{and} \quad \int_0^{\frac{1}{2}\pi} \frac{\log(1-\Delta\sin\theta)}{\Delta}\, d\phi$$

may be found from (1), and their difference from the formula

$$\int_0^{\frac{1}{2}\pi} \log\left\{\frac{1+\Delta\sin\theta}{1-\Delta\sin\theta}\right\}\frac{d\phi}{\Delta} = \pi\mathrm{F}(k', \theta)$$

which Mr. Roberts has demonstrated in the *Journal de Mathematiques*, May, 1846.

In conclusion it may be observed, that the particular results, (4), (6), (7), (8), are nothing more than immediate consequences of Mr. Jacobi's factorial developments of the trigonometrical functions of the amplitude of an elliptic function, in terms of the function itself.—*Traité des Fonctions Elliptiques*, tom. iii. page 97. It may be seen that they follow at once from these expansions, if we remember that

$$\int_0^\pi \log\,(1 \pm 2a\cos x + a^2)\, dx = 0$$

when a is less than unity; a theorem proved by Poisson in the seventeenth *cahier* of the *Journal de l'Ecole Polytechnique.*

Sir William R. Hamilton stated the following theorems of central forces, which he had proved by his calculus of quaternions, but which, as he remarked, might be also deduced from principles more elementary.

If a body be attracted to a fixed point, with a force which varies directly as the distance from that point, and inversely as the cube of the distance from a fixed plane, the body will describe a conic section, of which the plane intersects the fixed plane in a straight line, which is the polar of the fixed point with respect to the conic section.

And in like manner, if a material point be obliged to remain upon the surface of a given sphere, and be acted on by a force, of which the tangential component is constantly directed (along the surface) towards a fixed point or pole upon that surface, and varies directly as the sine of the arcual distance from that pole, and inversely as the cube of the sine of the arcual distance from a fixed great circle; then the material point will describe a spherical conic, with respect to which the fixed great circle will be the polar of the fixed point.

Thus, a spherical conic would be described by a heavy point upon a sphere, if the vertical accelerating force were to vary inversely as the cube of the perpendicular and linear distance from a fixed plane passing through the centre.

The first theorem had been suggested to Sir W. Hamilton by a recently resumed study of a part of Sir Isaac Newton's Principia; and he had been encouraged to seek for the second theorem, by a recollection of a result respecting motion in a spherical conic, which was stated some years ago to the Academy by the Rev. C. Graves. In that result of Mr. Graves, the fixed pole was a focus of the conic, and the polar was therefore the director arc; consequently, the sine of the distance from the polar was proportional to the sine of the distance from the pole, and, instead of the law now mentioned to the Academy, there was the simpler law of proportionality to the inverse square of the sine of the distance from the fixed pole or focus.

Professor Graves observed, that he had that morning, in conversation with the President, stated the theorem just announced, respecting the motion of a material point on the surface of a sphere. Sir William Hamilton having, at the last meeting of the Academy, kindly communicated to him his theorem of plane central forces, it occurred to Professor Graves to inquire whether two theorems, which he had stated in January, 1842,* relating to the motion of a point in a spherical conic, might not be included in a more general one, analogous to that first mentioned by Sir William Hamilton. This inquiry led him to perceive the truth of Sir William Hamilton's second theorem.

The mode of proof employed by Professor Graves rests, so far as regards the dynamical part of the question, on the two following elementary propositions:

If a material point, P, constrained to move on the surface of a sphere, be urged by a force acting along a great circle passing through a fixed point, S;

* See Proceedings of the Academy, vol. ii. p. 209.

1. Its velocity will vary inversely as the sine of the perpendicular arc let fall from S on the great circle which is a tangent at P to the trajectory described by the point.

2. The force in the direction of the arc SP is equal to $\frac{v^2}{\tan\gamma \sin\theta}$, v being the velocity of the point, γ the radius of the osculating circle, and θ the angle between SP and the tangent arc.

The proposition may be readily proved by means of these principles, taken in conjunction with the following property of spherical conics:

A tangent arc being drawn at any point on a spherical conic, if a perpendicular be let fall upon it from a fixed point, and if a second perpendicular be let fall from the point of contact on the polar of the fixed point, the quotient of the sines of these two perpendiculars will always be proportional to the tangent of the normal arc at the point of contact.

This very general theorem is its own polar reciprocal.

Mr. J. J. A. Worsăae, of Copenhagen, being requested to give an account of the formation of the Museum of Antiquities in that city, made a communication to the following effect:

"It is a very well known fact, that but few countries in the north of Europe escaped invasion or conquest by the Romans. Among those few, however, Ireland and Denmark are specially to be named; and on that account it is certainly more than a mere accident that these two countries are in possession of some of the best collections of national antiquities in Europe. I have had the opportunity of repeatedly inspecting the very interesting collection of the Academy, and it has been told me, that the comparatively large number of Irish antiquities there assembled has been brought together in a short time, but under circumstances of considerable difficulty. Our collection of national antiquities in Denmark has likewise been founded under great disadvantages; and perhaps it will not be without interest to the Academy, if only in that respect, to get a short history of its foundation, progress, and present state.

"About forty years ago, the general character of scientific pursuits was, in our country, much the same as in most other parts of Europe: great pains were spent in collecting all sorts of objects illustrating the changes of the globe upon which we live, and the distribution and habits of animals and plants, in short, all the departments of natural history; whilst, strange to say, people for the most part neglected *traces of men,* the remains, not only of their own ancestors, but also of all the different races who have been spread over the world. The antiquities, with the exception of those of Roman and Greek origin, were regarded as mere curiosities, without any scientific value; and they were generally found in collections mixed up with petrifactions and other objects, with which they had little or no connexion. It was not until after the French Revolution, that the value of ethnology, as a most important branch of science, was seen in its proper light. With a greater respect for the political rights *of the people,* there awakened in the nations themselves a deeper interest in their own history, language, and nationality. Since that time there have been formed antiquarian societies, and collections of national antiquities, in most European countries; in Germany alone there exist at present more than eighty societies, formed for the preservation and collection of national antiquities, which, as I hope, is sufficient to show that an earnest effort is now being made to do what undoubtedly has been too long neglected.

"Denmark was one of the first countries in which a collection of national antiquities was founded, and no wonder, because the olden time was that in which Denmark, together with the two other Scandinavian countries, Norway and Sweden, was in its greatest power. I shall only recall to your memory, that the weapons of the Scandinavian warriors had at that time conquered the coasts of the Baltic, a great part of the British islands, of France, and some parts of Spain and Italy; that, crossing the Atlantic so early as in the ninth and tenth century, they colonized Iceland and Greenland, and put their foot upon the mainland of America. It was immediately after great national calamities, that the attention of the Danish people was turned to that early period of their history, as a time from the contemplation of which their spirit of nationality might gain support, and in whose memories they found the hope of a new and equally glorious era again. The North, too, has this great advantage, that a complete picture of the life of the old time has been preserved in the remarkable Icelandic sagas, which, certainly, compared with other literary remains of that time, in regard to style and representation of character, are almost unique. In the year 1807, the Danish government, in compliance with the request of several literary men, appointed a Royal Committee for the Preservation and Collection of National Antiquities, but the unfortunate war with England hindered the Committee, for the first seven or eight years, from making much progress. After the restoration of peace, it happened that a young man, a merchant's son in Copenhagen, who, from his earliest childhood, had felt a great interest in all sorts of antiquities, was appointed Secretary of this Royal Committee. He found a few antiquities, mixed up with the most curious things, in a small room in the library of the University. He commenced with exceedingly small grants, and under very great difficulties. He had not only to contend with the prejudices of the unlearned, but also with the conflicting opinions and baseless theories of the learned men. Some believed that the antiquities of iron were the oldest, because they were most corroded; others believed that the antiquities of brass were older than the antiquities of stone; others, again, supposed, that the wealthy men had used iron, the middle classes brass, and the poor stone. However, he opened his small collection for public inspection; was always present on the public days for the purpose of showing and explaining the antiquities; and when peasants happened to visit the collection, he paid particular attention to them, 'because,' as he said, 'it is by them we shall have our collection enlarged.' For many years he continued to show the collection, and to diffuse an interest in the old remains throughout the country, and all this without receiving any pecuniary emolument, I ought rather to say, at very considerable expense to himself. At last, the collection became so large, that the room in the library was far from furnishing sufficient accommodation; and the constantly increasing interest in the collection, and fresh donations of antiquities, made its removal necessary. After many difficulties, he made a great step in advance, by getting rooms in the royal palace, 'Christiansborg,' in Copenhagen. He then fully carried out his idea of arranging the Pagan antiquities into three periods, the *stone*, *brass*, and *iron periods*, which he was the first to point out to antiquaries. It was not long before the collection acquired a great name on the Continent; all foreigners spoke about it as one of the most remarkable collections in the north of Europe. The Government evinced more and more interest in the Museum, and the public began to regard it as a national treasure. In the mean time, the Royal Society of Northern Antiquaries in Copenhagen had published many of the remarkable Icelandic sagas, through which the people got more knowledge of the importance of the olden time, than they had hitherto possessed. The Society published in its Annals descriptions of the antiquities of the Museum, and published separately popular tracts, illustrated with woodcuts, on the value and importance of preserving the antiquities, many thousands copies of which were spread over the country, among clergymen, schoolmasters, and peasants. From all sides and all parts of the country antiquities were presented to the Museum; and it has now been enlarged to such an extent, that when the new arrangement, which is now going on, is finished, it will occupy about ten rooms of the royal palace. His Majesty the present King of Denmark, whose great zeal for the promotion of literature and science is well known, and His Royal Highness the Crown Prince, are both most anxious to make this collection still larger and more important. The real founder of the Museum, about whom I spoke above, the present Councillor of State, *C. J. Thomsen*, has had the gratification of seeing his extraordinarily energetic efforts crowned with the most signal success. In order to give some idea of the extent of the Museum, I shall only mention, that it contains more than three thousand specimens of implements of stone; a very large room is filled with antiquities of brass, among which are complete shields, and several large trumpets of war, between two and three hundred complete swords and daggers of brass, several hundred celts and brass hatchets, lance-heads, ornaments, &c. As many specimens as possible, even of the most common things, are collected, because true historical results can be deduced only from a long series, showing that the various articles were in common use. Among the antiquities of the bronze and iron periods are to be seen a great number of rings, and other ornaments of silver and gold, I should say a larger number than I have found in any other collection. It was formerly a law in our country, that all antiquities of silver and gold, which were found in the earth, must be surrendered to the Crown, without any recompense to the finder, the effect of which was, that most of those things were melted and made away with. The King, therefore, ordered, that the finders of antiquities of silver and gold should receive the full value of the articles, when they sent them into the Royal Collection; and that they should get more than the real value when the specimens were uncommonly rare, or when particular pains had been taken to find or preserve them. I am happy to say, that the Museum now gets very nearly all the antiquities of silver and gold which are found in our country, particularly as they are paid for by the Government out of a peculiar fund.

"I have thought that it would not be without interest to the Academy, to see how a large collection has been formed, in about thirty years, by energetic exertions, continued in spite of great difficulties; and how the collection, after those difficulties have been overcome, now stands as a national monument, supported alike by the Government and by the people. I doubt not that the Collection of this Academy, which in a few years has attained such magnitude, will, if carried on with the same energy, be soon of so much importance, and gain so great a name in Europe, that it will receive that strong support, both from the Government and the inhabitants of Ireland, which it at present wants.

"If you will allow me I shall, at another meeting, institute a short comparison between the antiquities in the Irish and Danish collections. It is only through such a comparison of the antiquities in different countries, that a new light will be thrown over the many dark periods of the early history of Europe; and I hope that the connexion, which in ancient times existed between Ireland and Scandinavia, will give me a peculiar advantage in illustrating the origin and use of some antiquities in the collection of the Academy."

Rev. Samuel Butcher read a paper by Rev. Dr. Hincks, in continuation of his researches in the Persepolitan writing.

In this paper Dr. Hincks shows, that the general principles respecting the Persian writing, which he had laid down in his former communication on this subject,* are borne out by the Bisitun inscriptions, recently published by Major Rawlinson. The values which, in his former paper, Dr. Hincks had assigned to four of the characters, he admits to be erroneous, and, accordingly, now corrects them; but maintains that the values assigned by him to the remaining characters are the true ones, and adduces the new inscriptions in proof thereof. With re-

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* "On the first and second kinds of Persepolitan writing," by the Rev. E. Hincks, D. D. (*Vid.* Proceedings, vol. iii. p. 262).

spect also to the author's Median Alphabet, as given in the same paper, he now makes a few slight corrections, which, however, do not affect the general views there stated regarding that language. Dr. Hincks further gives a Babylonian alphabet or syllabary, exhibiting the values of sixty-five characters of the third Persepolitan writing, and of one hundred and twenty-eight of the Babylonian lapidary characters: placing the Babylonian characters in juxtaposition with the corresponding Persepolitan ones. He adds an analysis of fourteen proper names written in the latter character, and of two in the former; and he points out the mode of reading them in the different forms under which they appear in the inscriptions. Dr. Hincks moreover states, that, with the exception of a few letters, to which correct values had been assigned by Professor Grotefend, and a few others to which the same author had approximated, nothing in the right direction had hitherto been published concerning these two last-mentioned kinds of writing.

The Rev. Charles Graves read a paper on the date of the manuscript commonly called the Book of Armagh.*

Shortly after the Book of Armagh had been deposited in the Museum of the Royal Irish Academy, Mr. Graves observed, on a careful examination, that numerous erasures had been made in it. These occur at the end of the following writings contained in the volume:

1. The Confession of St. Patrick, fol. 24, *b*.
2. The Gospel of St. Matthew, fol. 52, *b*.
3. The Gospel of St. Mark, fol. 67, *b*.
4. The Gospel of St. Luke, fol. 89, *b*.
5. The Revelation of St. John, fol. 170, *a*.
6. The Acts of the Apostles, fol. 190, *a*.
7. The second Book of the Life of St. Martin of Tours, fol. 214, *a*.
8. A letter of Sulpicius Severus, fol. 220, *a*.

* For an account of this manuscript see Transactions, vol. xx. p. 329.

So effectually had the original writing been effaced in these places, that, in the first instance, Mr. Graves gave up the attempt to decipher it as utterly hopeless. But his attention was again urgently drawn to the subject by Mr. Eugene Curry, who had independently noticed the same fact. Being aware that it was usual for Irish scribes to insert, at the end of books written by them, their own names, and some notices of the date or occasion of the writing, he had been looking at these very places in the hope of finding such entries, and, to his disappointment, he had ascertained that they had been erased. Still he did not despair of their being ultimately read: and as he thought it probable that, like the body of the work, they were written in Latin, a language with which he is not well acquainted, he requested Mr. Graves to endeavour to make them out. One of the erasures to which he particularly directed attention was the one marked 7 in the list given above, and to this Mr. Graves first applied himself. He reads it as follows:

Pro Ferdomnacho ores.

A well-executed fac simile is subjoined, for the purpose of enabling those who have access to the manuscript to judge whether his reading be correct.

On turning to erasures 3, 4, and 8, he satisfied himself that the same words had been written in those places also. It is thus established that the whole volume was executed by the same scribe, as, indeed, the uniformity of the handwriting sufficiently proves. Erasures 6 and 7 are considerable ones; and there is good reason to apprehend that, in both these instances, we have to deplore the loss of much information respecting the manuscript.

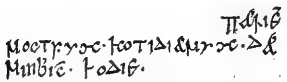
At all events, we know that it was written by a scribe named Ferdomnach. But it yet remains to be ascertained who this Ferdomnach was, and at what time he lived.

The Annals of the Four Masters contain entries respecting two persons of this name, both of them scribes.

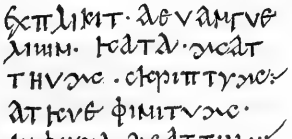
A. C. 726. Fſrdomnach sgriḃnſoir Arda Maċa d'ecc.	A. D. 727. Ferdomnach, Scribe of Armagh, died.
A. C. 844. Feardomnach eagnaiḋe ⁊ sgriḃniḋ toġaiḋe do ṁuintir Arda Maċa d'ecc.	A. D. 845. Ferdomnach, a sage and choice scribe of the church of Armagh, died.

The fact that both these persons were scribes of Armagh, where this manuscript was preserved for so many centuries, renders it in the highest degree probable that one or other was the writer. The names of between thirty and forty persons, who held the office of *Scriba* or *Scholasticus* in Armagh, are enumerated in the Annals of that see, given by Colgan in his *Trias Thaumaturga*. But of all these there were only two Ferdomnachs, the two already mentioned.

Assuming, then, as it seems safe to do, that one or other of these persons was the scribe of the manuscript, Mr. Graves proceeds to fix the actual year in which it was written. He thinks that he has effected this by partly deciphering the writing in the erasure No. 2. This erasure consists of four short lines; and the original writing was in a semi-Greek character, the nature of which is exhibited in the following passages, containing nearly all the letters of the Roman alphabet. The first is one of the petitions of the Lord's Prayer, as given in St. Matthew's Gospel, fol. 36, *a*. The second is a memorandum occurring in the very column at the foot of which is the erasure under consideration.



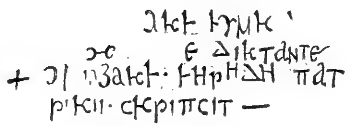
PANEM
NOSTRUM · COTIDIANUM · DA
NOBIS · HODIE ·



EXPLICIT · AEVANGVE
LION · KATA · MAT
TEVM · SCRIPTVM :
ATQVE FINITVM ·
IN FERIA · MATTEI · ·

After the latter passage comes a Collect appropriate to the Festival of St. Matthew, and then, at the bottom of the page, is the erasure.

By the use of a weak solution of gallic acid in spirits of wine, Mr. Graves revived the traces of the original writing a good deal; and, aided by a magnifying glass, he succeeded, at the expense of much time and labour, in deciphering the greater part of the erased writing. The following fac simile exhibits as much as can be read with any certainty:



Now, as the Heres Patricii undoubtedly meant the successor of St. Patrick in the see of Armagh, we at once gain this additional and positive information, that the scribe who wrote the book was contemporary with some Archbishop of Armagh whose name ended with *ach:* and this cannot be said of the earlier Ferdomnach, who died A. D. 727. It appears, from a passage in fol. 18, *b*, that Flann Febla had attained the primacy before this book was written, and he was succeeded by Suibne, who outlived this Ferdomnach. Nay, more, if we may trust the list of the Archbishops of Armagh contained in the *Leabhar Breac*, fol. 99, *b*, or that given by Colgan from the Psalter of Cashel, there had been no Archbishop of Armagh, whose name terminated thus, for more than a hundred years previous to the death of the first Ferdomnach. On the other hand, we know that, in the time of the second Ferdomnach, there were three Archbishops of Armagh whose names ended in *ach*, Foendelach, Connmach, and Torbach. But further, enough remains of the letter preceding the final *ach* to indicate that it was a *b*, certainly enough to show that it could not have been either an *l* or an *m*. Moreover, in the space occupied by the name, there is not room for more than seven or eight letters. On these grounds Mr. Graves concludes that the name was that of Torbach, whose death is thus recorded in the Annals of the Four Masters :

A. C. 807. Torbach mac Gormain Scribniḋ, Leġṫoir, ⁊ abb Arda Maċa eriḋe, do ċenel Torbaiġ, eaḋon, O Ceallaiġ Breaġ.	A. D. 808. Torbach, son of Gorman, Scribe, Lecturer, and Abbot of Armagh was he, of the Kinel Torbaigh, i. e. of Hy-Kelly of Bregia.

Introducing then the name of Torbach, Mr. Graves proposes to restore the whole passage thus:

F DOMNACH · HVNC · LIB
E RVM · E DICTANTE
R TORBACH · HEREDE · PAT
RICII · SCRIPSIT

Torbach held the primacy, according to the catalogues of the Psalter of Cashel and the *Leabhar Breac*, for a single year; and his death took place on the 16th of July ; "*colitur* 16° *Julii*," says Colgan, T. T. p. 294. Since, then, the writing of the Gospel of St. Matthew in the Book of Armagh was finished on St. Matthew's festival day, the 21st of September, and during Torbach's primacy, it must have been in the year 807.

If we could be quite sure that the half-erased name terminated in *bach*, there would remain no reasonable ground for doubting the conclusion at which Mr. Graves has arrived. For the satisfaction, however, of those who may not participate in the certainty which he feels as regards this point, he thinks it right to notice the following circumstances, which, although not deserving the name of proofs, tend in some degree to confirm the probability of his conjecture.

The Torbach abovementioned having been himself a scribe of Armagh, the copying of the precious manuscripts of the see was such a work as we might expect to find undertaken during his primacy: and of the second Ferdomnach we are informed, not only that he was a scribe of Armagh in Torbach's time, but that he was scriḃniḋ togaiḋe, a choice scribe, a fit person to be intrusted with so important a work. Certainly the penmanship of the Book of Armagh is of the most consummate excellence. The whole of the writing is remarkable for its distinctness and uniformity. All the letters are elegantly shaped, and many of the initials are executed with great artistic skill. The last verses of St. John's Gospel, fol. 103 *a*, may be especially referred to, as exhibiting a specimen of penmanship which no scrivener of the present day could attempt to rival.

It is also worthy of notice, that, about the time of Torbach's primacy, the inroads of the Danes in the north of Ireland, and the adjoining islands, were becoming so frequent and serious, that the ecclesiastics of Armagh might well have been anxious to take measures for the preservation of their records. In the year 802 the Scandinavian pirates plundered the monastery of Hy, on which occasion many of the inmates, both laymen and monks, perished. They again attacked it in 806, and put to death no less than sixty-eight of the monks. In 807 they effected a landing on the Irish coast, and, penetrating as far as Roscommon, destroyed it, and laid waste the surrounding country. But it was not till 831 that they entered Armagh. In that year, as we learn from the Annals of the Four Masters, they plundered it three times in the course of one month. It had never before been taken possession of by foreigners.

Mr. Graves stated that, on mentioning to his friend Mr. Petrie the fact of his having ascertained the name of the scribe of the Book of Armagh to be Ferdomnach, Mr. Petrie at once informed him, that he had, many years ago, made a drawing of a tombstone at Clonmacnoise, on which that name appeared. By his kind permission Mr. Graves is enabled to lay the following outline of it before the Academy:

The character of the inscription, and the style of the cross, belong, as Mr. Petrie thinks, to the ninth century. It is not unlikely that this may be the tombstone of the very person by whom the Book of Armagh was transcribed. His having been buried at Clonmacnoise rather than at Armagh, furnishes no argument to the contrary. We know that many distinguished ecclesiastics and learned men came from remote places to pass their last days as pilgrims at Clonmacnoise. It might be that Ferdomnach retired to that place when Armagh was plundered by the Danes in 831.

It is not a little remarkable, that the Book of Lecan, in the library of the Royal Irish Academy, furnishes us with the pedigree of a Ferdomnach, twenty-third in descent from Conary More, Monarch of Ireland, whose reign commenced A. D. 158. Allowing thirty years to a generation, we should bring the time of this Ferdomnach just down to the middle of the ninth century. For the discovery of this curious coincidence Mr. Graves is indebted to Mr. Eugene Curry, who, at his request, most kindly undertook the laborious task of making the necessary searches.

Sir William Betham, in his account of this manuscript,* has assigned to it an earlier date, assuming it to have been written by Aidus, Bishop of Sletty, who died A. D. 699. And in this he has been followed by Mr. Westwood, in his recently published *Palæographia Sacra.* Sir William Betham, wanting the positive evidences now brought forward, appears to have been led to that conclusion by a passage in the Life of St. Patrick, fol. 20, *b:* "*Hæc pauca de Sancti Patricii peritia et virtutibus* Muirchu Macc u Machthení *dictante Aiduo Slebtiensis civitatis episcopo conscripsit.*" But it would seem that these words were only intended to convey that the memoir of St. Patrick was originally drawn up at the desire or command of Aidus, just as the Gospel of St. Matthew, and probably the whole Book of Armagh, was transcribed by Ferdomnach *dictante herede Patricii*, at the bidding of the then Archbishop of Armagh.

The original Life of St. Patrick, by Muirchu, together with the annotations of Tirechan, were evidently becoming illegible at the time that Ferdomnach's copy of them was made. This is sufficiently indicated by notes in the margin, which show that the scribe found it difficult, in many places, to read the manuscript from which he was transcribing. Whatever abatement, therefore, has been made from the supposed age of the Book of Armagh, is fully compensated for by the knowledge that it is a copy from documents which were themselves old in the year 807.

* Irish Antiquarian Researches, vol. i. pp. 257, 270.

It is not easy to conjecture at what time the erasures now noticed were made in the manuscript. They seem not only to have concealed the name of the scribe from those scholars through whose hands the manuscript has passed at different times, but to have escaped their observation. At all events, they are not mentioned by those antiquaries who have hitherto published descriptions of the Book of Armagh. It is hardly possible to conceive how so intelligent a scholar as Lhwyd could have spoken as he does of the commonly received belief, that it was in the handwriting of St. Patrick, if the name of the real scribe, Ferdomnach, had appeared in eight or more places. And if he had not himself observed the signature of the real scribe, it could scarcely have passed unnoticed by Mr. Arthur Brownlow, who, on purchasing the book, after it had been left in pledge for £5 by Florentine Mac Moyre, carefully arranged and numbered the folios, and marked in the margin the beginnings of the chapters of the several books of the New Testament, a task, in the execution of which he must necessarily have examined every single page of the book. On these grounds Mr. Graves is inclined to believe, that the erasures were made before the manuscript came into the possession of Mr. Brownlow, that is to say, about the year 1680.

DONATIONS.

1. *Dictionary of the Roots of the Latin, according to the Method of A. F. Yazvenskago.* Presented by the Author.

2. *Memoirs of the American Academy of Arts and Sciences.* New Series. Vol. II. Presented by the Academy.

3. *Natuurkundige Verhandelingen van de Hollandsche Maatschappij der Wetenschappen te Haarlem.* 2e Verzameling. 3e Deel. 1e Stuk. Presented by the Society.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 58.

December 14th, 1846.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Rev. William Reeves, M. B., was elected a Member of the Academy.

The Rev. Charles Graves made some observations on the use of distributive signs of operation, both real and imaginary, in the construction of systems of algebra.

1. An algebra with two distributive signs, one real and the other imaginary, might be framed in the following manner:

We might define the sign $+$ as standing for a distributive operation, such that the repetition of it any integer number of times is still equivalent merely to $+$. We might suppose the nature of this operation known, and, on that account, call any quantity affected with the sign $+$ a *real* quantity. Along with this symbol we might employ a second, $+^{\frac{1}{2}}$, defining it as being the symbol of another distributive operation, such, that the twofold execution of it is equivalent to $+$. And the nature of this operation, denoted by $+^{\frac{1}{2}}$, might be supposed unknown, or in some sense transcendental as compared with $+$, so that it would be impossible to express a term affected with the sign $+^{\frac{1}{2}}$ by means of any combination of terms affected with the sign $+$. With this understanding we might call terms affected with $+^{\frac{1}{2}}$ *imaginary*, and the importance of the distinction between reals and imaginaries consists in this: that when we have an equation, $\mathrm{U} = 0$, involving quantities of both kinds, we may put the real and imaginary parts of U respectively equal to 0.

The following example will show the advantage of employing such a symbol. If we put $+^{\frac{1}{2}}\phi$ in place of $+\phi$ in the development of e^{ϕ}, we find

$$e^{+^{\frac{1}{2}}\phi} = \cos\phi +^{\frac{1}{2}} \sin\phi \tag{1}$$

where $\cos\phi$ and $\sin\phi$ respectively stand for the series

$$1 + \frac{\phi^2}{1.2} + \frac{\phi^4}{1.2.3.4} + \&c.$$

$$\phi + \frac{\phi^3}{1.2.3} + \frac{\phi^5}{1.2.3.4.5} + \&c.$$

multiplying equation (1) by the similar one

$$e^{+^{\frac{1}{2}}\phi'} = \cos\phi' +^{\frac{1}{2}} \sin\phi'$$

we have

$$e^{+^{\frac{1}{2}}(\phi+\phi')} = \cos\phi \cos\phi' + \sin\phi \sin\phi' +^{\frac{1}{2}} [\cos\phi \sin\phi' + \sin\phi \cos\phi'].$$

But again, from (1) we have

$$e^{+^{\frac{1}{2}}(\phi+\phi')} = \cos(\phi + \phi') +^{\frac{1}{2}} \sin(\phi + \phi').$$

And since we are entitled to compare the *real* and *imaginary* parts in the two last equations, we conclude that

$$\cos(\phi + \phi') = \cos\phi \cos\phi' + \sin\phi \sin\phi'$$

$$\sin(\phi + \phi') = \cos\phi \sin\phi' + \sin\phi \cos\phi'.$$

These are the fundamental equations for the comparison of hyperbolic sines and cosines.

2. The distributive symbols $+^{\frac{1}{4}}$, $+^{\frac{1}{2}}$, $+^{\frac{3}{4}}$, $+$, enable us to construct the ordinary double algebra. The equation $+^{\frac{1}{2}} 1 + 1 = 0$ makes $+^{\frac{1}{2}}$ *real* as well as $+$. But $+^{\frac{1}{4}}$ and $+^{\frac{3}{4}}$ are both *imaginary*, as compared with $+$ and $+^{\frac{1}{2}}$, though they admit of being compared together because of their relation

$$+^{\frac{1}{4}} 1 +^{\frac{3}{4}} 1 = 0,$$

which is a consequence of

$$+^{\frac{1}{2}} 1 + 1 = 0.$$

3. With the distributive symbols $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, supposed to be *heterogeneous*, we might construct an algebra of one real and two imaginary symbols. This algebra would be virtually equivalent to the triple system discussed by Mr. Graves in former communications to the Academy.

4. Starting with the primary symbol $+^{\frac{1}{9}}$ we might frame an algebra with two imaginaries, viz., $+^{\frac{1}{9}}$ and $+^{\frac{2}{9}}$, and three reals, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, related to one another by the condition

$$+^{\frac{1}{3}} 1 +^{\frac{2}{3}} 1 + 1 = 0.$$

Developing $e^{+^{\frac{1}{9}}\phi}$ we find it equal to

$$\lambda_{\phi} +^{\frac{1}{9}} \mu_{\phi} +^{\frac{2}{9}} \nu_{\phi},$$

λ_{ϕ}, μ_{ϕ}, and ν_{ϕ} representing series in which the signs of the terms are successively $+$, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, $+$, $+^{\frac{1}{3}}$, $+^{\frac{2}{3}}$, &c. Between these series and the ordinary trigonometric series, expressing the sine and cosine in terms of the arc, there exist many remarkable analogies.

Mr. J. J. A. Worsaae, of Copenhagen, in continuation of his former communication to the Academy, gave a review of the different descriptions of Danish and Irish antiquities, and of several historical events connected with the invasion of Ireland by the Danes.

Mr. Worsaae commenced by observing, that all the antiquities found in Ireland, as well as in other countries, are to be divided into two large classes. Those of the first are of the greater importance, being all of a time in reference to which we have no historical records. The monuments of the second class, belonging to a later period, could not give information of so much value, because we have from written records a certain degree of knowledge as to the civilization of the time ; but it is a remarkable fact, that the antiquities of the second class were, until lately, regarded with the greatest interest, because of the prevailing inclination to combine the study of antiquities with that of written records. It was long before archæologists could bring themselves to relinquish that mode of research, and come back to a critical examination of the monuments, without being influenced by written records ; but the time seems at length to have arrived, when it has become possible to enter upon an entirely new inquiry into the history of the earliest state of the European nations, by means of the antiquities alone.

§ 1. *The Stone Period.*

With regard to the existing collections of different kinds of stone implements, found in nearly all parts of Europe, it is interesting to compare those implements with the stone hatchets, knives, &c., found in America and Africa, and still used by the natives of the South Sea islands. Such a comparison indicates that they have been used by tribes which subsisted by fishing and hunting; and the striking resemblance of the forms is a direct proof that different people, in the same uncivilized state, use weapons and implements of exactly the same description for killing animals and building houses. It is well known that a great number of stone hatchets, arrow-heads, and lance-heads, have been found in Ireland, and particularly articles formed of flint, in the northern counties; but in Denmark there occurs a greater number and variety of implements of flint, probably because that stone is exceedingly common in that country. Most of these implements of stone seem to belong to a time before all history—to an aboriginal people, who lived by fishing and hunting upon the sea-coasts and along the large rivers of Europe. The tombs of these people give the best information on this point. There are in Ireland and England a number of stone structures called *Cromlechs, Druidical altars*, &c., which are generally regarded as religious monuments belonging to a historical time; but excavations in Ireland (in the Phœnix Park), in Jersey, Guernsey, and several other places, have shewn that they were tombs, containing skeletons, implements of stone or bone, vases of clay, and rude ornaments of amber and bone. Exactly similar monuments in Denmark and France contain, without exception, the same objects, and, like the Irish and English cromlechs, they differ from all other tombs. They are only found on the coasts of the Baltic and the German ocean, in Holland, France, Portugal, and on the coasts of the Mediterranean, but never in the interior parts of Germany, Austria, or Hungary. There is every reason to believe that those remarkable monuments were raised by a people who had no metal, and therefore were unable to penetrate into the interior of Europe, which was then covered with forests and morasses of an immense extent. It is only through a careful examination and comparison of the skeletons and skulls found in the tumuli just mentioned, that we can get information concerning the races to which this aboriginal people belonged. This discovery of a stone period, in the history of Europe, was the first important result arrived at by the study of antiquities alone.

§ 2. *The Bronze Period.*

Mr. Worsaae remarked further, that, in looking over the European collections of national antiquities, we next observe a great number of implements, weapons, and ornaments of bronze, together with ornaments of gold. It is impossible that all these could have belonged to the aboriginal people who built the cromlechs; first, because the tombs are quite different; and secondly, we do not find any sufficient transition from the antiquities of stone to the antiquities of bronze. The latter are of a mixed metal, and are so beautiful in their forms, and of such fine workmanship, that they must have belonged to a new people, who invaded Europe, followed the large rivers into the interior parts, where, with their implements of metal, they were now able to make their way through the thick forests. They began to drain the bogs and cultivate the soil, no longer living merely by fishing and hunting, as the people before them. There have been, however, different opinions about the origin of these implements and weapons of bronze. Some said they were all of Phœnician or Roman workmanship; some contended that they all belonged to an early Celtic population of Europe; but a comparison of the Irish and Danish antiquities will clearly shew that these opinions are not to be relied upon. There is certainly a general resemblance in the forms of the bronze antiquities, but many differences in details, which prove that the Irish antiquities of bronze were not brought from Ireland to Denmark, or from Denmark to Ireland. The handles of the Irish bronze swords were very nearly all of wood or horn: while in Denmark a great many had handles of bronze, ornamented with a peculiar sort of pattern, which in no instance appears on the Irish antiquities, and sometimes inlaid with gold. In Denmark there are several antiquities which are not to be found in Ireland; and in Ireland some have been found, which either did not exist at all in Denmark, or assumed another shape. The Danish antiquities of bronze are again different from the remains of the same period in the southern part of Germany, in Greece, in Italy, and France, which in their turn differ from the English and Irish antiquities; in all those countries moulds have also been found in which the implements and weapons of bronze were cast. This shews that different people in Europe, in the same state of civilization, had used the same implements of the same metal, only in slightly varying forms. The oldest accounts of Greek and Roman authors confirm this important result, which is principally due to the antiquities.

§ 3. *The Iron Period.*

It seems that the implements and weapons of bronze completely disappeared when the Romans had overrun the northern or north-western part of Europe; but Ireland and Denmark, the two countries which were never conquered by the Romans, continued longer to make use of them, and thus are peculiarly rich in antiquities of bronze, though poor in Roman remains. When the Roman empire fell, upon its ruins arose a new civilization, which commenced by imitating the Roman models; it is no wonder, therefore, that the same ornaments which appear on the oldest Irish ecclesiastical remains are also to be seen on remains from the iron period in Denmark, Sweden, Norway, France, Germany, &c. They were all barbarized imitations of Roman designs, and resembled one another, inasmuch as they were derived from a common archetype. It was only recently that Irish weapons belonging to the iron period were found, for the first time, in the course of excavations in the Shannon and at Dunshaughlin. In exhibiting some of these Mr. Worsaae remarked, that they were very small, compared with the large and heavy swords which were found, a short time ago, in cutting the railway at Kilmainham, and which, undoubtedly, were Danish, or rather Norwegian swords. The contrast between these Irish and Norse swords gives quite a picture of the time. It is a fact that not only Ireland, but other countries, England, France, Germany, &c., were, from about the eighth until the twelfth century, exceedingly weak, in consequence of internal wars; which accounts for the remarkable circumstance, that a small number of vikings from Scandinavia were able to take possession of large tracts of land in those remote countries, and keep them under their control for several centuries.

The swords which were found at Kilmainham are so like the Norse swords, that if they were mixed with the swords found in Norwegian, Swedish, and Danish tombs, and now in the collections of Christiana, of Stockholm, and of Copenhagen, it would be difficult to distinguish one from the other. The form of the handle, and particularly of the knob at the end of the handle, is quite characteristic of the Norse swords. Along with the swords found at Kilmainham, some other antiquities of undoubtedly Scandinavian origin were also discovered. Mr. Worsaae here exhibited a number of old draughtsmen of bone, of a hemispherical shape, and with a hole in the flat bottom, which were so constructed, that they could not tumble off the table if it was shaken. Great quantities of these draughtsmen are found in Norway, along with tesseræ, buried with the warrior in the grave. It might, perhaps, confirm what Tacitus said of the "Germanni," that they were exceedingly fond of gambling, so much so, that at last they staked their personal liberty, and thus sometimes became slaves. At Kilmainham were also found, besides the swords, large brooches of a peculiar sort, of a convex form, with a pin of iron, and ornamented with serpent-like devices; such brooches had never been found in other countries than in Scandinavia, or where Scandinavian people were settled; in Norway and Sweden they are most common. The existence of these brooches at Kilmainham along with the swords, would, therefore, furnish the strongest argument in favour of the Scandinavian origin of their buried owners. Similar brooches and weapons have been repeatedly found in the Phœnix Park and in College-green, memorials of the influence which the Norsemen had in Ireland, and particularly in Dublin; and it is a remarkable circumstance, that we should have in our hands, after so many centuries had elapsed, the swords by which a foreign people once ruled over Ireland, perhaps the very weapons by which Norsemen had shed Irish blood.

§ 4. *Early Civilization and Literature of the Northmen and Irish.*

It has often been said, and it is very well known, that the predatory attacks of the Danes or Norsemen on the English, Scotch, and Irish coasts, were attended with much bloodshed. There is scarcely a monument in the country which has not been attributed, by some at least, to the Danes; and Mr. Worsaae stated that he was well aware that in Ireland there still remains a traditional recollection of the plunderings of the Northmen; nor would he deny, what both the Irish annals and the Icelandic sagas often asserted, that the Northmen in their plunderings treated the Irish very ill, taking them prisoners, and carrying away both males and females from the country to be sold elsewhere as slaves. To illustrate these adventures he would endeavour to give, from memory, a story preserved in one of the Icelandic sagas. There was an Icelander of the name of Höskuld, who, at a fair in Halland (now a part of Sweden), bought Melkorka, the daughter of an Irish king, Myrkjartan, who had been made a prisoner in one of the Norse expeditions to Ireland. He took her to Iceland, where a son, Olaf, was the issue of their union. The Irish mother taught her son to speak Irish, and when he was grown up he fitted out a vessel, which was well manned, to go upon an expedition. The vessel was wrecked upon the coast of Ireland, and the crew were attacked and killed by the natives; but when Olaf spoke to them in their own language they spared his life; and, upon his telling them that his mother was the daughter of King Myrkjartan, they brought him to the king, who received him with the greatest kindness as his daughter's son. The king kept him at his court for some time, and when Olaf left, he gave him a new vessel. Afterwards, when Olaf returned to Iceland, and married there, he gave his son the name of *Kjartan*, after his grandfather. Several names in Iceland, as Niáll and Kjallak, are also said to have been brought over from Ireland.

The Irish annals contain many accounts of the Pagan Danes and Norwegians having burned or plundered churches and monasteries, and killed the monks, in Ireland. It has, therefore, been often said, that the Christian Irish were much more civilized than the Pagan Norsemen, and that before the invasion of the latter, a high state of civilization had prevailed in Ireland, which their barbarism interrupted and defaced. Mr. Worsaae quite agrees with his friend Mr. Petrie and the other Irish antiquaries, who say that many of the monuments in Ireland, such as the round towers, which had often been referred to as proving the civilization of the Danes or Norsemen, were in reality not of North origin at all. But he thinks, on the other hand, that some antiquaries have now and then detracted too much from the Northmen, making them out—according to the tradition of the country—to be only rude robbers and plunderers. Being convinced that these opinions were not founded upon historical truth, he would not omit this opportunity of trying to demonstrate their unsoundness.

It cannot be denied that Ireland was christianized at a very early period, and several centuries before Scandinavia. The Icelandic sagas show traces of that, and of the influence Ireland exercised in the christianizing of the North. When the Norsemen went first to Iceland in the latter half of the ninth century, they found no traces of inhabitants there, except of Irish monks, who had left croziers, bells, and Irish books. This account, in the sagas, of Irish monks in Iceland, is confirmed by the statement of an Irish monk, Dicuil, who wrote in the ninth century, and who mentions, that monks from Ireland had visited the Færö islands, before these were yet inhabited, and also Iceland, where the monks had stopped from the first of February until the first of August. (Dicuil's work was found in Paris, and published there in the years 1807 and 1814). The Norsemen called monks "*Papæ*," and thence they called the islands, where they found monks, "*Papey*" and "*Papeyar*." In the western isles of Scotland, in Orkney and Shetland, several islands bear that name; thus shewing, that monks had resided there at an early period. It is also curious to observe, that in Iceland there existed in the olden time a church which had been built in honour of St. Columba; and mention is made of an Icelander who was said to have been educated by an abbot named Patrick, in the western isles of Scotland or Ireland. The King of Norway, Olaf Tryggveson, who first tried to introduce Christianity there, was converted and baptized on an island near Ireland, or, as some sagas say, "west over in Ireland." Be that as it may, he was married to a sister of king Olaf Kvaran in Dublin, where he stopped for some time, and where, undoubtedly, the Christianity of Ireland must have influenced him. An Irish princess named Sunnifva is also said to have come to Norway, where she died, being persecuted by the Pagans. Her body, according to the legend, being afterwards found in a perfect state of preservation, she was canonized; and on the 8th of July the Norsemen afterwards had a mass in honour of her.

Such is the testimony borne by the Icelandic sagas to the fact of there having been an early Christian civilization in Ireland. But the question remains, whether this civilization was not limited, for the most part, to the *clergy*, and whether the mass of the people were not still very rude. From all the accounts in the Irish annals, it appears, as Mr. Worsaae thinks, that the people of Ireland, with the exception of the clergy, were at that time really not more civilized than the Northmen, perhaps even *less* so. It is true that the Northmen robbed and plundered, burned and killed; but the people of that remote period ought not to be judged according to the standard of modern usages and feelings. What is now regarded as a shame was then accounted an honour. The Northman who went upon an expedition in his vessel, acquired a large tract of land, or returned with gold and silver, after having killed many warriors in battle, was quite sure that the daughters of the highest nobles in the land would gladly give him their hand; but they never liked to marry a man who remained at home all his life. There is a curious story in one of the old Icelandic sagas which affords a very good idea of the state of society in the olden time. When King Olaf Tryggveson was stopping in England, after having been converted to Christianity, he bought a beautiful shield from a clergyman of the name Thangbrand. The clergyman, instead of purchasing holy books with the money, bought a handsome Irish girl, who some time before had been taken prisoner and made a slave. Not long afterwards the clergyman was obliged to leave England and proceed to Germany in company with a bishop Adelbert; nevertheless he carried the Irish girl along with him. In Germany, one of the followers of the Emperor becoming enamoured of her, attempted to take her from the priest, upon which the latter drew his sword and killed his rival. The priest, being expelled by the Emperor from his dominions, returned to England, where King Olaf Tryggveson received him with great kindness, and made him his chaplain.

When this was tolerated among the clergy, what was to be expected from the common people? It is true enough, that the Danes robbed and killed Irishmen, but they were, perhaps, not so much to blame as the native Irish, who constantly assailed each other, each tribe making incursions into the territory of its neighbour. If we take a view of the different countries of Europe, France, Germany, Spain, and Italy, we shall find that people of the same era called *Christian* were addicted to plunder and assassination as well as the *Pagan* Norsemen; nor were such practices considered in the least degree extraordinary. It would, however, be a great mistake to suppose that the Danes or Norsemen came to Ireland only to plunder and commit murder. The fact that the Norsemen in Ireland had their principal settlements in *towns*, as in Dublin, Limerick, Waterford, and Cork, is sufficient to shew that they must have carried on trade and commerce. The Icelandic sagas contain frequent accounts of men going, with their vessels, for trade to Dublin; and one saga adds, "as many now" (in the tenth century) "do." The Irish annals give some information relating to the Danish or Norse merchants in Dublin, where some families, supposed to be descendants of the old Norse merchants, still exist, and where a part of the town, called *Oxmantown*, or originally *Ostmantown* (in old documents, Villa Ostmannorum), to this day records the influence which they once possessed. The Norsemen were called by the Irish "Ostmen," and the Norsemen in return gave to the Irish the name of "Westmen." Some islands to the south of Iceland retain the name "Westmannaeyar," originally given because some Irish slaves, nearly a thousand years ago, were killed there.

It is often, though incorrectly, asserted, that "the Danes" were so completely defeated at the battle of Clontarf, that they never ventured to Ireland after that defeat. It is true that after that engagement the Danes, or rather Northmen, came less frequently to the shores of Ireland; but the reason was, not so much that they had been defeated at the battle of Clontarf, as that they became Christians, and their predatory excursions to the country became, on that account, less frequent. The battle was fought in the year 1014; at that time the Anglo-Danish king, Canute, succeeded his father, and completely introduced Christianity into Denmark. Norway also was Christianized about the same time. And it was the natural effect of Christianity to put an end to all single expeditions of vikings. In the year 1038, twenty-four years after the battle at Clontarf, the Ostmen appointed a bishop, Donatus, in Dublin, and Christ Church was founded by Sitric Mac Olaf, King of Dublin. The Ostmen continued to have bishops in Dublin until the English invasion; they had also bishops in Waterford and Limerick. The Bishops of Dublin and Waterford were, as an effect of the connexion between the Ostmen and their relatives in Kent, consecrated by the Archbishop of Canterbury; and that, perhaps, may have been a reason why the Archbishop of Armagh continued to hold the station of Lord Primate of all Ireland, rather than the Archbishop of Dublin, the metropolis of the country. There is no doubt but that the Norsemen possessed considerable influence in the towns above-mentioned, and in Cork, until the English invasion. In the year 1095, Godfrid Meranagh, King of the Ostmen, had ninety ships in the harbour of Dublin; and mention is made in the Irish annals of a meeting at Athboy of Tlactga, in the year 1167, when as many as 1000 of the Danes of Dublin were present. Both in Dublin and Cork they resisted the English; and in the year 1171, more than a century and a half after the battle of Clontarf, the Norse Prince of Dublin, Hasculf, who was expelled by the English Earl Strongbow, returned to Dublin with sixty ships, and tried to regain possession of the city. Even in the year 1263, the Irish applied to the Norse king, Hakon Hakonson, then on an expedition to the western islands of Scotland, to assist them against the English, which it is not probable they would have done, if there had not been remnants of the Ostmen in Ireland. Giraldus Cambrensis mentions a remarkable fact, that after the occupation of Dublin by the English, 400 of the Danes of Dublin were taken into the English army. One may well ask, how could the Ostmen have kept up their influence in the towns of Ireland after the expeditions from Scandinavia to Ireland had ceased, if they had not carried on trade and commerce, and been of great use to the Irish, whose daughters the Ostmen very often married?

Partiality might be imputed to Mr. Worsaae, because he was a Dane, but the ancient coins which have been discovered in this country fully bear him out in what he has stated. We do not find any particular Irish coins before the invasion of the Norsemen; but immediately after that event we see a great number of "Hiberno-Danish" coins, which were imitations of Saxon and other coins. A number of Arabian or Cufic coins have also been found in Ireland, a circumstance which could only be accounted for by the commerce with the Northmen. Arabian coins of the same kind have been found, with silver ornaments, in great abundance in Scandinavia, particularly in Sweden, on the island Gotland, upon the shores of the Baltic, and in Russia along the rivers as far as the Caspian sea, which shewed the course of trade with the East at that remote time. There is every reason to believe that the Norsemen, who were the only sailing people at that period, and who had connexions not only with Scandinavia, but with Russia, Scotland, England, Holland, Germany, and France, had highly improved the trade of Ireland. When we consider, moreover, that the Northmen had beautiful ornaments and well-made swords, inlaid with silver and gold,—that they could build vessels, in which, several centuries before Columbus, they sailed across the Atlantic,—we find strong arguments supporting the assertion, that the Northmen at that time were not so rude or so injurious to the civilization of Ireland as is generally supposed.

But what most sustains Mr. Worsaae's opinion is the *old Icelandic literature.* The Irish and Icelandic literature are so far alike, that they were written in the native tongue, the reason of which probably was, that Ireland and Iceland, in their remoteness, were unaffected by the many movements of Europe. But between them there was this great difference, that the Irish literature, and especially the historical part of it, was, for the most part, written *by the clergy;* but the Icelandic sagas or historical works are more characteristic monuments of the spirit of the people in the Pagan time. They preserve all the old re-collections which the clergy in other countries succeeded in destroying. The Irish annals, therefore, differ only in regard to language from the Latin annals of other countries; whilst the Icelandic sagas, like historical novels, are quite descriptive of the state, the manners, and the habits of the people at the periods to which they referred.

Mr. Worsaae here read a scene of the saga of King Harald Haardraade (cap. 99), which gives an idea of the origin of the sagas. It shewed that in Iceland there were men whose principal business it was to learn sagas and tell them to the people; such men were called *sagamen,* and they were very often bards (Skjalde). These bards accompanied kings on their expeditions, or lived at their courts, both in Scandinavia and in foreign countries. Several bards are known to have been in Ireland with the Norse Kings of Dublin. They had thus the best means of obtaining information respecting all events of importance. Both the bards, the vikings, and the merchants, after they had returned to their home, amused the people in the long winter evenings by describing the battles and other events they had been present at. This was an amusement in the house of the yeoman and in the hall of the king. The scene of the saga just quoted furnishes a proof that the sagamen and bards did not think it necessary to flatter the kings. On the other hand, the old Danish and Norse kings themselves were not very fond of flattery: the well-known story of the Anglo-Danish king, Canute, and the flatterers on the sea-shore, presents a striking example of this. We have thus an assurance of the general truth of the old sagas.

A few other extracts from the sagas, particularly illustrating the connexion between Ireland and Scandinavia, were read by Mr. Worsaae. The first was a dialogue between Eistein and Sigurd the Crusader, both Kings of Norway in the twelfth century, and sons of the Norse King Magnus Barfod or Barefoot (so called because he wore the Highland dress), who was killed in Ulster, A.D. 1103. Sigurd had been married, as the sagas relate, to Biadmynja, a daughter of Myrjartak, King of Connaught, or, as the Norsemen called it, "Kunnaktir." King Myrjartak was consequently an ally of King Magnus Barefoot on the occasion of his expedition to Ireland. But when Magnus was killed, Sigurd left his wife in Ireland, and afterwards sent an ambassador thither, requiring the Irish to pay him a large sum of money as a fine for the murder of his father, else he would again invade the country. It is said that the Irish, at a numerous meeting, when the ambassador delivered his message, resolved to pay the money.

The saga introduces the two kings comparing themselves (vid. *Laing's Translation of Snorre Sturleson's History of the Kings of Norway*, vol. iii. pag. 176, sq.), and gives in the two characters an admirable picture of the old and new time in Norway. Sigurd is the old viking, who cares only for warlike achievements, and viking expeditions, and looks with contempt upon the man who sits at home. Eistein has in the mean time been building churches, fishing villages, and lighthouses; he has made roads over the mountains, and restored peace in the country; and he says that the state of Norway has perhaps been more benefited by these domestic services than by his brother's victories over the infidels in the Holy Land.

The last fragment of the sagas read was a part of the history of an Irishman named Gillekrist or Harald Gille, who is said to have been a son of King Magnus Barefoot, and who, to satisfy King Sigurd the Crusader, proved his descent by undergoing the ordeal of treading over nine red-hot ploughshares; he afterwards became King of Norway, and left his sons as kings after him. The saga describes some quarrels between Harald and Magnus the son of King Sigurd, and gives a very spirited account of a race between Harald, who was on foot, and Magnus on horseback. The saga says: "Harald Gille was a tall, slender man, with a long neck and face, black eyes and dark hair, brisk and quick, and generally wore the Irish dress of short, light clothes. The Norse language was difficult for Harald, and he used expressions which many laughed at, but King Sigurd did not permit this when he was present." At the race Harald wore "an Irish hat," and he ran so swiftly, that Magnus was not able to follow him on his Gotland horse—"a beautiful animal, and very swift."—(Vid. *Laing's Translation of Snorre Sturleson's History of the Kings of Norway*, vol. iii. p. 193, sq.)

Mr. Worsaae pointed attention to the beautiful, simple, and lively descriptions in the sagas, which are full of the most entertaining accounts, representing the time, with all its favourable and unfavourable features. In the sagas one sees the family round the fire, the viking on board his vessel, the Parliaments at their meetings, the King and his retainers in his hall. When it is remembered that those sagas were written down immediately after the introduction of Christianity into Iceland in the twelfth and thirteenth centuries, so that the Christian element of civilization could have exercised but little influence upon them; and when those sagas are compared with the Irish historical works, what has been already said about the influence of the Northmen in Ireland being also borne in mind, Mr. Worsaae will, perhaps, be acquitted of the charge of partiality, when he contends, that the Pagan Northmen in the ninth, tenth, and eleventh centuries, were not less civilized than the Christian Irishmen, perhaps even a little more.

In conclusion, Mr. Worsaae made the following observations:

"I should be very sorry if any one should think that it was my intention, by these remarks on the Icelandic and Irish literature, to deny the worth of the latter; on the contrary, I have had the opportunity of seeing how many interesting and most important old manuscripts still exist in the collections of the Academy and Trinity College, and I have felt the greatest pleasure in observing with what energy the Irish Archæological Society has continued to print hitherto unpublished Irish manuscripts, which can scarcely be too highly valued. I must, at the same time, express my regret, that in Great Britain and Ireland public support is given in most instances to societies which publish their books *privately* for the use of their members, in consequence of which system the books are very little or not at all known on the Continent. I think it would be of great importance, if the old Irish literary remains could be more generally known in Europe than has hitherto been the case. Ireland has an immense advantage over Scotland, England, and all other countries, which are now partly, and were once completely inhabited by a Celtic people, in that it has preserved an entire literature, whilst other countries have preserved little or none at all. All nations of Celtic descent will, therefore, be obliged to turn their eyes to Ireland, when seeking information concerning the ancient manners and institutions of the genuine Celtic people. I have also learned, with the greatest surprise, that the most interesting and most important documents from the Celtic time—I should say the most remarkable documents in existence in Ireland—I mean, the old *Brehon laws*—have never been published. The Irish annals, and all the other existing Celtic remains, will scarcely throw so much light upon the real state of the institutions of Ireland and other Celtic countries, as those venerable laws; and I think now is the time to publish them, while the Celtic is still a spoken language, which it probably will not long continue to be, and while there are still men alive who are competent to do it. It is said that the publication of these laws will be a very expensive work: but if the British Government will not do for Ireland, what it has done for England, Scotland and Wales, by publishing their old institutions,—if Trinity College, in possession of which the manuscripts are, cannot do it,—then I hope that there are in Ireland men who will do honour to themselves and their native country, by publishing its most remarkable literary remains; and I hope that they will publish them in such a way, that literary men on the Continent also, will have the opportunity of becoming acquainted with them.

"I should be most happy, if my remarks could in any degree increase the interest which already exists in Ireland, for its remains, both literary and monumental; and if they also could shew how important it is that the antiquaries of the different countries should work together; that the Irish and Scandinavian, in particular, should unite their efforts more than they have hitherto done. I have tried to shew that our antiquities and sagas have an interest connected with Irish history. And I know now, better than before, how much light the Irish annals and other remains throw upon the invasions of the Danes and Norsemen in this country.

"The archæologists have not yet their general meetings like the naturalists. But I hope that the easy communication which now exists between Ireland and Denmark will soon bring an Irish antiquary over to us, who, addressing our Royal Society of Northern Antiquaries, may impart to us some portion of the rich store of information which not only we, but the whole of Europe, have every reason to expect from the unique Celtic National Museum, and the unique Celtic Literature of Ireland."

The President communicated to the Academy the result of the further researches of Dr. Hincks, in connexion with the third Persepolitan Writing, in which the Author has extended the number of ascertained characters from sixty-five to seventy-six.

Sir William R. Hamilton made a communication respecting a new mode of geometrically conceiving, and of expressing in symbolical language, the Newtonian law of attraction, and the mathematical problem of determining the orbits and perturbations of bodies which are governed in their motions by that law.

Whatever may be the complication of the accelerating forces which act on any moving body, regarded as a moving point, and, therefore, however complex may be its *orbit*, we may always imagine a succession of straight lines, or vectors, to be drawn from some one point, as from a common origin, in such a manner as to represent, by their directions and lengths, the varying directions and degrees (or quantities) of the velocity of the moving point: and the curve which is the locus of the ends of the straight lines so drawn may be called the *hodograph* of the body, or of its motion, by a combination of the two Greek words, ὁδός, a *way*, and γράφω, *to write* or *describe;* because the vector of this hodograph, which may also be said to be the *vector of velocity* of the body, and which is always parallel to the tangent at the corresponding point of the orbit, marks out or indicates at once the direction of the momentary path or way in which the body is moving, and the rapidity with which the body, at that moment, is moving in that path or way. This hodographic curve is even more immediately connected than the orbit, with the *forces* which act upon the body, or with the varying resultant of those forces, for the tangent to the hodograph is always parallel to the direction of this resultant; and if the element of the hodograph be divided by the element of the time, the quotient of this division represents (to the usual units) the intensity of the same resultant force; so that the whole accelerating force which acts on the body at any one instant is represented, both in direction and in magnitude, by the element of the hodograph, divided by the element of the time. We have also the general proportion, that the *force is to the velocity*, in any varied motion of a point, *as the element of the hodograph is to the corresponding element of the orbit.*

These general remarks respecting varied motion, under the influence of *any* accelerating forces whatever, having been premised, let it be now supposed that the force is constantly directed towards some one *fixed point* or *centre*, which it will then be natural to choose for the origin of the vectors of the hodograph. The straight lines drawn to the moving body from the centre of force being called, in like manner, the vectors of the orbit, or the *vectors of position* of the body, we see that each such vector of position is now parallel to the tangent of the hodograph drawn at the extremity of the vector of velocity, as the latter vector was seen to be parallel to the tangent of the orbit, drawn at the extremity of the vector of position; so that the two vectors, and the two tangents drawn at their extremities, enclose at each moment a *parallelogram*, of which it is easily seen that the *plane and area are constant*, although its position and its shape are generally variable from one moment to another, in the motion thus performed under the influence of a central force. If, therefore, this constant area be given, and if either the hodograph or the orbit be known, the other of these two curves can be deduced, by a simple and uniform process, on which account the two curves themselves may be called *reciprocal hodographs.*

The opposite angles of a parallelogram being equal, it is evident, that if the central force be attractive, any one vector of position is inclined to the next following element of the orbit, at the same angle as that at which the corresponding vector of velocity is inclined to the next preceding element of the hodograph. Also, if from either extremity of any small element of any curve, a chord of the circle which osculates to that curve along that element be drawn and bisected, the element subtends, at the middle point of this chord, an angle equal to the angle between the two tangents drawn at the two extremities of the element; that is, here, if the curve be the hodograph, to the angle between the two near vectors of position, which are parallel to the two extreme tangents of its element. We have, therefore, two small and similar triangles, from which results the following proportion, that the *half chord of curvature of the hodograph* (passing through, or tending towards the fixed centre of force) *is to the radius vector of the orbit* as the element of the hodograph is to the element of the orbit, that is, by what was lately seen, *as the force is to the velocity.**

But also, the radius of curvature of the hodograph is to the half chord of curvature of the same curve, as the radius vector of the orbit is to the perpendicular let fall from the fixed centre on the tangent to the same orbit; therefore, by compounding equal ratios, we obtain this other proportion: *the radius of curvature of the hodograph is to the radius vector of the orbit, as the rectangle under the same radius vector and the force is to the rectangle under the velocity and the perpendicular*, or to the constant parallelogram under the vectors of position and velocity. If, therefore, the law of the inverse square hold good, so that the second and third terms of this last proportion vary inversely as each other, while the fourth term remains unchanged, the first term must be also constant; that is, *with Newton's law of force* (supposed here to act towards a fixed centre), *the curvature of the hodograph is constant:* and, consequently, this curve, having been already seen to be *plane*, is now perceived to be a *circle*, of which the radius is equal to the attracting mass divided by the constant double areal velocity in the orbit. Reciprocally, we see, that *no other law* of force would conduct to the same result: so that the Newtonian law may be *characterized* as being the *Law of the Circular Hodograph.*

Another mode of arriving at the same simple but important result, is to observe, that because the radius of curvature of the hodograph is equal to the element of that curve, divided by the angle between the tangents at its extremities, or (in the case of a central force) by the angle between the two corresponding vectors of the orbit, which angle is equal to the

* By an exactly similar reasoning, the following known proportion may be proved anew, namely, that the force is to the velocity as that velocity is to the half chord of curvature of the orbit, whatever the law of central force may be.

double of the elementary area divided by the square of the distance (of the body from the centre of force), while the element of the hodograph has been seen to be equal to the force multiplied by the element of time, or multiplied by the same double element of orbital area, and divided by the constant of double areal velocity, therefore this radius of curvature of the hodograph must, for any central force, be equal to the force multiplied by the square of the distance, and divided by the double areal velocity.

The point on the hodograph which is the termination of any one vector of velocity may be called the *hodographic representative* of the moving body, and the foregoing principles show, that with a central force varying as the inverse square of the distance, this representative point describes, in any proposed interval of time, a *circular arc*, which contains the same number of degrees, minutes, and seconds, as the angle contemporaneously described round the centre of force by the body itself in its orbit, or by the revolving vector of position; because, whatever that angle may be, an equal angle is described in the same time by the revolving tangent to the hodograph. Thus, with the law of Newton, *the angular motion of a body in its orbit is exactly represented, with all its variations, by the circular motion on the hodograph;* and this remarkable result may be accepted, perhaps, as an additional motive for the use of the new term which it is here proposed to introduce.

Whatever the law of central force may be, if the square of the velocity in the orbit be subtracted from the double rectangle under the force and distance, which has been seen to be equal to the rectangle under the same velocity and the chord of curvature of the hodograph, the remainder is the rectangle under the segments into which that chord is cut by the centre of force, being positive when this section takes place internally, but negative when the section is external, that is, when the centre of force is outside the osculating circle of the hodograph. In the case of the law of the inverse square, this latter curve is its own osculating circle, and the rectangle under the segments of the chord is, therefore, constant, by an elementary theorem of geometry contained in the third book of Euclid; if, then, the square of the velocity be subtracted from the double of the attracting mass, divided by the distance of the body from the centre of force, at which that mass is conceived to be placed, the remainder is a constant quantity, which is positive if the orbit be a closed curve, that is, if the centre of force be situated in the *interior* of the circular hodograph.

In this case of a closed orbit, the positive constant, which is thus equal to the product of the segments of a hodographic chord, or the *constant product of any two opposite velocities* of the body, is easily seen, by the foregoing principles, to be equal to the *attracting mass divided by the semisum of the two corresponding distances of the body*, which semisum is, therefore, seen to be *constant*, and may be called (as in fact it is) the *mean distance*. The *law of living force*, involving this mean distance, may, therefore, be deduced as an elementary consequence of this mode of hodographic representation, for the case of a closed orbit; together with the corresponding forms of the law, involving a null or a negative constant, instead of the reciprocal of the mean distance, for the two cases of an orbit which is not closed, namely, when the centre of force is on, or is outside the circumference of the hodographic circle.

Whichever of these situations the centre of force may have, we may call the straight *line* drawn from it to the centre of the hodograph, the hodographic *vector of eccentricity;* and the *number* which expresses the ratio of the length of this vector to the radius of the hodograph will represent, if the orbit be closed, the ratio of the semidifference to the semisum of the two extreme distances of the body from the centre of force, and may be called generally the *numerical eccentricity* of the hodograph, or of the orbit (without violating the received meaning of the term).

Whatever the value of this numerical eccentricity may be, the constant area of the parallelogram under the vectors of position and velocity may always be treated as the sum or difference of two other parallelograms, of which one is equal to the rectangle under the constant radius of the hodographic circle and the varying radius vector of the orbit, while the other is equal to the parallelogram under the vectors of position and eccentricity; and hence it is not difficult to infer, that the length of the vector of position, or of the radius vector of the orbit, varies in a constant ratio, expressed by the numerical eccentricity, to the perpendicular let fall from its extremity, that is, from the position of the body, on a constant straight line or *directrix*, which is situated in the plane of the orbit, and is parallel to the hodographic vector of eccentricity. The *orbit*, therefore, whether it be closed or not, is always (with the law of the inverse square) a *conic section*, having the centre of force for a *focus*—a theorem which has, indeed, been known since the time of Newton, but has not, perhaps, been proved before from principles so very elementary.*

Conceive a diameter of the hodograph to be drawn in a direction perpendicular to the vector of excentricity; the extremities of this diameter correspond to the extremities of that chord of the orbit which is perpendicular to the shortest radius

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* The *hodograph of the earth's annual motion* may be considered to be exhibited to *observation* in astronomy, as the *curve of aberration of a star;* and it is known that this aberratic curve is a *circle,* notwithstanding the eccentricity of the earth's orbit; but the author is not aware that this circularity of the aberratic curve (for a star near the pole of the ecliptic) has ever been shewn before to be a *consequence of the law of the inverse square*, except by the help of the properties of the *elliptic orbit;* whereas the spirit of the present communication is to *derive that orbit from the circle*, and to regard that circle itself as a sort of geometrical *picture of Newton's law,* instead of being only one of many corollaries from the laws of Kepler.

vector, and which is called the *parameter ;* from which it follows, that the *semiparameter of the orbit is equal to the constant area of the parallelogram under distance and velocity, divided by the radius of the hodograph*, and, consequently, that it is equal to the *square of the constant double areal velocity, divided by the attracting mass.*

It is evident that these results agree with and illustrate those by which Newton shewed that Kepler's laws were mathematical consequences of his own great law of attraction. In applying them to the undisturbed motion of any binary system of bodies, attracting each other according to that law, we have only to substitute the sum of the two masses for the single attracting mass already considered, and to treat one of the two bodies as if it were the fixed origin of the vectors of a *relative hodograph*, which will still be circular as before. And even if we consider a ternary, or a *multiple system*, we may still regard *each* body as *tending*, by its attraction, to cause every *other* to describe an orbit of which the hodographic representative would be a perfect circle.

When there is *one predominant mass*, as in the case of the solar system, we may in general regard each other body of the system as moving in an orbit about it, which is, on the same plan, *represented* by a *varying circular hodograph*. For if, at any one moment, we know *the two heliocentric vectors* of position and velocity of a planet, we know the plane and area of the parallelogram under those two vectors, and can conceive a parallelepiped constructed, of which this momentary parallelogram shall be the base, while the volume of the solid shall represent the sum of the masses of the sun and planet; and then the height of the same solid will be equal to the radius of the momentary hodograph ; so that, in order to construct this hodograph, we shall only have to describe, in the plane, and with the radius determined as above, a circle which shall touch the side parallel to the heliocentric vector of position, at the extremity of the vector of velocity, and shall have its concavity, at the point of contact, turned towards the sun. The moon, or any other satellite, may also be regarded as describing, about its primary, an orbit, of which the hodographic representative shall still be a varying circle.

As formulæ which may assist in symbolically tracing out the consequences of this geometrical conception, Sir William Hamilton offers the following transformations of certain general equations, for the motion of a system of bodies attracting each other according to Newton's law, which he communicated to the Royal Irish Academy in July, 1845. (See Proceedings, vol. III, part 2, Appendix III. and V.)

The new forms of the equations are these:

$$\rho = \int \tau \, \mathrm{d}t; \quad \sigma = \frac{m'}{\mathrm{V}.(\rho' - \rho)(\tau' - \tau)}; \quad \tau = \Sigma \int \sigma \, \mathrm{d}\, \mathrm{U}(\rho' - \rho);$$

in which ρ and τ are the vectors of position and velocity of the mass m at the time t; ρ' and τ' the two corresponding vectors of another mass m' at the same time; σ is another vector, perpendicular to the plane, and equal in length to the radius of the *momentary relative hodograph, representing the momentary relative orbit*, which the attraction of the mass m' tends to cause the body m to describe; d, $\int$, Σ, are marks of differentiation, integration, and summation, and V, U, are the characteristics of the operations of taking respectively the vector and versor of a quaternion. Or, eliminating ρ and σ, but retaining the *hodographic vector* τ, and using Δ as the mark of differencing, the conditions of the question may be included in the following formula, which the author hopes on a future occasion to develope:

$$\tau = \Sigma \int \frac{(m + \Delta m)\, \mathrm{d}\, \mathrm{U}(\int \Delta \tau \, \mathrm{d}t)}{\mathrm{V}\,(\Delta \tau . \int \Delta \tau \, \mathrm{d}t)}.$$

Meanwhile it is conceived that any such attempt as the foregoing, to simplify or even to transform the important and difficult problem of investigating the mathematical consequences of the Newtonian law of attraction, is likely to be received at the present time with peculiar indulgence and interest, in consequence not only of the brilliant deductive discovery lately made of the new planet exterior to Uranus, but also of the extraordinary and exciting intelligence which has just arrived from Dorpat, of the presumed discovery, by Professor Mädler, of a *central cluster* (the Pleiades), and of a *central sun* (Alcinoe, called also Eta Tauri): around which cluster, and which sun or star, it is believed by Mädler that our own sun and all the other stars of our sidereal system, including the milky way, but exclusive of the more distant nebulæ, are moving in enormous orbits, under the combined influences of their own mutual attractions, all regulated by the same great law.

Sir William Hamilton exhibited Professor Mädler's work, *Die Centralsonne*, Dorpat, 1846, in which, as a first provisional attempt to determine the orbit of our own sun, with the help of the proper motions of a great number of stars, combined with Bessel's parallax of 61 Cygni, Mädler assigns to what he regards as the Central Sun, Alcinoe, a distance amounting to thirty-four million times the distance of our sun from us; concluding, also, but still only as first approximations, that the period of our sun's revolution is about eighteen millions of years, and that its orbit has now an inclination to the ecliptic of about 84 degrees, with an ascending node, of which the present longitude is nearly 237°.

A chart of observed places of Le Verrier's Planet was also exhibited by Sir William Hamilton; and was illustrated by comparison with Bremiker's Star-Map, which also was laid upon the table.

Mémoires présentés a l'Académie Impériale des Sciences de Saint Pétersbourg. Par divers Savans. Tom. 5me, Livs. 1–6; et Tome 6me, Liv. 1re.

Mémoires de l'Académie Impériale des Sciences de Saint Pétersbourg. 6me Serie. *Sciences Naturelles.* Tome 5me, Livs. 3me et 4me. *Sciences Mathématiques et Physiques.* Tome 4me, Liv. 2me. Presented by the Academy.

Introduction to Zoology, for the Use of Schools. Part I. By Robert Patterson, Esq. Presented by the Author.

Labour on Land, containing a plan for the Employment of the destitute Poor in Ireland. By Major S. W. Blackall. Presented by the Author.

Memorie della Società Italiana delle Scienze residente in Modena. Tomo XXIII.—Matematica. Presented by the Society.

An old Watch, found in the Bog of Allen, made in Paris by M. Gavdron. Presented to the Museum by John Lentaigne, M. D.

A Model of an ancient Frame or Vat, found in the Townland of Templemoyle, Parish of Banagher, County Derry, and also a number of the more portable parts of the structure discovered during the excavation.

A Model of an ancient Wooden Frame House, found in Drumkelin Bog, in the Parish of Inver, County Donegal, twenty-five feet below the surface. Presented to the Museum by Captain Larcom.

The Ordnance Survey of the County of Kerry, in 113 *sheets, including the Title and Index.* Presented by His Excellency the Lord Lieutenant.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 59.

January 11th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

William Brooke, Esq., Q.C.; Dominick Corrigan, M.D.; Leonard Dobbin, Esq.; Michael Donovan, Esq.; Charles Haliday, Esq.; William Neilson Hancock, Esq.; John Kells Ingram, Esq., F.T.C.D.; Charles P. Mac Donnell, Esq.; Right Hon. Louis Perrin; and Frederick John Sidney, Esq., were elected Members of the Academy.

On the recommendation of Council the following By-laws were adopted by the Academy:

"1. That the number of Honorary Members of the Academy shall be limited to sixty.

"2. That there shall be three Sections of Honorary Members, corresponding to the threefold objects of the Academy, and that the numbers in each Section shall be limited as follows:

"Section of Science, 30; of Literature, 15; of Antiquities, 15; = 60.

"3. That the number of Honorary Members in each Section, natives of Great Britain and Ireland, shall not exceed one-half of the total number in that Section.

"4. That the election of Honorary Members shall take place only at the Stated Meeting in November.

"5. That none shall be eligible as Honorary Members unless previously recommended by the Council, and that the choice of the Council shall be determined by Ballot.

"6. That the former By-laws of the Academy relating to Honorary Members be repealed, and the foregoing substituted in their place.

On the recommendation of Council, it was RESOLVED,—That the Committee of Antiquities be authorized to make a selection of specimens from the purchased articles in the Museum of the Academy, to be presented to the Royal Museum of Copenhagen, by the hands of Mr. Worsaae; and that this donation be accompanied by drawings of other antiquities.

The Rev. Charles Graves made a communication supplementary to his paper on the date of the Book of Armagh.

On the margin of fol. 64, b., and opposite to that part of the text which contains the twenty-first verse of the thirteenth chapter of St. Mark's Gospel, the proper name *Cellach* occurs, written in the peculiar Greek character of which examples have been already given.* As it was an ordinary thing for a scribe to make in the margin of a page a memorandum relating to some circumstance which took place at the time he was writing it, there is reason to suppose that the name *Cellach*, written here, was intended to record some event in which a person of that name bore a principal part. Unfortunately, as the name was a very common one amongst Irish ecclesiastics, we cannot, with any certainty, fix upon the in-

* Proceedings of the Academy, vol. iii. p. 318.

dividual here referred to, and so obtain grounds for determining the date of the manuscript *a priori*. On the other hand, if we are permitted to assume that it was written A. D. 807, good reasons for this conclusion having been already shewn, we are enabled to account in a satisfactory manner for the appearance of the name in question.

In the twenty-fourth chapter of Sir James Ware's *Antiquitates Hibernicæ*, where he gives an account of the acts of the Ostmen in Ireland, we find the following passage:

" Anno 807, Dani et Norwegi in Hiberniam appulerunt, et Roscomaniam, regionemque adjacentem ferro flammâque vastarunt. Eodem tempore Cellacus Abbas cænobii S. Columbæ Huensis, multis e suis, Norwegorum crudelitate, interfectis, in Hiberniam profugit, et Kenanusæ, alias Kenlisæ in Midia, monasterium in honorem S. Columbæ sive condidit, sive restauravit. Cum vero annos circiter septem ibi præfuisset Abbas, Dermitio quodam in dicto cænobio Abbate relicto, in Ionam sive insulam Huensem reversus est, ubi, post annum unum vel alterum mortem obiit."

Ware does not indicate the sources from whence he has derived this account. It is confirmed, however, as regards the date and the history of Cellach, by the Manuscript Annals in the Library of Trinity College, H. 1, 7, commonly called the Annals of Innisfallen; and mention is made in the Annals of Ulster, *ad. ann.* 806 (A. D. 807), of the building of the monastery at Kells.

To the history of this Abbot of Iona it is by no means improbable that allusion may have been intended by the scribe who wrote the name of *Cellach* in the margin: and perhaps he may have thought that our Lord's descriptive prediction of the miseries attending the destruction of Jerusalem (Mark xiii. 14–23) was not inapplicable to the sad visitation which had recently fallen upon Iona.

With respect to the practice of writing Latin in Greek characters, of which so many instances occur in the Book of

Armagh, it is to be observed that it does not necessarily indicate a greater antiquity than Mr. Graves has assigned to this manuscript. A decisive example of this kind occurs in the case of the great Bible of the Abbey of St. Germain des Prés, which, according to a note contained in it, was written in the eighth year of the reign of *Louis le Débonaire,* that is, A. D. 822. The scribe of that manuscript has inserted in it two Latin memoranda, which refer to the fact of its having been executed by him, employing in both a Greek character, similar to that used in the Book of Armagh.

His reason for doing so seems, in one case, to have been the desire to vindicate his right to the credit, of which some other scribe was depriving him. "*Obsecro te lector,*" he says, "*ne laborem manuum mearum despicias; sed quæso deprecor mellifluam charitatem tuam ut pro me Domini misericordiam exores. Evio idem fero laborem, alius tollit honorem.*" It must be inferred that the *alius* here spoken of was unacquainted with the Greek character, in which these words were written. In the other note the object of the scribe was plainly to exhibit his own skill and learning, and, at the same time, to test the intelligence of the reader. The passage, of which the first part is written in a strangely elongated cursive hand, and the last four words in Greek letters, runs as follows: "*Supplicamus omnibus in Christo fidelibus qui hunc libellum ad volvendum ad legendum accipitis meam ne reprehendito insipientiam;*" which is immediately followed by the pentameter,

"Me quicunque capit rusticitate caret,"*

written in the ordinary hand.

Nor was it only in the use of Greek characters that the scribes of those times displayed their pedantry. Sylvestre, in his *Paléographie Universelle,* vol. iv., describes a manuscript

* Fac-similes of these two passages are given in the *Nouveau Traité de Diplomatique* of the Benedictines, vol. iii., pp. 186–437.

of the ninth century, in the Royal Library at Munich, in which the Scribe has written the following sentence in Anglo-Saxon runes: "*Omnis labor finem habet, præmium ejus non habet finem. Madalfrid scripsit istam partem. Deo gratias quod ego perfeci opus meum.*" Thus recording his name, which he may not have been allowed to do without resorting to this artifice, and at the same time giving a proof of his learning.

Mr. Graves insists much upon the importance of determining, with precision, the date of a manuscript so ancient, and of so much interest, as the Book of Armagh. By effecting this, a great advance is made towards the establishment of principles of palæography, by which we may estimate the age of Irish manuscripts in general; and we are furnished with the means of refuting the assertion, still repeated, that Ireland has no manuscripts of a date more ancient than the close of the ninth century.*

The Secretary of Council read the abstract of a paper by the Rev. Dr. Hincks, on the third Persepolitan Writing, and on the Mode of expressing Numerals in Cuneatic Characters.

"When I laid before the Academy, at its last sitting, my alphabet of the third Persepolitan writing, with the corresponding lapidary characters, I by no means expected that it would prove perfectly correct; no first attempt at the alphabet of an unknown language has been so. I considered it, however, an approximation, and probably as near a one as could be attained by means of the data in my possession; and I looked forward to its being amended by those who had the command of more numerous inscriptions. There were some circumstances which left no doubt on my mind that error existed somewhere in it, though I could not discover where. The number of dentals was too small; there was no character

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* See Moore's History of Ireland, vol. i. p. 310.

for the word corresponding to the compound epithet, *wysadahyôsh,* in Du, were only *in part* legible; and the manner of writing the name *Ormusd* in the inscription H, and that of Artaxerxes on the vase at Venice, could only be explained by supposing the sculptors to have committed errors. All these for *tu* or *du;* the name in NRu, answering to *Harautish,* and difficulties, and others connected with the first inscription of the East India Company, have been removed by an important rectification, or series of rectifications, which I have made during the past fortnight; and the language has, moreover, been brought to exhibit a much greater similarity to the other Semitic ones than I had at first supposed. I have, therefore, to request leave to substitute the alphabet which I now send for that in my last paper. As the correspondence between the cursive and lapidary characters in the plate to that paper is correctly given, though the values of many of the characters are erroneous, and as the plate is, I believe, partly engraved, I propose to let it stand, with so much of the paper as is necessary for understanding it; but the transcriptions of Babylonian words into Roman characters, and the catalogue of Babylonian words, will be superseded by those which follow, which are much more correct. In the plate which I now send I give no lapidary characters, but instead thereof I give many additional Persepolitan ones; and at the foot of it I give a series of numbers from the rock inscription at Van, exhibiting the mode of expressing numbers in Cuneatic characters on to 100,000. These are so arranged as to require no comment; but it may be proper to state that the large numbers are those of men belonging to different nations which are named; and I presume they refer to the deportation of these nations, according to the Assyrian practice. The historic character of these inscriptions, of which I received a copy very recently, is obvious."

The President made a few remarks upon the present state of the researches connected with Persepolitan writing, and upon the position occupied by Dr. Hincks in these investigations.

The President presented to the Academy a set of coloured drawings of celts and other antiquities, found in Cornwall, which he had procured through the kindness of a friend residing in that country. It appeared from a comparison of them with the corresponding objects in the Museum of the Academy, that there was an exact resemblance of form. The Cornish antiquities of this kind are, however, comparatively rare.

DONATIONS.

Runamo og Braavalleslaget, med fem lithographerede Tavler, af J. J. A. Worsaae. Presented by the Author.

Tillæg til "Runamo og Braavalleslaget." 1845.

Giornale dell' I. R. Istituto Lombardo di Scienze, Lettere ed Arti. Tom. IV. *e* V. 1844 and 1845.

Memorie dell' I. R. Istituto Lombardo di Scienze, Lettere ed Arti. Vol. 2do. 1845.

Elogio di Bonaventura Cavalieri da Gabrio Piola. Presented by the Institute.

Philosophical Transactions of the Royal Society of London for 1846. Parts 1, 2, and 3.

Proceedings of the Royal Society. 1845. Nos. 62–65. Presented by the Society.

The Transactions of the Microscopical Society of London. Vol. I., Parts 1 and 2 ; Vol. II., Part 1. Presented by the Society.

Journal of the Asiatic Society of Bengal. Nos. 166, 167. 1845. Presented by the Society.

Bulletin des Séances de la Société Vaudoise des Sciences Naturelles. No. 12. 1846. Presented by the Society.

Oversigt over det Kongelige danske Videnskabernes Selskabs

Forhandlinger og dets Medlemmers Arbeider i Aaret 1842. Presented by the Society.

Quarterly Journal of the Geological Society of London. No. 8. Nov. 1, 1846. Presented by the Society.

Twenty-five School-books, published by direction of the Commissioners of National Education in Ireland. Presented by the Commissioners.

Proceedings of the Archæological Institute, Winchester. Presented by the Institute.

Archæological Journal. Vols. I. and II. Presented by the British Archæological Association.

Abhandlungen der Fürstlich Jablonowskischen Gesellschaft. Presented by the Society.

Det Kongelige Danske Videnskabernes Selskabs. Naturvidenskabelige og Mathematiske Afhandlinger. Niende deel. Presented by the Society.

Magnetical, Meteorological, and Astronomical Observations made at Washington. By Lieutenant J. M. Gilliss, U. S. Navy. Presented by the Author.

An Explanation of the Observed Irregularities of the Motion of Uranus on the Hypothesis of Disturbances caused by a more distant Planet. By J. C. Adams, Esq. Presented by the Author.

A long Sword, with four Guards, found in Malta; and a brass Puzzle Box, found in Ireland. Presented to the Museum by Major-General R. H. Birch, Royal Artillery.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 60.

January 25th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Secretary of Council read a letter from Edward J. Cooper, Esq., containing some recent observations of Le Verrier's planet, made with the meridian circle at Markree.

"*Merville, Stillorgan,*
"*Dec.* 28*th*, 1846.

"On the 9th of November my observations on the Le Verrier planet, made during the month of October, were presented to the Academy; I have now the pleasure to subjoin those which have been since obtained with the Markree meridian circle. My first Assistant, Mr. Graham, has annexed, in Table II., corrections of the British Association Catalogue places of those stars selected for comparison with the planet; and in Table III. corrections of the right ascensions and declinations of the planet as given in my former comunication, and in Table I. of the present.

"EDWARD J. COOPER."

Table I.

OBSERVATIONS OF LE VERRIER'S PLANET,

Taken with the Meridian Circle, at Markree Observatory, County Sligo.

Mean Time, Greenwich.	Apparent Right Ascen.	Apparent Declination. South.	Instrumental Correction from	Observer.	Remarks.
1846.	h. m. s.	° ′ ″			
Nov. 2.318324	21 51 20.50	13 34 9.6	1, 2, 3, 4, 5, 6, 7, 8, 9, 10.	E. J. C.	Stars blotty and unsteady. Planet better shewn.
4.312855	19.76	15.0	do.	do.	Good observation.
9.299204	19.87	12.3	do.	do.	Planet faint.
10.295787	20.43	6.3	do.	do.	
11.293756	21.04	7.0	do.	do.	Fixed stars much more unsteady than planet.
17.277445	27.17	33 29.8	5.	do.	Taken through auroral clouds.
24.258482	40.19	32 17.8	1, 2, 3, 4, 5, 6, 7, 8, 9, 10.	do.	Satisfactory observation of planet.
28.247681	50.73	31 20.7	6, 8, 9, 10.	do.	
Dec. 3.234210	52 6.37	29 57.7	1, 2, 3, 4, 5, 6, 7, 10.	do.	
4.231668	9.74	37.1	4, 5, 6, 7, 9, 10.	A. G.	Only four wires of planet. Observation is uncertain.
14.204703	52.14	25 54.6	1, 4, 5, 6, 7, 8.	do.	Planet and wires well seen by daylight.
15.202030	56.98	26.5	1, 4, 5, α.	do.	Do. do.
17.196684	53 7.02	24 36.0	4, 5, α, 6, 7, 8, 9, 10.	do.	Very faint. Daylight too strong.

ADDITIONAL NOTES.

Nov. 17th. The declination 13° 33′ 34″.7, deduced from the instrumental corrections of November 11th and 18th, is safer.

,, 24th. There were only three wires of 50 Aquarii obtained, in consequence of clouds; but these were well taken.

,, 28th. The stars 1, 2, 3, 4, 5 were observed by A. G. The comparison of the result from these with that from 6, 8, 9, 10, observed by E. J. C., tends to confirm the notion of a large personal equation. They give E. J. C.'s clock correction = A. G.'s —0s.57.

Dec. 4th. The planet was faintly seen through clouds. The stars were observed through openings; though all complete sets. Sky was totally clouded before 42 Capricorni, and after the last star.

The additional star of comparison, noted as "α", is μ Capricorni. The apparent places were deduced, as before, from the British Association Catalogue by Mr. Graham.

Table II.

CORRECTIONS OF BRITISH ASSOCIATION CATALOGUE PLACES OF STARS.

	Star.	In Right Ascension.	No. of Obser-vations.	In Declina-tion.	No. of Obser-vations.
1	*i* Capricorni, .	— 0″.034	15	— 1″.65	14
2	7451 B. A. C.	— 0.355	12	— 0.43	12
3	7487	— 0.021	12	— 0.21	12
4	42 Capricorni,	+ 0.161	17	— 0.41	17
5	δ	+ 0.074	17	— 0·69	17
a	μ	— 0.026	2	— 1.11	2
6	*i* Aquarii, . . .	— 0.007	17	+ 1.12	17
7	39	+ 0.030	15	+ 5.57	15
8	45	— 0.072	14	— 2.75	14
9	50	+ 0.102	13	+ 0.87	13
10	σ	+ 0.035	13	— 1.66	13

These corrections were obtained by subtracting each result from the mean of all the results for each night, and then by combining the remainders according to the weights of the determinations. They are to be applied, with the proper sign, to the Right Ascension and Declination of the Planet; the South Declination being, as usual, regarded as negative. On the night of November 9th, a mistake was made in reading the level attached to the declination circle in the instance of *i* Capricorni. In reducing the observations, what was, very probably, the reading was used instead of that given in the observing book. An anomaly still, however, appearing in the result; it has been thought better wholly to reject the observation of declination of this star for that night. The correction of declination for November 9th will consequently be $+ 0''.9$, instead of what would be obtained from the Table given above; or November, 9th $\delta = - 13° 34' 11''.4$.

Since the corrections given in the Table are *mutual*, any more accurate determination which may hereafter be obtained for one star will be applicable to all. —Andrew Graham.

Table III.

CORRECTIONS OF RIGHT ASCENSION AND DECLINATION IN TABLE I.

The Declination is regarded as *minus*.

	Oct. 3rd.	6th.	12th.	14th.	16th.	17th.	19th.	24th.	26th.	Nov. 2nd.	4th.
Right Ascension, . . .	s. + 0.03	s. + 0.03	s. + 0.03	s. — 0.03	s. + 0.03	s. — 0.02	s. + 0.04	s. — 0.01	s. + 0.03	s. + 0.01	s. + 0.01
Declination, ...	+ 5″.6	+ 5″.6	+ 5″.6	— 0″.4	+ 0″.1	+ 0″.1	+ 0″.6	+ 0″.2	+ 0″.8	0″.0	0″.0
	Nov. 9th.	**10th.**	**11th.**	**17th.**	**24th.**	**28th.**	**Dec. 3rd.**	**4th.**	**14th.**	**15th.**	**17th.**
Right Ascension, . . .	s. + 0.01	s. + 0.01	s. + 0.01	s. + 0.07	s. + 0.01	s. + 0.01	s. + 0.01	s. + 0.09	s. + 0.03	s. + 0.04	s. + 0.06
Declination, ...	+ 0″.9	0″.0	0″.0	— 0″.7	0″.0	— 0″.6	+ 0″.2	+ 0″.8	+ 0″.2	— 1″.0	+ 0″.1

Andrew Graham.

The following note by Professor Mac Cullagh, on the attraction of ellipsoids, was read.

"The object of the present note is to show how the final integrations by which the attraction of a homogeneous ellipsoid is found, when the force varies inversely as the square of the distance, may be performed geometrically; and thus to complete the synthetic solution of a celebrated problem. It has been always supposed that the utmost geometry can do is to arrive in a simple way at the differential expressions on which the attraction depends, leaving the further treatment of the question to the integral calculus; but we shall see that, by putting the differential of the attraction under a certain form, the integral is at once obtained, and that in a very elegant shape, by geometry.

"To avoid useless generality, we shall suppose the attracted point to be at the extremity of an axis of the ellipsoid, as it is well known that the solution of this particular case enables us find the attraction wherever the point is placed. Let O be the centre of the ellipsoid, and A, B, C the extremities of its semi-axes, the lengths of which are denoted by a, b, c respectively, a being the greatest, and c the least. And first, suppose the attracted point to be at C, the extremity of the least semiaxis. Let two right lines passing through C, and making respectively the angles ϕ and $\phi + d\phi$ with OC, revolve within the ellipsoid, describing two right cones, of which OC is the common axis, and which include between their surfaces a differential portion dM of the volume of the ellipsoid. The attraction of the matter contained in dM is evidently in the direction of OC.

"Now let us consider the focal ellipse having its centre at O, and lying in the principal plane which is at right angles to OC. Let E be the extremity of the major axis of this ellipse; and putting

$$\rho = \sqrt{c^2 + (b^2 - c^2)\cos^2\phi}, \quad \rho' = \sqrt{c^2 + (a^2 - c^2)\cos^2\phi},$$

take in OE produced a point P such that OP shall be to OE as ρ to c. Then, if a right line drawn from P touch the focal ellipse in the point T, and if the angle OPT be denoted by θ, it will be found that

$$\cos\theta = \frac{\sqrt{a^2 - c^2}}{\rho'} \cos\phi.$$

Suppose that the point P moves to p, when ϕ is changed into $\phi + d\phi$. Then it may be easily shown that the attraction of dM upon the point C is proportional to the interval Pp multiplied by the cosine of θ. In this form the differential of the attraction is immediately integrable. For if from p we draw a right line pt touching the ellipse in t, and if s denote the difference between the tangent PT and the elliptic arc ET, while $s + ds$ denotes the difference between the tangent pt and the arc Et, it will appear, by a lemma which I have frequently had occasion to use (see the Proceedings of the Academy, vol. ii. p. 508), that ds is equal to Pp multiplied by $\cos\theta$. The integral, beginning when $\phi = 90°$, or $\rho = c$, is therefore proportional to s. At the other limit we have $\phi = 0$, and $\rho = b$, which determines the extreme position of the point T. The difference between the tangent PT and the elliptic arc ET, corresponding to this position, is to be multiplied by a certain function of the semiaxes, in order to get the whole attraction of the ellipsoid on the point C.

"When the attracted point is at B, the extremity of the mean axis, we proceed exactly as before; but instead of the focal ellipse we make use of the focal hyperbola, whose plane is at right angles to OB. Putting now

$$\rho = \sqrt{b^2 - (b^2 - c^2)\cos^2\phi}, \qquad \rho' = \sqrt{b^2 + (a^2 - b^2)\cos^2\phi},$$

and calling E the extremity of the primary axis of the hyperbola, we take in OE a point P (which will lie between O and E) such that OP shall be to OE as ρ to b. Then, drawing from P the right line PT, touching the hyperbola in T, and denoting the angle EPT by θ, we have

$$\cos\theta = \frac{\sqrt{a^2 - b^2}}{\rho'} \cos\phi;$$

whence it may be shown that if p be the point to which P moves when ϕ becomes $\phi + d\phi$, the interval Pp multiplied by $\cos\theta$ will be proportional to the attraction of the matter dM contained between the surfaces of two right cones having B for their vertex and OB for their common axis, provided ϕ and $\phi + d\phi$ be the angles which the sides of these cones make with OB. The whole attraction is therefore proportional to the difference between the tangent PT and the hyperbolic arc ET, the position of T being that which corresponds to the supposition $\phi = 0$, or $\rho = c$.

"When the attracted point is at the extremity of the greatest axis of the ellipsoid, we cannot employ a similar method, because there is no focal curve perpendicular to that axis. But if A, B, C be the attractions at A, B, C respectively, we have the known relation

$$\frac{\text{A}}{a} + \frac{\text{B}}{b} + \frac{\text{C}}{c} = 4\pi,$$

of which a geometrical proof will be found in the Proceedings, vol. ii. p. 525; and from this relation we can find A in terms of B and C.

"The preceding method of treating the question of the attraction of ellipsoids was given at my lectures in Trinity College, in the beginning of last year. I have since observed that the same results may be obtained, and perhaps more readily, by dividing the ellipsoid into concentric and similar shells. For the attraction of dM is equal, in each case, to the attraction of the shell bounded by the surfaces of two ellipsoids whose semiaxes are $a\cos\phi$, $b\cos\phi$, $c\cos\phi$, and $a\cos(\phi + d\phi)$, $b\cos(\phi + d\phi)$, $c\cos(\phi + d\phi)$, these ellipsoids having O for their centre, and OA, OB, OC for the directions of their semiaxes. And the attraction of such a shell on an external point may be simply expressed by means of the semiaxes of a confocal ellipsoid passing through the point. (See a Memoir by M. Chasles in Liouville's Journal, vol. v.) The quantities which we have called ρ and ρ' are, in fact, semiaxes of an ellipsoid described through the attracted point (that is, through C in the first case, and through B in the second) so as to be confocal to the surface of which the semiaxes are $a\cos\phi$, $b\cos\phi$, $c\cos\phi$."

A note by Professor Mac Cullagh, on the rotation of a solid body, was read.

Let a solid body be made to revolve round a fixed point O, and be afterwards free from any external forces; and through O conceive a right line OI to be drawn perpendicular to the invariable plane (the plane passing through O and the direction of the primitive impulse). If O be the centre of an ellipsoid whose semiaxes are in the directions of the principal axes belonging to that point, and of such lengths that the square of each semiaxis is equal to the corresponding moment of inertia divided by the mass of the body, the motion will take place in such a way that the point I, in which the right line OI intersects the surface of the ellipsoid, will be fixed in space; and therefore OI will describe within the body a cone of the second order, condirective with the ellipsoid (that is, having its circular sections parallel to those of the ellipsoid), while the point I describes on the surface of the ellipsoid a certain spherical conic. In a former number of the Proceedings (vol. ii. p. 542), the author had alluded to a theorem for determining the time at which the point I occupies any given position on the spherical conic, and he now gave a particular statement of it as follows:

Conceiving a plane of circular section of the ellipsoid to be drawn through its mean axis, and the spherical conic to be projected on this plane, first by right lines parallel to the greatest axis of the ellipsoid, and next by right lines parallel to its least axis, each projection of the conic will be a circle having its centre at O. The two projections P and Q of the same point I will always be in a right line perpendicular to the mean axis; let this right line cut the mean axis in M. Then, while the point I describes the spherical conic, the points P and Q will move in their respective circles, in such a way, that the velocity of P will vary as the ordinate MQ, and the velocity of Q will vary as the ordinate MP.

From this theorem we immediately obtain the elliptic integral which represents the time. For, supposing the greater circle to be that described by Q, and taking its radius for unity, if we put c for the radius of the other circle, and denote by ϕ the complement of the angle which OP, in any position of P, makes with the mean axis of the ellipsoid, we have $MQ = \sqrt{1 - c^2 \sin^2 \phi}$, and therefore

$$dt = \frac{k d\phi}{\sqrt{1 - c^2 \sin^2 \phi}};$$

where dt is the element of the time, and k is a constant quantity. Hence, the time at which the point P attains any given position in its circle, and therefore the time at which the point I attains any given position in the spherical conic which it describes, is determined by an elliptic function of the first kind, the modulus and amplitude of which are exhibited geometrically. The modulus c of the function is obviously the ratio of the two moduli of the cone which the right line OI describes within the body; for the radius of each circle is found by dividing the distance OI by one of the moduli of the cone.

The preceding method of determining the time in the problem of rotation occurred to the author in the year 1831, and has since been given at his lectures in Trinity College.

It may be observed, that the motion of the axis of rotation within the body is known when that of the right line OI is known; for that axis is always perpendicular to the plane which touches the ellipsoid in the point I.

Mr. Donovan read the first part of a paper "on the supposed identity of the agent in the phenomena of ordinary electricity, voltaic electricity, electro-magnetism, magneto-electricity, and thermo-electricity." This part was introductory to the subject of the remaining portions, which will be devoted to an attempt to prove that the hypothesis at present accredited by philosophers, as adequate to explain the phenomena of the electric fluid, under its various aspects, does not accord with well-observed facts.

Its object was to render it probable that what is called the electric fluid, is not a simple element, as it is generally believed to be, but that it consists of several constituent elements, each exercising a separate function; and many facts were referred to in support of the opinion. Reasons were assigned for believing that this hypothesis is more consonant with the general analogy of nature, than the supposition that electricity is a homogeneous fluid. References were also given to the opinions of those who coincide in this view.

After a full consideration of the facts and arguments, Mr. Donovan summed up as his conclusion, that the electric fluid, in the comprehensive sense of the word, including frictional, voltaic, electro-magnetic, magneto-electric, and thermo-electric, does not consist of one homogeneous element, but of several, viz., heat, light, magnetism, electricity proper, chemical attraction, the physiological agent, and the deflecting agent: that the difference between the various exhibitions of it just mentioned depends on the proportions or energy of the constituent elements, or the influence of the modifications which, under different circumstances, they are capable of exerting on each other. This influence is probably of the same character as that which the forces of nature exercise on each other, on the great scale of creation, controlling, antagonizing, and modifying each other's effects, thus producing the diversified phenomena of the universe, but rarely acting independently.

Considerations were then adduced, with a view of rendering it probable that the electric fluid is matter in some extraordinary state of constitution. If it be matter, its constituents are probably endowed with chemical attraction, a property of which no known kind of matter is destitute: a kind of chemical combination of all the constituent elements would be the result. The affinity of the elements would keep them together, and this would explain the ready passage of *all* the constituents through conductors; one constituent, namely the "electricity proper," conveying the other constituents, some of which may not possess the same ready conductibility. Thus the electric fluid, either in its state of frictional, voltaic, or any other electricity, ought to pass with its known facility through the same bodies, and be intercepted by the same bodies, and so we find it.

In assuming that chemical affinity is thus transported to a distance, Mr. Donovan stated, that there is no innovation on opinions at present entertained by philosophers; and he referred to statements by Sir H. Davy, Faraday, Berzelius, Ampere, Schoënbein, and others, in support of that statement. That a distinct constituent element, possessing chemical powers, should exist in the compound called the electric fluid, associated with heat, prismatic rays, and magnetism, is neither less intelligible nor more improbable than that the very same elements should be found associated in the sun's rays. With regard to the existence of magnetism in the sun's rays, the author conceived that such a number of witnesses as Morichini, Carpi, Rodolfi, Davy, Playfair, Somerville, Baumgartner, and the Messrs. Knox, could not all have been deceived.

Admitting the agents in common and voltaic electricity to be compounds of certain constituent elements, the same in number and nature, but very different in proportions, or perhaps differently combined, they may be considered as fluids perfectly different; in the same way as chemists pronounce bodies composed of common matter to be different, when the constituents are similarly circumstanced.

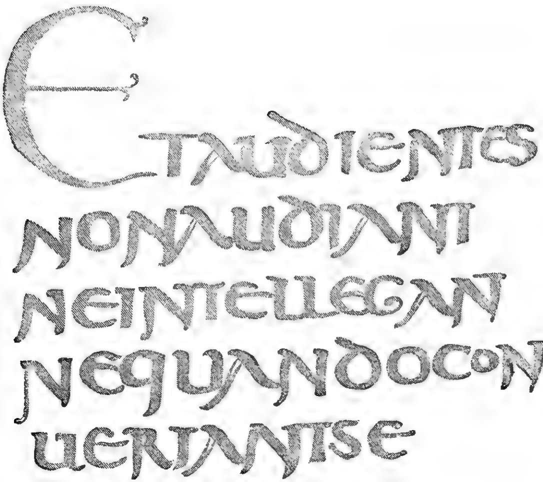
The paper (first part) thus concluded: "Aware that the identity of the agent, in all the phenomena called electric, is firmly established in the minds of the scientific, and that experiments of apparently so convincing a nature have been brought to bear upon the subject, that doubts seem to be no longer entertained, I scarcely know how to declare, in terms that shall protect me from the imputation of presumption, that I have never been able to view the matter in the same light. I have long hesitated to repeat, in advanced life, an opinion which, in my early days, I ventured to promulgate within the walls of this house, namely, that the agents in electricity and galvanism are different, and that the laws of one do not explain the phenomena of the other. Believing, however, that useful results have often sprung from humble causes; that moral cowardice is as little to be esteemed as moral rashness; that the influence of public opinion ought to have its limits in promoting and restraining human actions; I determined to bring my reasons for dissenting from the views of the philosophical world before a tribunal so competent to judge of their pretensions."

The Rev. J. H. Todd, D. D., gave an account of a fragment of an ancient purple manuscript of the Gospels, in Latin, which he supposes to have been written in the fourth, or early in the fifth century, and which he had purchased some years ago in Dublin.

The fragment is but a single leaf, containing a portion of the Gospel according to St. Matthew. It is written in double columns. Each column begins with a large capital letter, although in the middle of a sentence, or even (as in the case of the third and fourth columns) in the middle of a word. Capital letters are also used at the beginning of sections, which, however, do not always coincide with the ancient Ammonian sections, or *κεφαλαια*, employed in the Eusebian canons; nor are any traces of the Eusebian numbers to be found in this manuscript. Dr. Todd, having exhibited the manuscript to the Academy, proceeded to adduce some of the proofs of its great antiquity. These were derived,

1. From the character in which it is written, and the form of the letters, which agrees exactly with those manuscripts that are known to be of the fourth or beginning of the fifth century as, for example, the Codex Vercellensis and the Codex Veronensis, as also from the absence of all stops, divisions of the words, or *στιχοι*.

The following wood-cut is an accurate representation of the first five lines of the first column:



2. From its text, which is the ancient Italic version prior to St. Jerome's revision. This will appear from the following Table, in which the first column exhibits the text of Dr. Todd's fragment, divided exactly as in the original; the remaining columns exhibit the text, divided in a corresponding manner, of the Codex Vercellensis, the Codex Veronensis, and the modern Vulgate.

	Codex Vercell.	Codex Veron.	Vulgata hodierna.
ETAUDIENTES	et audientes	et audientes	et audientes non
NONAUDIANT	non audiant	audiant et	audiunt ne-
NEINTELLEGANT		non intellegant	que intelligunt.
NEQUANDOCON		ne quando con-	
UERTANTSE		vertantur	
ETTUNCREPLEBI	Et tunc imple-	Et tunc	Et adimple-
TURINEISPROFI	tur prophe-	in illis sermo	tur in eis prophe-
TIAESEIAEDICEN	tia eseiae dicen-	prophetae dicen-	tia Isaiæ dicen-
TESUADEETDIC	tis vade dic	tis uade et dic	tis
POPULOHUICAU	populo huic au-	populo huic au-	au-
DITUAUDIETIS	re audietis	re audietis	ditu audietis
ETNONINTELLE	et non intelle-	et non intelle-	et non intelli-
GITISETUIDEN	getis	getis et uiden-	getis, et viden-
TESUIDEBITISET		tes uidebitis et	tes videbitis et
NONUIDEBITIS		non uidebitis	non videbitis.
INGRASSATUM	Incrassa	Ingrassatum	Incrassatum
ESTENIMCOR	cor	est enim cor	est enim cor
POPULIHUIUSET	populi hujus	populi hujus et	populi hujus, et
AURIBUSGRAUI	aures	auribus suis gravi-	auribus gravi-
TERAUDIERUNT		ter audierunt	ter audierunt,

	Codex Vercell.	Codex Veron.	Vulgata hodierna.
ETOCULOSEO	et oculos eo-	et oculos su-	et oculos su-
RUMINGRAUA	rum grava	os gravaberunt	os clauserunt,
NECONUERTANT	ne quando convertan-	ne quando oculis videant	nequando videant oculis,
		et auribus audiant, et corde intellegant et	et auribus audiant, et corde intelli-
		convertant-	gant et convertantur
SEETSANEMEOS	tur et sanem eos	tur et sanem eos	et sanem eos.
		Incrassatum est enim cor populi	
		hujus et aures eorum obstrue et oculos	
		eorum grava ne quando convertantur	
		et sanem illos dicit Dominus	
VESTRAEAUTEM	vestri autem	Vestri autem	Vestri autem
BEATAEAURES	beati oculi	beati oculi	beati oculi
ETOCULIVESTRI	qui vident et aures	qui vident et aures	quia vident, et aures vestræ
QUIVIDENTAMEN	quae audiunt. Amen	quae audiunt. Amen	quia audiunt. Amen quippe
DICOUOBISQUO	dico vobis quod	dico vobis quod	dico vobis quia
NIAMMULTIPRO	multi pro-	multi pro-	multi pro-
FETAEETIUSTI	phetae et justi	phetae et justi	phetæ et justi
CUPIERUNTUI	cupierunt vi-	cupierunt vi-	cupierunt vi-
DEREQUAEUIDE	dere quae vide-	dere quae vidis-	dere quæ vide-
TISETAUDIRE	tis et non audierunt	tis et non viderunt	tis et non viderunt,
	et audire	et audire	et audire
QUAEAUDITISET	quae auditis et	quae vos auditis et	quæ auditis et
NONAUDIERUNT	non audierunt	non audierunt	non audierunt.
VOSAUTEMAUDITE	Vos ergo audite	Vos ergo audite	Vos ergo audite
PARABOLASSE	parabolam se-	parabolam se-	parabolam se
MINANTISOM	minantis om-	minantis om-	minantis. Om-
NISQUIAUDITUER	nis qui audit ver-	nis qui audit ver-	nis qui audit ver-

	Codex Vercell.	Codex Veron.	Vulgata hodierna.
BUMREGNI	bum regni	bum regni	bum regni
ETNONINTELLE	et non intelle-	et non intelle-	et non intelli-
GITUENITMALUS	git venit malus	git venit malus	git, venit malus
ETRAPITQUOD	et rapit quod	et rapit quod	et rapit quod
SEMINATUMEST	seminatum est	seminatum est	seminatum est
INCORDEEIUS	in corde illius;	in corde illius;	in corde ejus;
HICESTIUXTAUIAM	hic est qui secus viam	hic est qui secus viam	hic est qui secus viam
SEMINATUSSU	seminatus est qui autem	seminatus est qui autem	seminatus est. Qui autem
PERAUTEMPE	supra pe-	supra pe-	super pe-
TROSAMSEMI	tra semi-	trosa loca semi-	trosa semi-
NATUSHICEST	natus est hic est	natus est hic est	natus est, hic est
QUIAUDITVER	qui verbum audit	qui verbum audit	qui verbum audit
BUMETCUM	et continuo cum	et continuo cum	et continuo cum
GAUDIOSUSCI	gaudio acci-	gaudio acci-	gaudio acci-
PITILLUMETNON	pit illum, sed non	pit illud, sed non	pit illud; non
HABENSRADI	habet in se radi-	habet in se radi-	habet autem in se radi-
CEMINSESED	cem sed	cem sed	cem, sed
ESTTEMPORALIS	est temporalis.	est temporalis.	est temporalis:
FACTAAUTEMAN	Facta autem tri-	Facta autem tri-	facta autem tri-
GUSTIAAUTPER	bulatione vel per-	bulatione vel per-	bulatione et per-

	Codex Vercell.	Cgdex Veron.	Vulgata hodierna.
SECUTIONEM	secutione	secutione	secutione
PROPTERUERBUM	propter verbum	propter verbum	propter verbum
CONTINUOSCAN	continuo scan-	continuo scan-	continuo scan-
DALIZATUR	dalizatur	dalizantur	dalizantur.
QUIAUTEMINSPI	Qui autem in spi-	Qui autem in spi-	Qui autem semi-
NISSEMINATUR	nis seminatur	nis seminatus est	natus est in spinis
HICESTQUIAUDIT	hic est qui verbum	hic est qui verbum	hic est qui verbum
UERBUMETSOLLI	audit et solli-	audit et per solli-	audit et solli-
CITUDOSAECULI	citudine saeculi	citudinem saeculi hujus	citudo seculi istius,
ETDIUITIARUM	et voluntates	et voluptates	et fallacia
UOLUNTASSUF	diuitiarum suf-	divitiarum suf-	divitiarum, suf-
FOCATUERBUM	focat verbum	focat verbum	focat verbum,
ETFITSINEFRUC	et infructuosus	et sine fructu	et sine fructu
TU	fit.	efficitur.	efficitur.
INTERRAMAUTEM	Qui vero in terram	Qui vero in terram	Qui vero in terram
BONAQUISEMI	bonam semi-	bonam semi-	bonam semi-
NATUSESTHICEST.	natus est hic est	natus est hic est	natus est, hic est,
QUIAUDITVER	qui audit ver-	qui audit ver-	qui audit ver-
BUMETINTELLE	bum et intelle-	bum et intelle-	bum et intelli-
GITTUNCFACIT	git tunc fructum	git tunc fructum	git, et fructum
	adferet et facit	adferet et facit	affert, et facit

The Codex Vercellensis is shown by Blanchini, and generally believed, to be the autograph of Eusebius, first bishop of Vercelli, in the diocese of Milan, who died in the year 371. Having been banished from his see by the Arians, he employed his retirement, at the suggestion of Pope Julius, in the revision of the Latin versions then in use, which were, for the most part, full of errors, interpolations, and solecisms; and his recension became afterwards very generally received throughout the West, having been adopted by St. Hilary of Poictiers as the text from which he quotes in all his writings.

The Codex Veronensis is a purple manuscript, written in letters of gold and silver, and is assigned by Blanchini to the beginning of the fifth century: its text is generally considered to belong to the Eusebian recension, but it has manifestly been corrected by the Greek text of Hesychius, and is no where indebted to Jerome's revision.

An examination of the foregoing table, in which these two very ancient specimens of the Vetus Itala, or old italic Latin version, are compared with the fragment, will prove that Dr. Todd's fragment is also a manuscript of that version; and many of its peculiarities are such as would naturally be expected in a manuscript of the same age. It is curious that in all the three manuscripts the word *intelligo* is uniformly spelt *intellego*, showing that this variation from the usual spelling was not the mistake of a copyist, but the spelling of the same period and locality.

Again, it will be seen that the fragment sometimes agrees with the Verona manuscript and differs from the Vercelli manuscript; sometimes agrees with the Vercelli manuscript and differs from the Verona manuscript; and, in some cases, even where the two other manuscripts agree with the modern Vulgate, it differs from them all.

The conclusion, therefore, is inevitable, that this is a leaf of a purple manuscript of the fourth or early part of the fifth century, of the Eusebian revision,—one of those which were in use before the Hieronymian Vulgate, and from which Jerome made the recension now known as the Latin Vulgate. It was probably written, like the Vercelli manuscript, in gold or silver letters, but the metallic surface, if what are called gold and silver letters in this class of manuscripts be metallic, has long since been rubbed away, and nothing now remains but the traces of the original ink with which the letters were described before the golden substance was applied to them. Of this, however, we have no certain proof.

It will be observed that this fragment is full of solecisms, mistakes of the scribe, and misspellings, a circumstance very common in the more magnificent manuscripts of the class to which Dr. Todd supposes it to belong; for the artists who excelled in penmanship and decorative skill were often very incompetent as biblical scholars; and the very costliness of the material, and elegance of the writing, were obstacles to correctness, for the scribe preferred leaving a mistake to spoiling the beauty of his penmanship by attempting to correct it. Thus we find *dicentes* for *dicentis; parabolas* for *parabolam; illum* for *illud; persecutionem* for *persecutione; bona* for *bonam.*

DONATIONS.

Antiquities from Dunshaughlin, viz.: A fragment of an Iron Chain, consisting of twenty-seven double-looped links, one Ring, and part of a Staple. A large Steel Knife or Dagger. A Draughtsman made of Bone, mounted with Bronze Pin. A Bronze Spear-head with double Blade, and two lateral loops. Three Boar Tusks. The Bone of a Cock's foot.

A Stone Celt, from the county Antrim.

A similar Stone Celt, found near the Falls of Niagara.

A Steel Spear and Ferule, from the Gambia.

A similar Spear, but larger, with small Trowel for Foot of Shaft, used by the Mandingoes, from the Gambia.

A Steel Spear and Ferule, from the Gold Coast.

A pair of Slave Shackles from the Gold Coast, supposed to be of African manufacture.

Two Scarabei; three small Mummy Figures; one large Mummy Figure; two Copper Coins, with Arabic Inscriptions; from Egypt.

Three African Idols in Brass or Bronze, viz.

1. *Sitting Figure, from Bight of Benin.*
2. *Ornamented Head, with Spirals, from Gold Coast.*
3. *Ichneumon, from Mandingo Country.*

Ingot of Bronze.

All the foregoing antiquities were presented by Dr. Madden.

Miscellany of the Irish Archæological Society. Vol. I. Presented by the Society.

Ancient Iron Hatchet found in the Bed of the River Boyne. Presented by Charles Cheyne, Esq.

Legal Education. By Henry Holmes Joy, Esq., M.R.I.A. Presented by the Author.

Medicines, their Uses, and Mode of Administration. By J. Moore Neligan, M.D., M.R.I.A. Presented by the Author.

Memoir of the Family of French. By John D'Alton, Esq., Barrister, M.R.I.A. Presented by the Author.

A Copy of a Document preserved in the Chapter House, Westminster, with impressions of the Seals attached to it. Presented by Sir William Betham.

ERRATA.—Page 353, lines 7 and 19, *for* "Alcinoe" *read* "Alcyone."

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 61.

February 8th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Rev. Henry Tibbs and John O'Donovan, Esq., were elected members of the Academy.

A paper by the Rev. M. Roberts, on the lines of curvature on the surface of the Ellipsoid, was read by Mr. Ingram.

The analogy between the lines of curvature on an ellipsoid, and a system of homofocal plane or spherical conics, was first remarked by Mr. M. Roberts, in a note communicated by M. Liouville to the French Academy of Sciences, where he has shewn, among other things, that the sum (or difference) of the geodetic distances from the umbilics to any point of the same line of curvature is constant. The following properties, which he has recently obtained by assigning a significant geometrical meaning to the constant introduced by M. Jacobi in the integral of the differential equation of the geodetic line, appear worthy of the attention of geometers:

1. Assuming the distance between the umbilics (interior) as the base of a system of triangles, whose sides are geodetic

lines, and of which the product of the tangents of the semi-angles at the base is constant, the locus of the vertex will be a line of curvature.

2. And if the ratio of the tangents of the aforesaid angles be constant, the locus of the vertex will be a line of curvature of the orthogonal system.

3. As the arc s of a plane curve is expressed in polar co-ordinates by the equation

$$ds^2 = d\rho^2 + \rho^2 d\omega^2,$$

and the arc of a spherical curve by the equation

$$ds^2 = d\rho^2 + \sin^2\rho \, d\omega^2,$$

so let the arc of a curve on the surface of an ellipsoid, referred to the geodetic distance (ρ) from one of the umbilics, and the angle (ω) made by ρ with the section containing the umbilics, be given by the equation

$$ds^2 = d\rho^2 + \mathrm{P}^2 d\omega^2,$$

and, as Mr. Roberts has demonstrated,

$$\mathrm{P} \sin \omega$$

will be the perpendicular distance of the point (ρ, ω) from the plane of the umbilics. Hence, P', ω' denoting the same things for the contiguous umbilic, we have

$$\mathrm{P} \sin \omega = \mathrm{P}' \sin \omega', \text{ or } \frac{\mathrm{P}}{\mathrm{P}'} = \frac{\sin \omega'}{\sin \omega},$$

which may be regarded as an extension, to the surface of an ellipsoid, of the fundamental property of plane and spherical triangles, that the sides (or the sines of the sides) are proportional to the sines of the opposite angles.

4. Let ω be a right angle, and the corresponding geodetic vector will pass through the vertex of the mean axis, and its length comprised between this point and the origin (the umbilic) will be equal to the quadrant of the elliptic section containing the umbilics. The function P of this arc will be equal to the mean semiaxis of the surface, in the same way as the sine of the quadrant is the radius.

5. Let geodetic lines issuing from the same point upon a line of curvature, and passing through the umbilics O, O′, meet the line of curvature again in the points P, P′. Then will the locus of the point of concourse of the geodetic lines OP′, O′P, be a line of curvature of the same species as the given one.

The following note, by Mr. M. Roberts, on a theorem relating to the Hyperbola, was also read:

Let S denote the difference between the infinite arc and the asymptote of the hyperbola, whose equation is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\,;$$

and let S′ be the length of the quadrant of the curve which is the locus of the feet of perpendiculars dropped from the centre upon its tangents; also, let Σ, Σ′ denote the same things in reference to the conjugate hyperbola

$$\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1,$$

and we shall have

$$SS' + \Sigma\Sigma' = \tfrac{1}{2}\pi\left\{\frac{a^3}{b} - \sqrt{a^2 - b^2}\,s\right\}$$

where we suppose $a > b$, and denote by s an arc of the first hyperbola, measured from the vertex to the point whose coordinates (x', y') are

$$x' = \frac{a^2}{b}, \qquad y' = \sqrt{a^2 - b^2}.$$

If $a = b$ the hyperbola is equilateral; the derived curve is the common lemniscate, $S = \Sigma$, $S' = \Sigma'$; and

$$SS' = \tfrac{1}{4}\pi a^2,$$

a theorem proposed by Mr. W. H. Talbot, and proved by M. Sturm, in vol. xiv. of Gergonne's *Annales de Mathematiques*, page 17.

Professor Harrison read the following paper on the anatomy of the elephant:

"Having had, within the last few weeks, an opportunity of examining the body of an elephant which died in this city, I have dissected different portions of it with some care, and shall lay before the Academy, at the present Meeting, a section of the recent skull, also the brain, and a minute dissection of the nerves of the proboscis on one side. As my time has been limited, and much engaged with other matters, I shall not enter into a minute examination of these parts, but merely exhibit such striking characters and peculiarities as are likely to engage the attention of those who are not fully acquainted with the subject. I may observe that, in the course of my dissections, I have ascertained some points which I do not find noticed by preceding writers, and others which have been stated differently from what I have found. Thus, among the ocular appendages, the membrana nictitans, or third eye-lid, presents some interesting relations; in the orbit I do not find any retrahens oculi muscle, as is to be inferred from the descriptions in books; I also observe a second external rectus muscle. In the ear also I find a peculiar arrangement of the muscles of the ossicula auditus; and I see no evidence of the muscular structure of the membrana tympani, so accurately described by Sir E. Home in the *Philosophical Transactions*, and mentioned by subsequent writers, who seem to have adopted his opinions, rather than to have examined the organ for themselves. In the thorax I have remarked the absence of the pleural or serous membranes; and I find the lungs are connected to the parietes by a great quantity of filamentous and yellow tissue, yielding and elastic. I have also discovered a remarkable muscle extending from the back part of the trachea to the œsophagus, near its passage through the diaphragm, of which I can find no previous notice. The rudimentary condition of the gall bladder in the abdomen, as well as the mucous membrane of the small intestines, and the reproductive male organs, present certain peculiarities, which do not appear to me to have received sufficient attention. I shall not allude further to any of these points at present, but as time permits me to complete their examination, and to prepare suitable illustrations, I hope to place some additional remarks before the Academy, which may tend to enlarge our knowledge of the anatomy of this very interesting animal.

"One of the earliest dissections of the elephant on record is that which was made in Dublin, in 1682, by A. Moulin, a medical graduate of Trinity College. This animal was destroyed by a fire which accidentally occurred in the city. In the volumes of the *Philosophical Transactions* several papers have been published on the anatomy of particular parts of this animal. Camper's description and plates have added much to our knowledge; but the most complete and concise description is to be found in the *Encyclop. Method.* vol. iii. p. 173. In Cuvier's system of comparative anatomy several of its peculiarities are noticed; and in his splendid work, *Ossemens Fossiles*, tom. i. p. 12, its osteology has been minutely and carefully described.

"The subject of this examination was an Indian or Asiatic elephant, one of the family of the *Pachydermata,* which may be regarded as a distinct genus, under the name of *proboscidean,* as no other animal possesses the true and perfect proboscis or trunk. Of this genus there are only two living species, the Asiatic and the African, which are distinguished by certain well-marked differences: the head of the Asiatic is large and oblong, the forehead concave, the external ears small, and the molar teeth present undulating transverse ridges of enamel, which are the separations of the laminæ which compose them, worn down by trituration; the head of the African is round and smaller, the forehead convex, and the ears very large, and the molar teeth are marked with lozenge-shaped ridges of enamel.

"The present animal was not full grown; he was supposed to be nine or ten years of age, was about six feet high, and six and a half from the top of the head to the root of the tail; he had latterly increased considerably in height and size, had always enjoyed perfect health until within a few days of his death, which was the result of an acute fever. No organic disease could be detected in any part of his system.

"The large expanse of forehead, the peculiar expression of countenance of the elephant, together with his well-known docility, lead to the presumption that this animal must possess a brain of considerable magnitude. Although this is really the case in a remarkable degree, yet the external skull is by no means a measure of the organ within. I now place before you a horizontal section of the cranium of this animal; and it exhibits two remarkable facts, first, the small space occupied by the brain, and secondly, the beautiful and curious structure of the bones of the head. To the latter we may first direct our attention. The two tables of all these bones, except the occipital, are separated by rows of large cells, some from four to five inches in size, others very small, irregular, and honey-comb-like; these all communicate with each other, and through the frontal sinuses with the cavity of the nose, also with the tympanum or drum of each ear; consequently, as in some birds, they are filled with air, and thus, while the skull attains a great size, in order to afford an extensive surface for the attachment of muscles, and a mechanical support for the tusks or the enormous incisor teeth, it is at the same time very light and buoyant in proportion to its bulk; a property the more valuable, as the animal is fond of the water, and frequently takes to it, and swims and bathes in deep rivers. All these cells are lined by a delicate mucous membrane of a light rose colour, being slightly vascular, like that in the frontal sinuses, of which cavities these cells may be considered as an extension, rather than as analogous to the diploe in other animals. The septa between the cells are vertical, and pass from the outer table to the inner; they are very hard and vitreous, whereas the outer table is of a coarse and porous texture. These septa strengthen the whole fabric, the outer table abutting against them; some rows are separated from others by horizontal shelves. This structure also extends into some of the bones of the face, and into those at the base of the cranium, the pterygoid processes, and the condyles of the occipital; but all the superior part of the last-named bone is devoid of them, the two tables being close and thin, and the bone diaphanous. A deep depression exists in this region, at the bottom of which is a slightly prominent crest, and on either side a very rugged surface for the attachment of the ligamentum nuchæ, which substance I now also place before you; it is a yellow, elastic tissue of immense strength, attached by thick roots to the spinous processes of the vertebræ; ascending thence it divides into two thick vertical and diverging plates, which are inserted into the rough surfaces of the occipital bone already alluded to, and it is worthy of remark, that each fasciculus or lamina of these plates of yellow tissue first ends obliquely in a round tendinous cord, and it is through the medium of an infinite number of these tendons the attachment to the skull takes place. This peculiar structure is well seen in the preparation on the table; its design, most probably, is to effect a more intimate union with the bone than the elastic tissue could obtain. The internal table of the cranium is thin, but very hard and vitreous, and the base is rough and irregular; the cribriform plate is very broad and deeply depressed, with numerous foramina for the passage of the olfactory nerves, which are also numerous and large; the foramen for the nasal branch of the opthalmic is also very large; the optic holes are small; there is little or no sella Turcica, and there is no distinct pituitary body attached to the brain; some vascular and fibro-cellular tissue corresponds to its situation: the foramen rotundum is very large, to transmit the superior maxillary nerve, which is of prodigious size.

"I next place before you the cast of the encephalon, and two drawings, one of its upper, the other of its under surface, both of the full size; also portions of the organ hardened in spirits. The brain, though very large, forms a diminutive contrast to the immense cranium. On examining the three divisions of the encephalon, I found the anterior lobes of the cerebrum to be but of moderate size, narrow anteriorly, and arched a little downwards; beneath each is the olfactory lobe, of considerable size; its rounded oval ganglion was so depressed into the ethmoidal recesses, that it was necessary to cut through each of them in removing the brain; each of these lobules or ganglions contained a large ventricle with smooth surface, and communicating with the lateral ventricle. The middle lobes of the cerebrum are very large and of great transverse breadth, like those of the cetacea, as may be seen by comparing them with the brain of the porpoise upon the table: there are no posterior lobes. The cerebellum is of considerable extent, both transversely and vertically, and abuts against the posterior inferior margin of the cerebrum; the mesocephale or cerebral protuberance is very large and broad, and the crura cerebri on leaving it are of great thickness; the medulla oblongata is highly developed, and not only the anterior pyramids, but also the olives, are of great size; the tubercula quadrigemina and pineal body are small, not much larger than the human; the optic nerves also are small, and the fourth pair are about the same size as in man; the fifth nerves are of prodigious size; the seventh also are rather large. The weight of the encephalon was eleven pounds ten ounces, but, allowing for the loss of some portions of the surface injured in the removal, also for the olfactory lobes, and the empty state of the vessels, it may be fairly stated as twelve pounds, Troy weight. On weighing each part separately, the cerebrum was seven pounds and a half, the cerebellum was four pounds, and the mesocephale and medulla oblongata half a pound.

"Thus the brain of this young elephant weighs twelve pounds, while that of the full-grown horse does not exceed two pounds, and that of man seldom equals four pounds. From an examination of the brains of 150 men, the average weight of this organ was found to be about three pounds eight ounces, but exceptions occasionally occur, thus the brains of Cuvier and Dupuytren are recorded as nearly five pounds Troy weight. From an examination of ninety females, the average weight of the brain was three pounds four ounces, that is, about four ounces lighter than that of man. The brain of the idiot is seldom more than one pound and a half. These are the general results of the observations of Tiedemann,* and of the extended series of inquiries of Sir W. Hamilton† of Edinburgh, Mr. Sims,‡ and Dr. Reid.§ In considering the brain, however, in relation to the nervous function, many other circumstances are to be attended to beside the actual size or weight of the organ, namely, its weight compared with that of the body, the relative proportion of one part of the organ to another, as the cerebellum to the cerebrum, the relative size of the brain to that of the nerves connected to its base, and, above all, the structure of the organ, the size, number, and depth of the convolutions, and the extent and thickness of the encrusting lamina of the vascular, or grey vesicular neurine, which there are good reasons for believing to be the essential dynamic agent in the function of innervation. The weight of the human brain, compared to the weight of the whole body, is as one to forty-five or fifty, supposing the former to be about four pounds, and the latter to vary from 180 to 200 pounds; whereas, the proportion in the elephant will be as one to 400 or 500, supposing the former to be twelve pounds, and the weight of the body to be only from two to three tons. The weight of the present specimen, which was by no means full grown, was about two tons. Therefore, the human body is only forty-five to fifty times heavier than the brain, whereas the elephant's body is four or five hundred times as heavy as its brain: then again the human cerebellum is much smaller than the cerebrum, being in the proportion of one to eight or nine, but in the elephant, the proportion is as one to two: therefore, the human cerebrum is eight or nine times larger than the cerebellum, whereas the cerebrum of the elephant is only twice as large as his cerebellum. These relative proportions are interesting and important, if, as we believe to be the case, the cerebellum be connected with the functions of the general muscular system, and the cerebrum with the manifestations of the mental principle. It must be admitted, however,

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* Phil. Trans. 1836. † Munroe's Anat. of Brain, 1831, p. 4.

‡ Med. Chir. Trans. vol. xix. p. 359.

§ Lond. and Edinb. Monthly Journal of Med. Sci., April, 1843.

that conclusions drawn from the relative weight of the brain and of the body are open to many objections; as the weight of the latter must be influenced by the previous state of health, of fulness or of emaciation; the weight of the brain too must be materially affected by the amount of fluid it contains, or which may have escaped during the operations of removal and of weighing. Attention to the relative structure of the brain, therefore, is also necessary. The convolutions of the elephant's cerebrum are very numerous, but rather small, and but few of the sulci are deep; the fissure of Sylvius is closed, and therefore that numerous group of convolutions forming the "island of Reil" are absent; the grey neurine is not so thick as in man: on the whole, the cerebrum bears more analogy to that of the porpoise than to that of man, in whom the convolutions are large and numerous, the sulci deep, and many of them involuted over and over again. In man, too, the fissure of Sylvius is very deep, and when opened out presents a prodigious number of convolutions; his posterior cerebral lobes are extensive, and overlap the cerebellum; the grey neurine forms a thick investing lamina, the superficial extent of which is increased to a wonderful extent, and to a degree superior to what it is in any other animal.

"I shall next place before this meeting a dissection of the *proboscis* or trunk of the elephant, the organ which forms the striking and characteristic feature of this group of animals, no other possessing it in a perfect state, though in many it is rudimentary, as in the tapir and in the pig. It is essential to the existence of this animal, as the instrument for taking its food, and hence its name (προ, βοσκω). Its length varies from four to six feet, according to the height of the animal; it is of a conical form, the base is attached to the nose, of which it may be regarded as a continuation, and is about two feet in circumference; the apex is from four to six inches; the fore-part and sides are convex, and marked by rugged, transverse folds, which admit of extension and change of form; the posterior surface is flat and rough, and bounded on each side by a row of rough tubercles : the whole is covered by a coarse, hard skin ; this being removed, the proper structure comes into view. The proboscis consists of two long tubes separated by a median septum ; these tubes open above into the nose, and below at the extremity of the trunk ; they are somewhat contracted at each of their extremities ; they are composed of a whitish membrane, not very vascular or sensitive, covered by a dense elastic tissue, which preserves their calibre ; inferiorly the skin is continuous with the lining membrane. The point of the proboscis is worthy of attention, a thick lip surrounds it, with a groove below, and a thumb-like projection above ; this little conical appendix enjoys free motion, and can be brought into contact with every part of the border. The skin being removed from the proboscis, the muscular tissue which composes the principal portion of the entire mass is exposed ; this is arranged, partly in superficial strata, and partly in deeper seated, radiating, and decussating fasciculi. The first layer consists of strong, red, longitudinal fasciculi, which extend, from the frontal and nasal bones, the entire length of the organ ; some of the fasciculi are short, and end between two others ; some are intersected by tendinous lines, which adhere to the integuments : on each side of the trunk also are seen longitudinal muscles extending downwards from the superior maxillary bones and commissures of the lips. As these fasciculi descend they spread out obliquely, some forwards, others backwards, or to the under surface ; some intermingle with the other superficial muscles, others are inserted into the tendinous interlacements of the deeper muscles, and some into the skin ; the posterior or flat surface also is covered by muscular fibres, which take a decided oblique course, attached above to the intermaxillary bones in front of the mouth ; they descend in different laminæ, some obliquely inwards, and join those of opposite sides, in a median tendinous raphé, which extends the whole length of this flat surface of the trunk ; this muscular lamina is not so red or longitudinal as those on the front and sides ; this lamina may also be partially divided into two layers, the fibres of the deeper layer passing obliquely downwards and outwards, and decussating the former. All these investing muscles which you now see exposed on one side, must have the effect of moving, bending, and curling this organ in every direction ; the animal can thus bend it upwards over his head, or downwards between his legs, or on either side along his neck.

"On one side we have raised these longitudinal muscles, and in doing so we perceive some of the principal nerves descending in tendinous sheaths or canals. To these nerves we shall more particularly allude directly. These sheaths are connected to the muscular fibres on either side, and resemble the tendons of digastric muscles ; there are several laminæ of these, through and between which the principal nerves descend, and are thereby protected ftom the pressure of the contracting muscles, as, during the action of the latter, these canals will be enlarged rather than diminished. Beneath all these long muscles, but intimately united to them, we meet another portion of muscular structure, which extends from the longitudinal fibres, and from these tendinous canals obliquely inwards, to be inserted into the parietes of the tubes. If we look at the upper extremity of the proboscis, which has been cut off from the skull, the course of these fleshy fibres is evident; they present a radiated appearance, as they pass from the central tubes outwards to be inserted, some into the longitudinal fibres, others into the tendinous canals for the nerves just mentioned, and others pass between the longitudinal fasciculi to the subcutaneous aponeurosis. Some have attempted to count the number of these muscles, but such an attempt is totally useless. These radiated fibres, by their contraction, can approximate the parietes of the tubes to the skin, and at the same time compress the general structure of the trunk, and thus tend to its general elongation, when the longitudinal fibres are relaxed ; while by this arrangement, also, circular compression or constriction of the tubes is avoided, which must have been the effect if the fibres pursued a circular course. At the upper or cranial extremity, however, some strong fasciculi take a semicircular course around its anterior and lateral surfaces, and are attached to the bones on either side; and around the lower end also are some oblique and partly annular fibres, which are attached to the central appendix and to the border on either side.

"I shall next allude to the nerves which are distributed to the proboscis, and which extend its entire length: they are four in number, that is, two on each side, and, like the muscles, are symmetrically arranged; these nerves are the facial, or the portio dura of the seventh and the infra-orbital or supramaxillary division of the fifth cerebral nerves. These nerves are remarkable for their size and length, and for the numerous plexuses they form with one another. The portio dura, or the seventh, is about the size of the little finger, and between the point of its exit from the stylo-mastoid foramen in the base of the skull, and its entrance into the proboscis, pursues a curved course about two feet in length; in this course it sends many branches to the glands and muscles on the side of the face, and then enters the upper and lateral part of the proboscis between the anterior and lateral longitudinal muscular fasciculi, and is joined in this situation by the superior maxillary nerve, the second division of the fifth pair; this great nerve, fully the size of the middle finger, escapes from the infra-orbital foramen, gives off a few branches to the lower eye-lid and to the integuments, and then descends into the same sulcus in the proboscis with the seventh. The two nerves now unite, butthey soon separate and divide, the divisions reuniting so as to form a most intricate nervous plexus; and, as you may observe in the dissection before you, these nerves pursue a similar arrangement during their long course down the trunk, until within a few inches of the extremity. This chain of plexuses resembles a thickly tangled skein of silk extending from one end to the other. When the dissection is carried deeper, the same plexiform appearance is observed, the nerves running in the tendinous sheaths, already described, in courses one beneath the other. From the larger nerves innumerable fibrillæ cross in every direction to supply the surrounding muscular fasciculi. Language can convey no adequate idea of the number and plexiform arrangement of these proboscidean nerves; and, although in this dissection many hundred are brought into view, I have no doubt as many more could be displayed, if sufficient time could be afforded to the tedious process of exposing them. The plexiform arrangement of the seventh and fifth nerves on the human face, though anatomically and physiologically analogous to these, yet bears no comparison as to size, number, or complexity. From the intimate union between the seventh, which is the nerve of motion, and the fifth, the nerve of sensation, the greater number of the branches derived from these plexuses must be compound filaments, and, therefore, supply the parts to which they are distributed with the two endowments, motion and sensation. In this dissection, however, I have in several situations unravelled the nerves in the plexus to their respective sources, and traced fine filaments from the fifth or sentient nerve, inwards to the lining membrane; and have also pursued some very large branches of the same nerve, undivided, down to within two or three inches of the proboscis, where they separate into fine hair-like branches, about twenty of which are exposed in one situation, all descending in parallel lines to the very border of the opening, where they branch off into minute filaments, and terminate in the subcutaneous tissue.

"I shall not delay you with any minute account of the blood-vessels of this organ, and shall only observe, that they are very large and very numerous. Some of the principal trunks accompany the nerves, but many others run in channels through the muscular substance, and distribute their branches to it in every direction.

"I shall next place before you a dissection of the cartilages of the true nose, which are connected above to the nasal bones, and below to the proboscis. These cartilages present a long, curved tube, which is convex forwards, and divided into two tubes by a median cartilaginous septum. The tubes of the proboscis communicate with these, and so with the nares and throat. The lateral cartilages are very elastic, and lie so close to the septum as partially, but not completely, to close this communication; but between the lower border of each cartilage and the proboscis are some strong muscular fibres, which can compress the connecting membrane, and thereby perfectly close the upper end of each proboscidean tube, and so prevent the fluid which the animal draws into these from rising into the nose and flowing through the nares into the throat.

"The lining membrane of these cartilaginous nasal tubes is somewhat of the same nature as that which lines the nares, which is thick, soft, and vascular, and very superior in organization to that which lines the proboscidean canals. In the latter there are no olfactory nerves, and no sense of smell; this sense resides, as in other animals, in the true nose.

"From the anatomical examination of the complex structure of this very curious appendix, we can understand the powers it displays, and the purposes it effects in the economy of the animal to which it appertains. Its muscularity and its pliancy render it, as a weapon of offence and defence, powerful and effective. With it he can strike down an animal, or can raise it into the air and dash it to the earth; or he can bend it round its body or its neck, and crush it by powerful compression. It can tear down large branches of trees, and raise and propel great weights; hence the great value of the elephant in warfare, and in marshy countries, as a beast of burden, for the transport of heavy guns and cumbrous baggage. In such services his exertions are well-known, and have often excited admiration and surprise; the more so, as the docility and intelligence of the animal enable him to direct his physical strength with the degree of energy and skill exactly suited to accomplish the desired object.

"For the prehension of food the proboscis is indispensable to his existence; by its means he plucks off branches and leaves, blossoms and fruits, and tears away the herbage, gathers all into a mass, and, as with a hand, places it within his mouth. With it he sucks up a large draught of fluid, closes the communication with the nose, and then turns its spout-like extremity into the mouth, the anterior part of the latter being so shaped as accurately to receive the contents.

"As the prehensile organs for food in other animals possess the sense of touch, whether they be lips or anterior extremities, so the tapering end of this organ enjoys exquisite sensibility and delicate motor power. By its thumb-like appendix he can pick up the smallest substance from the ground, hold it, and turn it in every direction, to examine it with accuracy before he commits it to his mouth. He can also execute (especially in his captive state, when taught and encouraged by kindness) a variety of delicate manipulations with astonishing dexterity.

"This organ also enables him to remain in deep water, his whole body immersed and concealed from view, except the point of the proboscis, which appears just over the surface, and through which respiration is conducted, occasionally drawing in a little fluid, and then ejecting it with force, not unlike the "jet d'eau" from the blow-hole on the head of the cetacea. By it also, when on land, he can dash water, sand, and mud all over his body, so as to cool and refresh the surface, and remove any source of irritation. Finally, by this instrument he can modify his voice, and increase its tone, so as to cause it to be heard at a distance of one or two miles, and, according to some, still more. Through it he can send forth trumpet sounds, loud, harsh, and discordant, but varying according as they are indicative of social or sexual feeling, or of terror, anger, or satisfaction."*

* Since the above went to press I have found, in the library of Trinity College, A. Moulin's pamphlet on the dissection of an elephant, published in the form of a letter to Sir William Petty, Lond. 1682. He has correctly noticed the absence of pleuræ or pulmonary serous membranes. The brain, in his specimen, which was older and larger than mine, weighed ten pounds (I presume avoirdupois), or between thirteen and fourteen pounds Troy weight.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 62.

February 22nd, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The President read the following letter from Mr. Cooper:

"*Merville, February* 19*th*, 1847.

"MY DEAR LLOYD,—On Saturday last I received a letter from my First Assistant at Markree Observatory, Mr. A. Graham, dated 11th inst., in which he states, that having on the previous day heard from Mr. Hind of his discovery of a new comet on the 6th, he had the night before observed it thus:

Feb. 10.49768, G. M. T. $\alpha = 21$h. 42m. 39.4s. $\delta = 69° \; 1' \; 44''$.

"Mr. Hind's place on

Feb. 7th. 6h. 54m. 5s. $\alpha = 21$h. 16m. 58.8s. $\delta = 70° \; 56' \; 42''$.

"Again, on the 15th, I heard from Mr. Graham, under date the 12th, stating that he had attempted an orbit from the observations of Mr. Hind of the 6th and 7th, and his own of the 10th, which, he conceived, could not be worth much, as the heliocentric arc was only $0° \; 38' \; 20''$ between the extreme observations. The result he obtained was:

Perih. Passage, 1847, March 30th, 9h. G. M. T.

Long. Perihel. .	284° 42′
Asc. Node . . .	34 53
Inclination . . .	49 40
Log. Per. Dist. =	8.4087

Motion direct.

" A letter has reached me this morning from Mr. Graham, dated yesterday, in which he states that he has been fortunate enough to obtain, with the circle, the place of a second star, with which he had compared the comet on the 10th, and which was not in any of the catalogues. It is :

$$\alpha = 21h.\ 44m.\ 16.42s. \qquad \delta = 69^\circ\ 26'\ 51''.5.$$

The other star of comparison on the 10th was 7610, B. A. Catalogue. On the 15th he was enabled to compare the place of the comet with the stars 7786, B. A. C., and 7810, B. A. C. Result :

Feb. 15.45391, G. M. T. $\alpha = 22h.\ 16m.\ 26.9s.$ $\delta = 65^\circ\ 36'\ 42''$.

" The elements deduced by him from Mr. Hind's observations of the 6th, and his own of the 10th and 15th, are :

Perihel. Passage, 1847, March 30.3483 G. M. T.

Long. Perih. . .	276° 50′ 32″	To app. Equinox, Feb. 10th.
Asc. Node . . .	22 38 16	
Inclination . . .	48 44 2	
Log. Perih. Dist. =	8.60520	

Motion direct.

These agree near enough with the middle observation. He says that he can find no former comet that comes near this orbit : that of 1680 is the most similar, but still wide. His first set of elements seems to have been by no means bad. It strikes me, from the short perihelion distance, that this comet may prove ere long a very fine one.

" Ever sincerely your's,

" Edward Cooper."

The President also exhibited to the Academy a diagram, representing the diurnal changes of temperature during the late remarkable depression, which occurred in the week commencing February 7. The observations from which it is taken are those made at the Magnetical Observatory of Trinity College, at six stated hours during the day, together with those of the maximum and minimum temperature, furnished by self-registering thermometers. The following Table exhibits the *mean temperature* for each day; the *maximum* and *minimum* of temperature; and the difference of the latter, or the *diurnal range*. The mean temperature is deduced from the observations at 10 A. M. and 10 P. M., except on the two Sundays, when it is inferred from the maximum and minimum temperatures. The Table includes the day preceding and that following the depression:

Day of Month . .	6th.	7th.	8th.	9th.	10th.	11th.	12th.	13th.	14th.
Mean Temp. . .	45.0	33.8	29.6	28.4	30.8	31.9	24.2	35.6	45.5
Maximum	50.5	34.5	34.8	33.2	35.5	34.6	30.3	47.5	51.2
Minimum	40.0	33.0	26.0	25.2	25.7	24.7	21.5	16.7	39.8
Range	10.5	1.5	8.8	8.0	9.8	9.9	8.8	30.8	11.4

The greatest depression took place on the morning of the 13th, the minimum temperature being 16°.7.* A registering thermometer, exposed to radiation towards the sky, exhibited at the same time the temperature 12°.0. Temperatures so low as these, although not uncommon in England, have not occurred for many years in this country. The range of temperature, otherwise pretty constant, was remarkably affected on the first and last days of the depression (the 7th and 13th), the decrease of temperature on the former day reducing it to 1°.5, while the increase on the latter raised its amount to 30°.8. In fact, on this day (the 13th), the rise of temperature was so rapid, and so continuous, that the day maximum was entirely

* At Mr. Yeates' house, on the south side of Dublin, the minimum thermometer registered 11°.

obliterated, the absolute maximum occurring during the night. In six hours, namely, from 7 A. M. to 1 P. M., the increase amounted to nineteen degrees.

Michael Donovan, Esq., read a continuation of his paper "on the supposed identity of the agent in the phenomena of ordinary electricity, voltaic electricity, electro-magnetism, magneto-electricity, and thermo-electricity."

After briefly noticing the circumstances of the discovery of galvanism, the author remarked, that as soon as even a few of the facts were discovered, an hypothesis was invented to account for them ; they were at once attributed to the agency of an animal electricity secreted by the brain, of which the muscles are the depositories, and the nerves the conductors. Volta admitted that the agent is electricity, but maintained that it is generated by the contact of the conductors concerned; the contraction being produced by the restoration of the equilibrium through the animal. Hundreds of facts have been since discovered, but, wonderful to say, the hypothesis invented before they were known has been adhered to, and is still used for their explanation.

The vast difference of properties observable in electric and voltaic phenomena has been conceived to be explicable on the supposition that in the former the quantity of electricity is small, and the intensity great, while in the latter the quantity is great and the intensity low. Several quotations from authorities were adduced to this effect, and also with a view of determining the exact meaning of *quantity* and *intensity*.

It was then argued, that the efficiency of great quantity, at a low intensity, to account for the difference between common and voltaic electricity, has been received upon grounds which have not been sufficiently canvassed; that we know nothing of quantity of electricity but by its intensity; that the two terms represent ideas which are inseparable; that intensity is the only significant condition of electricity of which our senses take cognizance; that the expression "quantity of electricity" aims at conveying to the mind a condition which cannot be comprehended, and that, therefore, no clear idea of any explanation founded on the notion of "quantity" can be attained. Several considerations, in support of these positions, and an experiment to the same effect, were adduced. In fine, it was concluded, that there is not a known phenomenon the explanation of which receives any real assistance from the assumed agency of quantity. M. Biot, probably perceiving this defect in its alleged operation, has substituted the influence of "velocity."

Those who sought to establish identity of the different forms of electricity had long been embarrassed by the failure of all efforts to produce deviation of the galvanometer needle by means of common electricity, although it is so easily effected by voltaic. M. Colladon, imagining that this want of success was occasioned by an insufficient quantity or supply of the electric fluid, or by imperfect insulation of the coil of the galvanometer, employed one in which particular precautions were taken to insure insulation. With this instrument, placed in the circuit of a very large Leyden battery, a deviation of twenty-three degrees was obtained; the deviation increased with the intensity of the charge; it sometimes amounted to forty degrees. When the galvanometer was made part of the circuit between the conductors of a Nairne's electrical machine, the deviation was three or four degrees only; but when a coil of 500 turns, of the same construction, was substituted, a maximum deflection of thirty-five degrees was produced, provided the cylinder was made to revolve three times in a second.

Mr. Donovan then stated an experiment of his own in relation to this subject, the object of which was to prove, that in Colladon's experiments it was intensity and not quantity that acted. Other experiments were adduced to prove that, when the electricity is of the voltaic kind, the most feeble intensities are far more efficient in causing deflection of the needle than the most powerful intensities of common electricity; showing, as it was suggested, that in the latter case the agent was electricity, which, being evolved by chemical action, contained much of the deflecting constituent element; and that in the former the agent was electricity, with its natural minimum of the deflecting constituent, because it was developed without chemical action, by mere friction; and hence the necessity of the presence of such electricity in considerable abundance to produce the required effect.

The same thing was stated to be evidenced by an experiment, in which two voltaic apparatuses were made to act separately on a differential gold-leaf electrometer, one of them producing divergence, the other none; yet the effect of the latter on the galvanometer needle was powerful, that of the former null.

Professor Faraday's repetition of Colladon's experiments on deflection by common electricity were then reviewed, and the remarkable circumstance was adverted to, that one of his deflections was produced by one pole, contrarily to the laws of voltaic electricity, in which the operation of two poles is indispensable. If it be admitted as proved, that common electricity does not require a twofold polar arrangement in order to produce deflections, it becomes a question, what is the use of the two poles used in Colladon's and Faraday's experiments with the Leyden battery? One of them must be superfluous. If this be so, we arrive at this general proposition, that voltaic electricity is composed of elements existing in such ratio, and so combined and modified, that it must be brought to bear upon the subject of its action by means of two poles simultaneously and equally energetic; while the proportions and mode of combination in the common electric fluid are such, that it produces the same effect with one pole only. Thus a difference, instead of an identity, would be proved by these experiments.

It was further observed, on Faraday's deflection of the galvanometer needle by common electricity, that no less than

2000 one-inch sparks were required to produce a deviation of forty degrees, while, in an experiment with a minute pair of zinc and platinum plates, the zinc not weighing more than the head of a pin, and probably not the thousandth part of a grain dissolved during the action of an acid on it, the needle, nevertheless, whirled round the circle twice. Thus, a chemical action, almost inconceivably small, produced an effect eighteen times greater than 2000 sparks of electricity from a powerful plate-machine. The inference drawn was, that the agents could not be the same in both.

In furtherance of the objects above detailed, Professor Faraday has made experiments to determine the quantity of electricity associated with the particles or atoms of matter; from which it may be calculated, that to decompose a single grain weight of water, 800,000 discharges of an electric battery, each discharge consisting of 300 one-inch sparks, would be required; which Faraday conceives is equal to a powerful flash of lightning: and he estimates that the electricity, that is the affinity, which maintains the oxygen and hydrogen of the grain of water in combination, is of the same amount. Thus, according to him, there is the electricity of a flash of lightning in every grain or drop of water, that is, if the electricity of a drop of water could be collected in one spark, it would be 454545 miles in length.

But Faraday neglected to compare his results with those of MM. Paets Van Troostwick and Deiman, and also with those of Dr. Pearson. These philosophers, who made experiments with the greatest care, represent the matter very differently. Many calculations were entered into, which proved that, according to the experiments of the Dutch chemists, the quantity of electricity necessary to decompose a grain of water is thirty-eight times less than Faraday's estimate, and, according to those of Pearson, forty-two times less. The vast difference of Faraday's estimate leads to some suspicions of the universality of the law as laid down by that philosopher, namely, that if water be subjected to the influence of the electric current, no matter what the intensity or acting surface, the quantity decomposed will be exactly proportionate to the quantity of electricity which has passed. All this may be very true, when applied to the voltaic influence, but, if so, the law seems to individualize common electricity, and to dissever it from its alleged identity with voltaic electricity. When we find two estimates of an effect to agree pretty well, while a third is forty-two times greater than one, and thirty-eight times greater than the other, it is plain there is a monstrous error somewhere; and hence, before we venture to draw any conclusion, it will be necessary to investigate the grounds on which the discordant opinion has been formed. This becomes the more necessary, when it is recollected that the stronghold of those who maintain the identity of the voltaic and electric agents is the almost unlimited supply of the latter at a low intensity, which, they affirm, can be brought into action during the exhibition of any phenomenon caused by the former.

Faraday has affirmed, as already observed, that one grain of water, decomposed by four grains of zinc, can evolve electricity to an enormous amount, no less than 240 millions of one-inch sparks. To test this, an experiment was made, in which diluted sulphuric acid was made to act on a voltaic pair consisting of four grains of zinc foil and a plate of platinum, the metals being separately connected with a differential electrometer with insulated, detached, and moveable gold leaves. The solution of the zinc occupied one minute and a half, and during this period the gold leaves were rapidly approached until they touched, and then rapidly withdrawn; there was not the slightest attraction or repulsion, although, according to Faraday's estimate, the equivalent of 240 millions of one-inch sparks was passing between them at the time. Yet, when the same electrometer was subjected to the action of a voltaic series, consisting of twenty pairs of three-quarter-inch plates, both attraction and adhesion took place.

Lest it should be supposed that this quantity had been really evolved, but was lost by dissipation, an experiment was made, in which a voltaic pair, the zinc weighing four grains, was made to act as above described, the whole being contained in a hermetically sealed glass vessel, with an electrometer so constructed that it would indicate the smallest quantity of dissipated electricity; but there was not the slightest appearance of such.

Should it be affirmed that the alleged enormous quantity of electricity was produced, but, being in the positive and negative states, they neutralized and destroyed each other, the following experiment was opposed to the supposition. A plate of zinc was connected with a plate of platinum, by means of an inch of platinum wire $\frac{1}{100}$ inch thick: this was included in a glass sphere with dilute sulphuric acid, the glass being hermetically sealed. When the acid was made to act on the zinc, there was not the least appearance of heat in the platinum wire, resulting from the neutralization of the alleged enormous quantity of the two states of electricity; nor were any traces of electrical action on the electrometer discoverable. If, according to the hypothesis, the equivalent of 240 millions of positive and negative one-inch sparks had passed through the platinum wire, at the rate of 1,600,000 per second, need it be inquired what would have become of the wire and the whole apparatus. Van Marum, with one discharge of his battery, melted forty feet of thin iron wire.

The supporters of the doctrine here objected to may maintain that the alleged quantity of electricity was really in operation, but that it was retained and concealed in the constitution of the resulting gases; and this also seems to be the opinion of Faraday, by his adoption of the electro-chemical theory of Berzelius, wherein he expresses his belief that the light and heat evolved during combination are produced by the discharge of positive and negative electricity which at that moment takes place. If in the seven or eight cubic inches of mixed oxygen and hydrogen, which result from the decomposition of a grain of water, there be electricity concealed equal to 240 millions of one-inch sparks, when the mixture is detonated, so as to recompose water, a flash of lightning and clap of thunder ought to be the consequence, instead of the little bright flame and the trivial crack which occur.

But it is not merely this immense quantity of electricity that is unaccounted for. Professor Faraday conceives that the electricity which holds the elements of a grain of water in combination, enormous as its quantity is affirmed to be, can only be overcome, during decomposition, by an equal quantity of electricity. What then becomes of this second portion? What has become of the first? We have not been able to discover traces of either. No less than 480 millions of one-inch sparks are concerned in the decomposition of one grain or drop of water, and we can find no account of any portion of them.

Mr. Donovan thus concludes this portion of his paper: "I conceive that the rules of discussion warrant my running this hypothesis as closely to the impossible as I can. The higher the authority, the stronger must be the argument to give it any chance of success. It is on this account that I take the liberty of reasoning thus freely on the opinions of so celebrated a philosopher."

The President and Dr. Apjohn made some remarks on Mr. Donovan's communication, in opposition to his views, and confirmatory of the received doctrine of the identity of electricity from different sources.

DONATIONS.

A Silver Hiberno-Danish Coin and a Bronze Celt, found at Newington, County Kildare. Presented by James Forbes, Esq.

The Twenty-sixth Report of the Leeds Philosophical Society, for 1845–6. Presented by the Society.

The Head of a Cow and the Horn of a Bull; *two portions of Deer Horn and two Bones*; *fragments of an ancient Iron Spear*; *a Hammer and Ring*; *fragment of a Crucible*; *from a Rath in the townland of Cullanagh, parish of Ballyroan, Queen's County.* Presented by Joseph Ferguson, Esq.

Nieuwe Verhandelingen der eerste Klasse van het Koninklijk-Nederlandsche Instituut van Wetenschappen, Letterkunde en Schoone kunsten te Amsterdam. 12 vols.

Précis Historique des Opérations Géodesiques et Astronomiques, faites en Hollande.

Beschrijving van eenen Toestel ter Verwarming van een Uitgestrekt Gebouw; door A. Van Beek.

Verhandeling over eene nieuwe wijze om Afstanden te Meten, door wijlen den heere Hendrik Aeneae. 1812.

Over de meetkundige bepalingen; door J. F. Schröder.

Onderzoekingen aangaande het Zwart in de Melisbrooden; door C. M. van Dijk en A. van Beek. 1829.

Verhandeling over het verschil tusschen de Algemeene grondkrachten der Natuur en de Levenskracht; door C. G. Ontijd.

Nadere waarnemingen en proeven over de onlangs geheerscht hebbende ziekte der aardappelen; door G. Vrolik. (Parts I. and II., 1845 and 1846).

Presented by the Royal Institute of Sciences, Belles Lettres, and Arts, of the Low Countries.

A Silver Reliquary. Presented by Dr. A. Smith.

An ancient Irish War Axe, made of Iron, found in the bed of the River Boyne, near Castle Rickard. Presented by General Birch, R. A.

Fragments of an old Irish Folio Manuscript, on Paper; part of a larger Work in the Library of the Academy, described at page 157 in the Catalogue of the Collection of Irish Manuscripts lately purchased from Messrs. Hodges & Smith.

"The fifteen unpaged loose leaves at the end of this volume appear to be part of a translation into Irish, from some other language, of the ancient history of the Greeks, Romans, Persians, &c. These leaves are not in the same handwriting as the preceding treatise on medicine, nor do they appear to have been at any time part of the volume. The second page is marked Chapter III., between which and Chapter VI., on the fifth page following, no other chapter intervenes, so the leaves are not consecutive. The style of the translation is that of the early part of the sixteenth century."

The fragment now presented begins with the end of Chapter I. It contains the whole of Chapter II., and the beginning of Chapter III.

The following is a translation of the heading of Chapter II.:

"The second chapter, in which Belus, Ninus, and Semiramis, are spoken of, and of their successors in the kingdom of Babilon, until the fifth year of the reign of Sethos."

The chapter then opens in words to this effect: "Syncellus says, following the opinions of Halanicus, Ctesias, Halicarnassus, and Hephalion, that Abraham was fourteen years dwelling in the country of Canaan, when Belus came to conquer Babilon."

Page 7 contains a list of the Judges and Chiefs of the Children of Israel, and of the contemporary kings of Egypt.

The third chapter: "Of the kings of Asiria from the time of Laostines to the nineteenth year of the reign of Nabuchadonosor, who plundered and destroyed the Temple."

Presented by Eugene Curry, Esq.

ERRATA IN PRECEDING NUMBER.

Page 383, line 3, *for* "the Rev. M. Roberts" *read* "Mr. M. Roberts."
„ 385, „ 6, *for* "Mr. M. Roberts" *read* "the Rev. W. Roberts."
„ 385, „ 15, *for* $\frac{x^2}{b^2} - \frac{y^2}{a^2} = 1$, *read* $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 63.

March 16th, 1847. (Stated Meeting.)

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Secretary of the Academy read the following Report:

The Council, on the expiration of their year of office, are happy to be able to express their satisfaction at the continued prosperity of the Academy.

During the past year the first part of the twenty-first volume of the Transactions has been published, containing some valuable papers in the departments of Science and Polite Literature. The second Part is in the press, and is considerably advanced.

The Proceedings have been published with great regularity. The Council are happy to be able to state, that the strict enforcement of the rule which requires a complete abstract of each paper to be furnished before leave to read it can be obtained, has enabled the Secretary of Council to bring out the Proceedings after each meeting without delay. It would be unjust, however, to that gentleman, not to admit, that this success is also very much due to the zealous manner in which he has carried out the intention and spirit of the regulation, as well as to the willing co-operation he has received from the authors of papers.

The Proceedings, therefore, being now in the hands of all Members up to the present date, render unnecessary any minute recapitulation of what has occurred at our public meetings; it must suffice to observe, that there has been no lack of important and valuable communications in any of the three departments of the Academy's labours.

The formation of a Museum of Irish Antiquities is an object to which the attention of the Academy has been directed for the last seven years; and the Council have great pleasure in being able to report, that, during the past year, this important department has not been neglected. The value of such a Museum is now recognised by every Member of the Academy, and continues to be regarded with great interest by the public.

The Council are most anxious that the Museum should be thrown open to public inspection as fully as is consistent with its safe preservation. But they are impeded in this desire by the narrow accommodation afforded by our present rooms, and also still more by want of funds; for it is obvious that a greatly increased staff of attendants would be necessary if the public were admitted to the Museum as fully as is desirable. For these reasons, admissions have hitherto been restricted to parties accompanied by Members, or bearing their orders, although it is the wish of the Council that no respectable person who applies for admission should be excluded.

The Museum, as is well known, is deeply indebted to the liberality of Members, and other friends, who have, from time to time, by private subscription, raised large funds for the purchase of antiquities. About £800 have also been contributed in various sums at different periods, during the last seven years, from the funds of the Academy, for the promotion of this object, and donations of antiquities of great interest and value have been, from time to time, received. It would be very desirable that a full and exact account of all these purchases and donations should be placed on record; and the Council would, therefore, strongly recommend it to their successors to have a brief historical account of the formation of the Museum drawn up, and presented in the form of a Report to the Academy.

During the past year a subscription has been set on foot for the purchase of the Domhnach Airgid, a most interesting relic, whose history is well known to the Academy from Mr. Petrie's paper, which has appeared in our Transactions. The purchase-money agreed upon was £300, and the sum of £285 7*s.*, including £50 subscribed by the Academy, has already been received, leaving a balance of about £15 (exclusive of some costs for printing, &c.), which, it is hoped, will without difficulty be collected.

During the past year very little progress has been made in the Catalogue of the Museum, which must necessarily be a work of time, and of considerable expense; but the Pictorial Catalogue has been continued, and every article added to the Museum has been drawn as it was obtained, so that the Academy will at all times possess the means of ascertaining the identity of every thing that now forms a part of the Collection.

Some few articles of interest and value have been purchased and added to the Museum during the past year, at a cost to the Academy of £22 in all. Several donations have also been received, which have been acknowledged from time to time in the Proceedings, and it is, therefore, unnecessary to give a list of them here.

Next to the donors who have presented to the Museum articles of interest and value, the thanks of the Academy are due to those who have deposited there, for the inspection of the Members and of the public, such valuable relics as they do not wish to part with. It will be in the recollection of the Academy that, some time ago, Sir Richard O'Donnell deposited in the Museum, under the safe keeping of the Academy, the Cathach of St. Columbkille, and Dr. William Stokes deposited the Fiachall Phadruig.

During the past year the Rev. Francis Brownlow has deposited, in the same way, the celebrated Book of Armagh, a manuscript of great antiquity, and of the utmost importance to the ancient Church History of Ireland.

The Council would beg leave to call the attention of the Academy, and of the public generally, to the example set by these gentlemen. Those who are in possession of such relics of antiquity, by depositing them in the safe keeping of the Academy, retain the full power of recalling them whenever they think fit, whilst, at the same time, they give an opportunity to those who are best qualified, of discussing and investigating their claims to antiquity; and thus, in the end, the value of these curious relics is greatly enhanced, by the light that is thus thrown on their history, and the additional evidence that is collected in confirmation of their genuineness.

The sum expended on additions to the Library during the past year has necessarily been very small. The funds of the Academy are so nearly absorbed in the unavoidable expenses of the House and Officers, and in the publication of our Transactions and Proceedings, that the expenses of binding, together with the annual subscription to periodical publications, generally exhaust almost all that can be spared for the support of the Library.

The Library, however, has received several valuable donations during the past year, which have been acknowledged from time to time in the Proceedings.

We have also added the following to the list of Societies with whom we are to exchange Transactions:

1. The Society of Zurich.
2. The Society of Agriculture, Lyons.
3. Academia della Crusca of Florence.
4. Societé des Sciences Naturelles du Canton de Vaud.
5. The Imperial Observatory, Pulkova.
6. L'Institut Royal des Pays Bas, Amsterdam.
7. The Library of the Museum of Economic Geology, London.

No changes have been made during the past year in the by-laws or rules for the management of the affairs of the Academy, except in the mode of electing Honorary Members. It was found that some inconvenience was experienced from the vagueness of the former rules, and that a somewhat too great facility was given by them for the admission of Honorary Members; and, as this tended to the injury of the Academy, by diminishing the value of that distinction, the Council submitted to the Academy the regulations which received your approbation on the 11th of January last, and which will be found in the Proceedings under that date. The principle of these new regulations, which, it is hoped, will prove satisfactory in practice, is twofold; first, to limit the number of Honorary

Members; and secondly, to divide them into sections corresponding to the threefold objects of the Academy, so that the respective branches of Science, Polite Literature, and Antiquities, may be fairly represented among the Honorary Members.

During the past year an opportunity presented itself, from the visit of an intelligent Danish antiquary to this city, of opening a communication between the Academy and the Royal Museum of Northern Antiquities of Copenhagen, from which we may reasonably expect the most beneficial consequences. From the connexion which subsisted in ancient times between the Norsemen and this country, a comparison of Scandinavian and Irish antiquities has long been a great desideratum. The Academy, therefore, in accordance with the recommendation of Council, requested Mr. Worsaae to be the bearer of some drawings of the objects which seemed most likely to be interesting to Northern antiquarians in our Collection, to be presented, in the name of the Academy, to the Royal Museum of Copenhagen. They selected also, for the same purpose, under the superintendence of the Committee of Antiquities, a few duplicates from the Museum, which, it is hoped, may be regarded at Copenhagen as worthy of a place in the Royal Museum. They will, at all events, serve as a testimony of our good-will, and as an expression of our desire to establish a friendly intercourse between the learned men of two countries, whose early history was once so closely and so singularly united.

During the past year the Academy has to lament the death of the following Members:

Abraham Palmer, Esq., ...	elected	1838.
George Digges La Touche, Esq., . .	,,	1838.
Goddard Richards, Esq., ...	,,	1843.
Maxwell M'Master, Esq., ...	,,	1844.

The death of George Downes, Esq., also occurred during the year. He died on Sunday, August 23, 1846; and although, from the state of his health, he had, a short time before, ceased to be a Member of the Academy, the Council cannot refrain from paying this brief tribute to his memory. His attainments in Polite Literature were of a high order; and his extensive knowledge of the northern languages was favourably displayed in his valuable paper on the Norse Geography of Ireland, which has appeared in the Transactions of the Academy.

Thomas Moore, Esq., the distinguished Lyric Poet of Ireland, a gentleman whose name is too well known to need any eulogy on this occasion, was the only Honorary Member elected during the past year.

The Academy has also been increased by the election of twenty-six ordinary Members, whose names are as follows:

1. John Alcorn, Esq.
2. Abraham W. Baker, Esq.
3. Philip Bevan, M. D.
4. J. O. Curran, M. D.
5. Matthew D'Arcy, Esq.
6. James Birch Kennedy, Esq.
7. M. H. Stapleton, M. B.
8. Hon. and Rev. W. Wingfield.
9. John Aldridge, Esq.
10. George Lefroy, Esq.
11. Thomas John Beasly, Esq.
12. Thomas Percy Boyd, Esq.
13. Rev. Robert J. M'Ghee.
14. Rev. William Reeves, M. B.
15. William Brooke, Esq.
16. D. J. Corrigan, M. D.
17. Leonard Dobbin, Esq.
18. Michael Donovan, Esq.
19. Charles Haliday, Esq.
20. W. Neilson Hancock, Esq.
21. John Kells Ingram, Esq.
22. Charles P. M'Donnell, Esq.
23. Right Hon. Louis Perrin, Justice, Queen's Bench.
24. Frederick John Sidney, Esq.
25. Rev. Henry Tibbs.
26. John O'Donovan, Esq.

It was resolved, that the Report of the Council be adopted, and printed in the Proceedings.

The ballot for the annual election having closed, the Scrutineers reported that the following Gentlemen were elected Officers and Council for the ensuing year:

President.—Rev. Humphrey Lloyd, D. D.

Treasurer.—Robert Ball, Esq.

Secretary to the Academy.—Rev. James H. Todd, D. D.

Secretary to the Council.—Rev. Charles Graves, A. M.

Secretary of Foreign Correspondence.—Rev. Samuel Butcher, A. M.

Librarian.—Rev. William H. Drummond, D. D.
Clerk and Assistant Librarian.—Edward Clibborn.

Committee of Science.

Rev. Franc Sadleir, D. D., Provost of Trinity College; James Apjohn, M. D.; James Mac Cullagh, LL. D.; Robert Ball, Esq.; Sir Robert Kane, M.D.; George J. Allman, M.D.; Sir William R. Hamilton, LL. D.

Committee of Polite Literature.

Samuel Litton, M.D.; Rev. William H. Drummond, D. D.; Rev. Charles W. Wall, D. D.; John Anster, LL. D.; Rev. Charles Graves, A. M.; Rev. Samuel Butcher, A. M.; Rev. James Wilson, D. D.

Committee of Antiquities.

George Petrie, Esq., R. H. A.; Rev. James H. Todd, D.D.; J. Huband Smith, Esq., A. M.; Captain Larcom, R. E.; William R. Wilde, M. D.; F. W. Burton, Esq.; Samuel Ferguson, Esq.

The President then appointed, under his hand and seal, the following Vice-Presidents:

Captain Larcom, R. E.; Sir William R. Hamilton, LL. D.; Rev. Franc Sadleir, D. D., Provost, Trinity College; Rev. Charles W. Wall, D. D.

The following note by Sir W. R. Hamilton, announcing a theorem of hodographic isochronism, was read:

If two circular hodographs, having a common chord, which passes through, or tends towards a common centre of force, be cut perpendicularly by a third circle, the times of hodographically describing the intercepted arcs will be equal.

DONATIONS.

Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin. Aus dem Jahre 1844.

Bericht über die zur Bekanntmachung geeigneten Verhandlungen der Königlichen Preussischen Akademie der Wissenschaften zu Berlin. From July, 1845, to June, 1846.

Specimens in Silver and Copper of a Medal struck in honour of Leibnitz. Presented by the Academy of Sciences of Berlin.

The Seal of the Right Rev. Dr. Sandes, late Bishop of Cashel. Presented by the Rev. Charles Mayne, M. R. I. A.

Three Lectures on Political Economy. By W. Neilson Hancock, Esq., LL.B., Whateley Professor of Political Economy. Presented by the Author.

Extracts from the Introduction to the Observations made at General Sir T. M. Brisbane's Observatory in the Year 1843. *By J. Allan Brown, Esq.* Presented by General T. M. Brisbane.

Comptes rendus Hebdomadaires des Séances de l'Académie des Sciences. From 16 Novem. 1846, to Feb. 1, 1847. Presented by the Academy.

Transactions of the Geological Society of London. Second series. Vol. VII., Part 2. Presented by the Society.

Journal of the Asiatic Society of Bengal. No. CLXIX. 1846. Presented by the Society.

Journal of the Royal Asiatic Society of Great Britain and Ireland. No. XVII., Part 2. Presented by the Society.

The Quarterly Journal of the Geological Society of London; for February 1, 1847. Presented by the Society.

Memoires de la Société Géologique de France. 2me Serie. Tome II. 1re partie. Presented by the Society.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 64.

April 12th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Abraham Whyte Baker, Esq.; James W. M. Berry, Esq.; Richard V. Boyle, Esq.; Sir Thomas Esmonde, Bart.; Nathaniel Hone, Esq.; and Philip Jones, Esq.; were elected Members of the Academy.

Dr. Allman made a communication on the Structure of the Fruit and Mechanism of Dehiscence in some of the *Hepaticæ*, and on the probable Origin of the Organs known by the name of Elaters in these Plants.

In this paper the author demonstrated the existence, in the sporangia of *Jungermannia*, *Marchantia*, and *Fegatella*, of spiral cells similar to those found in the lining membrane of the anthers of flowering plants. The sporangium of these genera is composed of a single layer of such cells immediately enclosing the spores and elaters. The author described the circumstances which he had observed to attend the phenomenon of dehiscence in *Fegatella conica*. In this plant the peduncle of the sporangium is connected with the fundus of the calyptra by a mass of loose, succulent, cellular tissue. At the period of dehiscence this mass becomes rapidly enlarged, apparently by engorgment of fluid, and, by acting against the more resisting receptacle, forces the sporangium with its pedicle through the mouth of the calyptra, beyond the margin of the receptacle. The peculiar spiral cellular tissue of the sporangium is thus brought into direct contact with the atmosphere, and the hygroscopic properties of this tissue placed in a condition for manifesting themselves. The result of this is the dehiscence which takes place in the exposed portion of the sporangium, in consequence of the balance of tension in the opposite walls of the cells being disturbed. The author was, moreover, of opinion that the presence of elaters in the hepaticæ might be accounted for by supposing the separation, in an early period, of certain cells from the sporangium walls; that these, falling into the cavity, retain for a certain time their independent vitality, and become developed into the peculiar organs in question, lying at last free among the spores. He could not assent to the doctrine so generally promulgated, that the elaters assist in the dispersion of the spores.

Dr. Allman also read a paper on the external anatomy of *Chelurus*, Phil., a genus of *Amphipodous Crustacea,* destructive to submarine timber works.

In this communication the author gives a detailed description of the external anatomy and zoological relations of an amphipodous crustacean, recently discovered by Mr. Mullins, C. E., in the timber jetty at Kingstown Harbour, where it has been doing a vast amount of mischief by destroying the timber of the submarine works in that locality. It proves, on investigation, to be referable to a genus established by Philippi, under the name of *Chelurus*, for a crustacean discovered by this naturalist at Trieste.* The Irish specimens, however, differ

* See Wiegmann's Archiv. für Naturgesch., 1839.

in certain points from the animal as described by Philippi; and Dr. Allman thought it not impossible that these differences would indicate a distinct species, which might be noted under the name of *C. destructor*, as distinguished from the *C. terebrans* of Philippi. As, however, it is by no means improbable that the differences in question are referable to slight inaccuracies in the memoir of the continental zoologist, the proposed distinction was considered purely provisional, to be confirmed or rejected according as actual comparison of specimens shall decide. The following summary of the generic characters of *Chelurus*, more condensed than that given in the memoir of Philippi, was proposed by Dr. Allman :

CHELURUS, *Philippi.*

Gen. Cha.—Body not compressed; *head* distinct; superior *antennæ* shorter and more slender than the inferior, and consisting of a peduncular portion, which supports two unequally developed rami.* Inferior *antennæ* large, not divisible into a distinct peduncle and ramus. *Mandibles* strong, palpigerous; furnished with a molar tubercle, with transverse ridges. First pair of maxillæ strong, pyramidal, palpigerous; second pair lamelliform. *Maxillary feet* large, bearing a palp-like stem, and united at their origin, so as to constitute a great opercular lip, covering all the other organs of the mouth. *Thorax* composed of seven distinct segments, with the epimeræ distinct, and moderately developed. First two pairs of *thoracic legs*, didactyle; five remaining pairs terminated by a small, unop-

* Philippi takes no notice of this character of the superior antennæ, and simply describes them as setaceous, with seven articulations; whereas the number of articulations, in the animal I have examined, are three in the peduncular portion, and six in the larger ramus. I cannot, however, help believing that this excellent naturalist has, in the hurry of examination, overlooked the real form of these antennæ; otherwise I would feel well inclined to consider the character in question as affording sufficient grounds to separate generically the Irish from the Mediterranean amphipod.

posable articulation. First three segments of *abdomen*, each bearing a pair of biramous, natatory feet; remainder of abdomen consisting of one very large trunk, supporting anteriorly a pair of large, foliaceous, lobed appendages, and a pair of cylindrical false feet, and terminated posteriorly by two lamellar leaping organs, and an intermediate leaf-like lobe.

1. *C. terebrans*, Philippi.—Last three pairs of thoracic feet with the terminal joint foliaceous. *Hab.*—Coast of Trieste.

2. *C. destructor*, Mihi (provisional species).—All the thoracic feet, except the first two, terminated by a small hooked claw. *Hab.*—Irish coast.

The author, after entering into the details of the external anatomy of *Chelurus*, maintains the necessity of considering this genus as the type of a distinct family among the amphipoda, assuming as the grounds of the assigned rank the remarkable condition of the posterior region of the abdomen, the first, second, and third ring of this region being consolidated into one great trunk, bearing three pairs of *heteromorphous* appendages. Availing himself, therefore, of the characters derived from these considerations, the families of amphipodous crustacea were analytically arranged as follows:

			Family.
AMPHIPODA.	Fourth and fifth abdominal segments confluent; abdominal appendages of the fourth and fifth pair very different in form (heteromorphous).		CHELURIDÆ.
	Fourth and fifth abdominal segment distinct; abdominal appendages of the fourth and fifth pair nearly similar in form (isomorphous).	Mouth concealed by the maxillary feet.	GAMMARIDÆ.
		Mouth not concealed by the maxillary feet.	HYPERIDÆ.

Dr. Allman also read a paper on a new Genus and Species of *Entomostraca*.

The little animal which formed the subject of this communication inhabits the branchial sac of *Ascidia communis*, in which it may be observed swimming freely during, perhaps, all seasons of the year. The author describes it under the name of *Notodelphys ascidicola*, and assigns to it the following generic characters:

NOTODELPHYS,* *Mihi.*

Gen. Cha.—*Body* elongated; *head* scutiform, and bearing in front a solitary median eye; *antennæ* two, filiform, multiarticulate; *mouth* with a pair of mandibles, and surrounded by five additional pairs of appendages, of which the anterior, as well as the last two pairs, are prehensile; *thorax* having but two rings distinct, the anterior one being confounded with the head. Female with a large dorsal ovigerous receptacle immediately behind the last distinct thoracic ring. Locomotive *feet*, four pairs; biramous natatory, each pair having an intermediate plate; *abdomen* with about five rings, the last of which is terminated by two setigerous appendages.

Species unica.—*N. ascidicola.* *Hab.*—swimming freely in the branchial sac of *Ascidia communis*, Irish and English coasts.

The author, after entering into numerous details relative to the anatomy of the new entomostracon, and describing four distinct phases in its development, maintained that the ovigerous receptacle was formed by a peculiar development of the dorsal arches of a certain number of posterior thoracic rings, and that it was the true representative of the singular, elytroid, dorsal appendages of the thorax in *Anthosoma, Cecrops*, and certain other suctorial crustacea. He was, moreover, of opinion, that the genus *Notodelphys* presents us with a most interesting transitional form between the true entomostraca and the suctorial crustacea. Its perfect mandibulate mouth will at once place it with the former,—a position, indeed, which its highly developed natatory feet and active habits, as well as its general

* From νῶτος, *tergum*, and δελφύς, *matrix*.

physiognomy, would, in the first instance, suggest. The form, on the other hand, of the accessory oral organs, or maxillary feet, which are here constructed so as to constitute organs of attachment, as well as the singular development of the dorsal arch of the posterior thoracic ring, and the connexion of the feet of opposite sides, through the intervention of a large intercoxal plate,—a striking feature in the greater number of the suctorial crustacea, but not found in the true entomostraca,—unite with the semi-parasitical habits of *Notodelphys* in indicating an affinity not to be mistaken with the true suctorial tribes.

Dr. Allman read another paper on a new Genus and Species of *Tracheary Arachnidans.*

In this paper the author described a *Tracheary arachnidan* discovered by Dr. O'Brien Bellingham in the posterior nares of a seal (*Halichœrus gryphus*). It turns out, on examination, to be generally distinct from all the forms hitherto on record, and Dr. Allman assigned to it the appellation of *Halarachne Halichœri,* with the following generic characters:

HALARACHNE,* *Mihi.*

Gen. Cha.—*Palps* free, filiform; *mandibles* didactyle; sternal *lip*, bifid; *legs* with the last joint terminated by two hooks, and an intermediate three-lobed caruncle; *body* entire, elongated, sub-cylindrical, furnished anteriorly with a dorsal plate; *eyes*, none.

Species unica.—*H. Halichœri.* *Hab.*—Infesting the posterior nares of *Halichœrus gryphus,* Irish coast.

H. Halichœri possesses a very distinct tracheary system, opening externally by means of two spiracles, which are placed one at each side of the anterior extremity of the abdomen. The alimentary canal, just before its termination at the pos-

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* From ἅλς, *mare,* and ἀράχνη, *aranea.*

terior extremity of the abdomen, receives two long cæca, which may be traced forwards, one at either side of the body, till they terminate anteriorly by entering the basal joint of the first pair of legs. A large central nervous mass may be easily demonstrated. It is placed near the middle of the cephalothorax, of a somewhat stellate figure, with four pyriform lobes, from which nervous filaments pass off to the surrounding parts.

The reproductive system is very obscure; to it, however, may probably be referred a pair of tubular organs, placed in the anterior part of the abdomen, terminating at one end in a *cul de sac*, which contains a striated, curved body, and apparently opening at the other upon the surface. The larva is hexapod. The structure of the oral organs is similar to what occurs in *Gamasus*.

Sir Robert Kane read a paper on the composition of the essential Oil of the Laurus Sassafras, and of certain compounds derived from it.

This oil, which has a specific gravity of 1087, boils at 438° Fahr. By fusion with potash it is not acidified. Its composition was found to be expressed by the formula $C_{20}\ H_{10}\ O_4$, the same as that obtained by Saint Evre for the stearopten, with which it is consequently isomeric.

When this oil is treated with chlorine, a great deal of muriatic acid gas is produced and given off, and finally a very thick liquid is produced, which requires to be kept for a long time at a temperature of boiling water to free it from the excess of muriatic acid gas; the same may be done by washing with a small quantity of water of ammonia, and then exposure to a moderate temperature, when the chlorine compound can be obtained again, quite anhydrous. Its formula is found to be $C_{20}\ H_8\ O_4\ Cl_4$. When this body is heated to about 350° of Fahr., it suddenly and violently decomposes, gives off much muriatic acid, and deposits a large quantity of charcoal.

When the oil of sassafras is put into contact with bromine, great heat is evolved, much hydrobromic acid is evolved, and a solid product obtained, which, washed with alcohol until the excess of bromine and some secondary products are removed, is a brilliant white, crystalline powder. If this be gently heated it becomes yellowish pink. It fuses at about 300°, and soon after decomposes, giving off hydrobromic acid, and depositing charcoal. It is moderately soluble in ether, and by the cooling, or the evaporation of its ethereal solution, it can be obtained in brilliant oblique rhombs. The formula of this body is $C_{20}\ H_7\ O_4\ Br_5$.

When oil of sassafras is treated with nitric acid, very violent action is produced, and a large quantity of oxalic acid is formed; but by diluting the acid, and avoiding much elevation of temperature, a resinoid substance is obtained, of a pale yellow colour, soluble in alcohol and ether, slightly soluble in boiling water, and depositing on cooling. This body is quite destitute of acid characters, but it dissolves in alkalies, and may be precipitated by solutions of earthy or metallic salts, with the bases of which it forms definite compounds. The formula of this substance, which I term nitro-sassafras, is $C_{16}\ H_6\ O_8\ N$, and it is evidently formed by the action of

$$C_{20}\ H_{10}\ O_4 . + N_4\ O_{20}$$

giving

$$C_4\ O_6 . + H_4\ O_4 \text{ and } N_3\ O_6,$$

besides

$$C_{16}\ H_6\ O_8\ N.$$

When oil of vitriol is put in contact with oil of sassafras, an intense and beautiful crimson colour is produced. This is so remarkable as to constitute a very decisive means of recognising this essential oil. On studying this reaction more minutely, it is found that the oil of sassafras enters into a direct compound with oil of vitriol, and there is produced an intensely deep purple resinoid body, soluble in alcohol and ether, a little soluble in water, destitute of acid reaction, yet dissolving in alkalies, and being precipitated by the alkaline and earthy salts. On analysis this substance gives the formula

$$C_{20}\ H_{16}\ O_{16}\ S_2,$$

corresponding to

$$C_{20}\ H_{10}\ O_4 + S_2\ O_6 + H_6\ O_6.$$

That the sulphuric acid exists in this body, as such, is shown by the fact that, on treating it with pure potash, the whole of the sulphur separates as sulphate of potash. This substance is denominated sulpho-sassafras by Sir Robert Kane.

During the reaction of the sulphuric acid and oil of sassafras, in order to produce this body, very small quantities should be operated on, and the mixture kept perfectly cool, so that no effervescence, or violent reaction, should take place. If there be the slightest elevation of temperature, the substance in question is decomposed, sulphurous acid is evolved, and a resinoid black material is produced, which does not contain sulphuric acid, but for which, from the complexity of the reaction accompanying its formation, and the great difficulty of obtaining it absolutely pure, Sir Robert Kane does not at present wish to propose a formula.

M. Donovan, Esq., continued the reading of his paper on the Nature of the Agency which produces the Effects called Galvanic, Electro-magnetic, Magneto-electric, and Thermo-electric.

To support the opinion of identity, and to effect other objects, one of the chief of which is to show the absolute quantity of electricity with which matter is associated, Professor Faraday makes use of the following law, viz.: "If the same absolute quantity of electricity pass through the galvanometer, whatever may be its intensity, the deflecting force upon the magnetic needle is the same." The general method of proof of the truth of this law was to charge a Leyden battery with a certain number of turns of a powerful plate electric machine, varying the number of jars employed from eight to fifteen; to transmit the charge through a galvanometer, and to note the deflection. The charge was transmitted through various media, all intended to retard it more or less, and thus to affect the galvanometer with various intensities of electricity. In all cases the deflection of the needle was the same, no matter what the intensity: hence Faraday concluded that his law was proved.

To invalidate the inferences and proofs thus drawn, Mr. Donovan brought forward a number of considerations to show that, in all Faraday's experiments, the intensity of the electrical discharges employed was the same or commensurate with the deflection of the needle; and that it is the intensity of the electricity which passes through the galvanometer, and not its quantity, that determines the degree of deflection, the highest intensities producing the greatest deflection.

We should be cautious, therefore, Mr. Donovan observed, in applying Faraday's law: and if the law fail, the comparison drawn by him between the quantity of electricity produced during chemical action, to be immediately noticed, and that discharged from an electric machine, cannot be considered as proved. The comparison is this: Faraday found that by connecting a galvanometer with a wire of platinum and a wire of zinc, each being $\frac{1}{18}$ inch in diameter, and plunging their other ends $\frac{5}{8}$ inch deep in a mixture of four ounces of water and one drop of sulphuric acid, during $\frac{8}{150}$ of a minute, the deflection of the galvanometer amounted to exactly the same degree as when, in a former experiment, he passed a charge of common electricity through the galvanometer, amounting to thirty turns of the large plate-machine received in fifteen jars. Each turn of the machine afforded 300 or 360 dense sparks. Hence, according to the law, Professor Faraday inferred the equality of the two "absolute quantities" of electricity from the equal deflection of the needle in both cases. The double purpose of this experiment was still further to support the inferred identity of voltaic and frictional electricity, and to establish the estimate, already alluded to, of the enormous quantity of electricity with which matter is naturally associated.

In order to discover how far the experiment supports either of these positions, Mr. Donovan adduced counter-experiments, in which combinations of zinc and copper were acted on by dilute acid of different strengths until dissolved. The solution took place in different periods of time, and, consequently, the electricity evolved during any given period was unequal in quantity, in some cases very much so; yet in all of them the effect on the galvanometer was the same.

These experiments appear incompatible with Faraday's law of equal quantities of electricity producing equal deflections, irrespectively of other circumstances. Support is, consequently, withdrawn by them from his estimate of the enormous quantity of electricity naturally associated with matter.

The following note by Professor Mac Cullagh was read.

Let a surface A of the second order be represented by the equation

$$\frac{x^2}{\mathrm{P}_0} + \frac{y^2}{\mathrm{Q}_0} + \frac{z^2}{\mathrm{R}_0} = 1,$$

its primary axis being that of x. Through a given point S, whose coordinates are x', y', z', conceive three surfaces confocal with A to be described, and let P, P', P'' be the squares of their primary semiaxes. Then, if normals drawn to these surfaces respectively at the point S be the axes of a new system of coordinates ξ, η, ζ, and if we put

$$\mathrm{P} - \mathrm{P}_0 = k, \qquad \mathrm{P}' - \mathrm{P}_0 = k', \qquad \mathrm{P}'' - \mathrm{P}_0 = k'',$$

$$\frac{x'^2}{\mathrm{P}_0} + \frac{y'^2}{\mathrm{Q}_0} + \frac{z'^2}{\mathrm{R}_0} = f,$$

the equation of the surface A, referred to the new coordinates, will be

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = (f-1)\left(\frac{\xi_0\xi}{k} + \frac{\eta_0\eta}{k'} + \frac{\zeta_0\zeta}{k''} - 1\right)^2, \qquad (a)$$

where ξ_0, η_0, ζ_0 are the coordinates of its centre.

From the form of this equation it is evident, that if the surface be intersected by the plane whose equation is

$$\frac{\xi_0 \xi}{k} + \frac{\eta_0 \eta}{k'} + \frac{\zeta_0 \zeta}{k''} = 1, \qquad (b)$$

it will be touched along the curve of intersection by the cone whose equation is

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = 0. \qquad (c)$$

This mode of deducing, in its simplest form, the equation of a cone circumscribing a surface of the second order, is much easier than the direct investigation by which the equation (c) was originally obtained.

Let a right line passing through S intersect the plane expressed by the equation (b), in a point whose distance from S is equal to ϖ, while it intersects the surface A in two points, P and P′, the distance of either of which from S is denoted by ρ. Let the surface B, represented by the equation

$$\frac{\xi^2}{k} + \frac{\eta^2}{k'} + \frac{\zeta^2}{k''} = f - 1, \qquad (d)$$

be intersected by the same right line in a point whose distance from S is equal to r, the distance r being, of course, a semidiameter of this surface. Then it is obvious that the equation (a) may be written

$$\frac{1}{r^2} = \left(\frac{1}{\varpi} - \frac{1}{\rho}\right)^2;$$

so that, if ρ and ρ' represent the distances SP and SP′ respectively, we have

$$\frac{1}{\rho} = \frac{1}{\varpi} \times \frac{1}{r}, \qquad \frac{1}{\rho'} = \frac{1}{\varpi} - \frac{1}{r}; \qquad (e)$$

and therefore

$$\frac{1}{\rho} - \frac{1}{\rho'} = \frac{2}{r}. \qquad (f)$$

This result is useful in questions relating to attraction. For if A be an ellipsoid, every point of which attracts an external point S with a force varying inversely as the *fourth* power of the distance, and if the point S be the vertex of a pyramid, one of whose sides is the right line SPP′, and whose transverse section, at the distance unity from its vertex, is the indefinitely small area ω, the portion PP′ of the pyramid will attract the point S, in the direction of its length, with a force expressed by the quantity

$$\left(\frac{1}{\rho}-\frac{1}{\rho'}\right)\omega, \quad \text{or } \frac{2\omega}{r};$$

and, putting θ for the angle which the right line SP makes with the axis of ξ, the attraction in the direction of ξ will be

$$\frac{2\omega\cos\theta}{r}. \qquad (g)$$

Now, supposing the axis of ξ to be normal to the confocal ellipsoid described through S, it will be the primary axis of the surface B, which will be a hyperboloid of two sheets; and, the surface being symmetrical round this axis, it is easy to see, from the expression for the elementary attraction, that the whole attraction of the ellipsoid will be in the direction of ξ. Therefore, when the force is inversely as the fourth power of the distance, the attraction of an ellipsoid on an external point is normal to the confocal ellipsoid passing through that point.

Hence we infer, that if υ be the sum of the quotients found by dividing every element of the volume of an ellipsoid by the cube of its distance from an external point, the value of υ will remain the same, wherever that point is taken on the surface of an ellipsoid confocal with the given one.

The question of the attraction of an ellipsoid, when the law of force is that of the inverse square of the distance, has been treated by Poisson, in an elegant but very elaborate memoir, presented to the Academy of Sciences in 1833 (*Mé-*

moires de l'Institut, tom. xiii.) In the preceding year I had obtained the theorems just mentioned, by considering the law of the inverse fourth power; and, as well as I remember, they were deduced exactly as above, by setting out from the equation (a). But I did not then succeed in applying the same method to the case where the law of force is that of nature, probably from not perceiving that, in this case, the ellipsoid ought to be divided (as Poisson has divided it) into concentric and similar shells. This application requires the following theorem, which is easily proved:

Supposing A′ to be another ellipsoid, concentric, similar, and similarly placed with A, let the right line SPP′ intersect it in the points p and p', respectively adjacent to P and P′; then, if the direction of that right line be conceived to vary, the rectangle under Pp and P′p (or under Pp' and P′p') will be to the rectangle under SP and SP′ in a constant ratio.

Denoting the constant ratio by m, and combining this theorem with the formula (f), we have

$$\frac{\mathrm{P}p \times \mathrm{P}'p}{\mathrm{PP}'} = \frac{mr}{2}. \qquad (h)$$

Now let the two surfaces A and A′ be supposed to approach indefinitely near each other, so as to form a very thin shell, then ultimately P′p will be equal to PP′, and we shall have

$$\mathrm{P}p = \mathrm{P}'p' = \frac{mr}{2},$$

where m is indefinitely small. Therefore, if the point S, external to the shell, be the vertex of a pyramid whose side is the right line SP, and whose section, at the unit of distance from the vertex, is ω, the attraction of the two portions Pp and P′p' of this pyramid, which form part of the shell, will be equal to $mr\omega$. Hence it appears, as before, on account of the symmetry of the surface B round the axis of ξ, that the whole attraction of the shell on the point S is in the direction of that axis, and consequently (as was found by Poisson) in the direction of the internal axis of the cone whose vertex is S, and which circumscribes the shell.

To find the whole attraction of the shell, the expression

$$mr\omega \cos \theta \tag{i}$$

must be integrated. Let ϕ be the angle which a plane, passing through SP and the axis of ξ, makes with the plane $\xi\eta$; then

$$\omega = \sin \theta d\theta d\phi,$$

$$\frac{1}{r} = \sqrt{\left(\frac{\cos^2\theta}{k} + \frac{\sin^2\theta \cos^2\phi}{k'} + \frac{\sin^2\theta \sin^2\phi}{k''}\right)} \frac{1}{\sqrt{f-1}}.$$

When these values are substituted in (i), that expression may be readily integrated, first with respect to θ, and then with respect to ϕ.

It is evident that, by the same substitutions, the expression (g) may be twice integrated.

An investigation similar to the preceding has been given by M. Chasles, for the case in which the force varies inversely as the square of the distance (*Mémoires des Savants Etrangers*, tom. ix.) He uses a theorem equivalent to the formula (f), but deduces it in a different way.

From what has been proved it follows that, if v be the sum of the quotients found by dividing every element of the shell by its distance from an external point S, the value of v will be the same wherever that point is taken on the surface Σ of an ellipsoid confocal with the surface A of the shell.

Let Σ' be another ellipsoid confocal with A, and indefinitely near the surface Σ. The normal interval between the two surfaces Σ and Σ', at any point S on the former, will be inversely as the perpendicular dropped from the common centre of the ellipsoids on the plane which touches Σ at S. Hence, supposing the point S to move over the surface Σ, that perpendicular will vary as the attraction exerted by the shell on the point S, when the force is inversely as the square of the distance, or as the attraction exerted by the whole ellipsoid A on the point S, when the force is inversely as the fourth power of the distance.

When the point S is on the focal hyperbola, the integrations, by which the actual attraction is found in either case, are simplified, for the surface B is then one of revolution round the axis of ξ, and its semidiameter r is independent of the angle ϕ.

From the expression for the attraction of a shell we can find, by another integration, the attraction of the entire ellipsoid, when the law of force is that of nature. And thus the well-known problem of the integral calculus, in which it is proposed to determine directly the attraction of an ellipsoid on an external point, without employing the theorem of Ivory to evade the difficulty, is solved in what appears to be the simplest manner.

The preceding note having been read, Mr. Graves observed that the mention therein made, of the equation which represents so simply a cone circumscribing a given surface of the second order, reminded him of a circumstance which he thought it right to state; as that remarkable equation had been in circulation among geometers long before it appeared in print, and thus its origin, though generally known, was sometimes mistaken. Mr. Graves stated that he still retains a large part of the memoranda, in which he set down, from day to day, the substance of Professor Mac Cullagh's lectures, delivered in Hilary Term, 1836, and that the part preserved contains the equation in question (the equation (c) of the preceding note). In the memoranda it is deduced directly; that is, the equation of the cone is first given in the usual form, and is then reduced to the form (c) by a transformation of coordinates.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 65.

April 26th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

The Rev. Dr. Robinson presented an autograph of Euler.

Dr. Apjohn drew the attention of the Academy to some researches in Thermo-chemistry, with which he has been recently occupied.

"It is well known" (he observed) "to chemists, that muriatic acid gas and ammoniacal gas are absorbed by water in large quantity, and that during their absorption much heat is developed. The experiments which I have undertaken had for object, to submit to exact estimation the heat so evolved; and the results being, as far as my information extends, quite novel, and, I think I may add, theoretically and practically interesting, I am anxious to present them to the public with as little delay as possible.

"The apparatus employed in these experiments, though necessarily somewhat complex, proved admirably adapted to the purpose to which it was applied. It consisted of a flask, in which the gas was developed, of an exsiccation tube, and of a pair of tubes of small bore attached to the latter, by one of which the gas, when dried, could be conveyed into a cup containing mercury covered with a stratum of water, and, by the other, into a small cylinder of very thin copper containing the liquid to be heated. The cylinder was furnished with two necks, into one of which a very delicate thermometer was fitted by means of a cork, while the second received the tube by which the gas was to be introduced. By turning a stopcock attached to the latter tube, the gas could at any instant be conveyed into the copper cylinder, or excluded from it, an assistant at the same moment raising or depressing the vessel containing the mercury, so as to prevent or permit of the issue of the gas at this part of the apparatus. The temperature of the air at the instant of the admission of the gas, and the temperature of the liquid in the cylinder, having been accurately noted, and, in addition, the time m, which elapsed between the introduction and exclusion of the gas, as also the time m', which intervened between the latter manipulation and the second reading of the thermometer, data were obtained for calculating the total rise of temperature, and hence, for estimating the heat evolved by the gas as a consequence of its absorption. Thus, if n be the number of degrees by which w grains of water are heated, by the absorption of G grains of gas, $\frac{nw}{G}$ will be the heat extricated, i. e., the number of degrees that the caloric given out by the gas would heat an equal weight of water.

" Such is an outline of the general course pursued. The experiments required the greatest attention, with a view to the management of the apparatus, and the accurate mensuration of time and temperature; and I am not a little indebted to my scientific friend, Dr. Head, for the valuable assistance which he has rendered to me.

" The observations being made, other difficulties remained to be overcome. How, it will be asked, is the true value of n to be determined ? For the observation of temperature made at the close of the experiment is obviously less than the truth, for two reasons: 1st. Because it was not taken until m' minutes after the gas was cut off. 2nd. Because, during the m minutes that the gas was being absorbed, the copper cylinder was hotter than the surrounding air, and, therefore, constantly losing heat. After a good deal of reflection on the subject, it finally occurred to me to adopt the following method of calculating the necessary corrections.

Having introduced into the copper cylinder, furnished with its thermometer, 4.66 cubic inches of water, very nearly the quantity used in all the experiments, and raised the whole to 93.4° (the air being 53.7°), the temperatures were noted at intervals of a minute, until the thermometer indicated 66.5°. To those I then applied the expression for the velocity of cooling, deducible from the Newtonian law, viz.:

$$v = \frac{\mathrm{T}(h.l.\mathrm{A} - h.l.\mathrm{T})}{t},$$

A being the excess of temperature of the cylinder over the air at any instant, and T the excess after t minutes, and thus obtained the velocities of cooling corresponding to the successive values of T separated by intervals of a minute. These being reduced to a tabular form, furnish, by mere inspection, the means of applying the first of the two corrections already indicated, or of ascertaining the temperature which the thermometer would show, had it been read at the instant the gas was cut off, or m' minutes previous to the actual time of observation.

" But the rise of temperature actually produced is less than that which we are in search of, in consequence of the cooling power exercised by the joint influence of radiation and atmospheric contact during the time m. The method I have adopted of determining the effect of refrigeration, and which, as far as I am aware, has not been previously used, I shall now explain.

" The velocity of cooling at any instant while the gas is passing in, is, adopting the Newtonian law, proportional to the rise of temperature at such instant. But the gas having been always introduced in my experiments at a uniform rate, the rise of temperature is proportional to the time. Hence the velocity of cooling at any instant is proportional to the time. Such being the case, the well-known theorems, which relate to the motion of a material point actuated by a constant force, are here strictly applicable; and, amongst the rest, that the space (number of degrees) through which the cylinder cools in the time m, is equal to half the rectangle under the time and the last acquired velocity. This theorem, in fact, immediately gives the correction in question, not, I may observe, in an approximate, but in a complete manner, and, in practice, I have every reason to be satisfied with it.

"In what precedes it will be seen, that I have employed the Newtonian law of cooling, which the researches of Dulong and Petit have shown not to represent observations with rigour, except when the excesses of temperature are small. My results, however, are not on this account appreciably less accurate, for the thermometer which I employed only read to tenths, and the divergence of the Newtonian law from the truth, within the range of my experiments, is only observable in the second decimal place.

"Having explained every thing necessary to enable the Academy to judge of the accuracy of my results, I shall now state the numbers at which I have arrived:

	Equal weights.	An atom.
Ammoniacal gas passed into water,	940°	940°
Muriatic acid gas passed into water,	885°	1900°

Weight for weight, then, ammonia gives out more heat than muriatic acid; but an atom of the latter gives out almost exactly the double of the heat evolved by an atom of the former.

"The number for ammonia, it will have been observed, does not materially differ from that for aqueous vapour of maximum density at 212°, the latter having been fixed, by the recent experiments of Brix, at 972°. The numbers, however, are not strictly comparable. For, the heat evolved by the steam is truly its latent heat, or is due solely to its change of state, while the caloric evolved by ammoniacal gas, or muriatic acid gas, undoubtedly consists of two distinct parts, viz., of the heat of *compression* of these gases, and of that due to the chemical action exerted between them, supposed in the liquid condition, and water.

"Though I did not entertain any doubts as to the accuracy of the results just stated, it was obviously desirable to resort to some experiments, if any such could be devised, by which they could be tested; and none appeared better suited to the purpose, than to pass ammoniacal gas into liquid muriatic acid, and muriatic acid gas into liquid ammonia, and determine, by the means already explained, the heat developed in each case. Such experiments were accordingly performed, care being taken that the gas introduced did not in amount exceed what would be necessary for saturating the opposite principle contained in the liquid, and subjoined are the numbers to which they have conducted:

Heat of ammoniacal gas passed into liquid muriatic acid,	2523°
Heat of muriatic acid passed into liquid ammonia, . . .	1527°

Deducting from the former 940°, and from the latter 885°, the remainders are 1583° and 642°. Now, according to Andrews (Transactions, Royal Irish Academy, vol. xix. part 2), .129 of a gramme of ammonia, in the form of aqua ammoniæ, in combining with liquid muriatic acid, evolves sufficient heat to raise 31.09 grammes of water 5.58°, from which it is easy to calculate that it would raise an equal weight of water 1344°. But the heat evolved by equal weights of ammonia and muriatic acid, in combining with each other, are obviously reciprocally proportional to their atomic weights, so that, 1344 being the number for ammonia, $1344 \times \frac{17}{36.5} = 626$ will be the number for muriatic acid. The following, therefore, are the comparative results, the numbers in first column being the *differences;* those in the second, the heat of the chemical action between aqueous ammonia and muriatic acid, as inferred from the experiments of Andrews; and those in the third, the excesses of the former over the latter:

	Differences.	Andrews.	
Ammoniacal gas into aqueous acid, .	1533°	1344°	239°
Muriatic acid gas into aqueous ammonia,	642°	626°	16°

"A glance at these numbers is sufficient to show that, when muriatic acid gas is passed into aqueous ammonia, the heat extricated exceeds that obtained when the gas is passed into water, by almost exactly the heat of the chemical action of aqueous muriatic acid and ammonia; while, as respects the case of ammoniacal gas passed into water and aqueous acid, this equality is wanting, the estimate by difference exceeding the direct determination by 239°.

"This is a very curious result, and I was so startled by it, that it was not my intention to give publicity to these experiments until I had more frequently repeated them, and executed others, which I had planned with the view of throwing additional light upon the subject of my inquiry. In entering, however, upon this new investigation, I had the misfortune to lose, by an accident, both my thermometers, and as I do not anticipate being able to return to it for a considerable time, I gladly avail myself of the permission of the Council, to submit these researches, in their present state, to the judgment of the Academy. I entertain, indeed, a very confident hope, that the numbers at which I have arrived will eventually be found to be very close approximations to the truth. Assuming such, for a moment, to be the case, and, in addition, that the results of Dr. Andrews, in relation to the heat arising from the chemical action of aqueous ammonia and aqueous muriatic acid, are rigorously correct, it has occurred to me that my results admit of the following interpretation:

"Let $a + b =$ heat given out by ammoniacal gas when absorbed by water, a representing the heat of *compression*, and b that of the chemical action between compressed or liquid ammonia and water. When ammoniacal gas is passed into liquid muriatic acid, the heat represented by b will be wanting, and that actually developed will be $a + c$, c being the chemical heat determined by Andrews. The difference of these, therefore, or $a + c - (a + b)$, will be $c - b$. But this difference we have actually found to be greater than c. b must, therefore, have a negative sign; or, in other words, when *compressed* ammonia is brought into contact with water, *cold*, not heat, is the result.

"This may appear a very paradoxical supposition, but I am not aware of any fact which would prevent us from entertaining it; and the great expansion which water experiences when absorbing ammoniacal gas, even confers upon it some degree of probability. I may add, that this view of the matter gives us 239° as the value of b, and suggests an experiment, which, though difficult, it would not be impossible to perform, and the result of which would at once elucidate completely the subject under consideration."

The Rev. Dr. Todd exhibited an ancient Irish brooch, belonging to the Rev. Richard Butler, of Trim.

Mr. Petrie having been called on for his opinion respecting the style, workmanship, and age of this beautiful relic of antiquity, stated, that he considered it as the most elegant specimen of Irish workmanship *in silver* which he had hitherto seen, but believed its age to be not so great as that of most, or perhaps any, of the brooches in the Museum of the Academy, or the other collections in Dublin; its minor ornaments being peculiarly those characteristic of the early portion of the twelfth century, to which period he referred it; though the type of many of its general forms might be found in earlier examples.

Mr. Petrie then proposed that the thanks of the Academy should be given to the Rev. Richard Butler, for his kindness in sending this brooch for the inspection of the Meeting. The thanks of the Academy were accordingly voted to Mr. Butler.

Mr. Ingram read the following note on certain Properties of the Surfaces of the Second Degree.

"Mr. Salmon, Fellow of Trinity College, has given a mode of generating certain of the surfaces of the second degree, which is in a remarkable way supplementary to the *modular* method of Professor Mac Cullagh, and which has been called, for distinction's sake, the *umbilicar* method. In it the surface is had as the locus of a point moving so that the square of its distance from a fixed point is proportional to the rectangle under its distances from two fixed planes. Out of this generation arise many highly interesting properties of the surfaces in question, to some of which it is the object of the present communication to call the attention of the Academy.

"The fixed point is called the Focus of the surface, the two fixed planes the Directive Planes, and their line of intersection the Directrix.

"1. Two right lines, reciprocal-polars with relation to the surface, meet a directive plane in two points such that the vectors drawn to them from the focus are at right angles.

"2. A similar theorem holds for two conjugate tangents at any point of the surface.

"3. Two right lines, reciprocal-polars with relation to the surface, seen from the pole of a directive plane, appear to cut at right angles.

"4. Let a cone be described, passing through two plane sections of the surface; it will intersect a directive plane in a certain conic: let a second cone be described, passing through this conic, and having its vertex at the focus; the cyclic planes of this latter cone will pass respectively through the two right lines in which the planes of the two sections meet the directive plane.

"5. Hence, through a right line, situated in a directive plane, draw two planes cutting the surface in two conics; a cone passing through these two curves intersects that directive plane in a certain conic: now let a cone be described, passing through this conic, and having its vertex at the focus; this latter cone will be one of revolution, and its principal plane will pass through the right line assumed in the directive plane.

"6. Hence, a cone enveloping the surface intersects a directive plane in a certain conic; a cone passing through this conic, and having its vertex at the focus, is one of revolution; and its principal plane passes through the right line in which the plane of contact of the enveloping cone meets the directive plane.

"7. A plane curve is traced on the surface, and through it a cone is described, having its vertex at the pole of a directive plane; this cone cuts the directive plane in a certain conic: now let a second cone be described, passing through this conic, and having its vertex at the focus; the latter cone is one of revolution, and its principal plane passes through the right line in which the plane of the original curve intersects the directive plane.

"8. If a cone be described, having its vertex at the focus, and passing through a plane section of the surface, the cyclic planes of this cone will pass respectively through the two right lines in which the plane of the section meets the two directive planes; and the *directive* axis of the cone will therefore be the right line drawn from its vertex to the point where the directrix is cut by the plane of the section.

"9. Hence, any plane passing through the directrix intersects the surface in a curve such that the cone passing through it, and having its vertex at the focus, is one of revolution;

and the principal plane of that cone passes through the directrix.

"The above properties are true for *every* point on the *umbilicar* focal of the surface, with the directive planes and directrix corresponding to that point. When the two directive planes coincide, these theorems, suitably modified, reduce to known properties of the *non-modular* surfaces of revolution. For that particular case they have been demonstrated by M. Chasles, in the Transactions of the Royal Academy of Brussels (*Nouveaux Mémoires*, tom. v.) In the general shape in which they are, I believe, now for the first time* given, they appear to me of sufficient elegance to merit the attention of geometers."

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* "Since the above note was read, my attention has been directed to a paper 'On the Focal Properties of Surfaces of the Second Order,' by Dr. Booth, in the Philosophical Magazine for December, 1840. In that paper he considers as analogous to the foci in conic sections four points, which he calls the foci of the surface, situated, two by two, on the umbilical diameters, at distances from the centre equal to each other and to $u\epsilon$, where u is the length of the umbilical semi-diameter, and $\epsilon^2 = \frac{a^2 - c^2}{a^2}$ $(a > b > c.)$ The polar planes of these points he terms the 'directrix planes' of the surface, and of these planes the two which intersect in a directrix of the principal section (a, b) are 'conjugate directrix planes.' The foci of the same section (a, b) he calls the 'focal centres' of the surface. These definitions being premised, he states the theorem, that if from any point of the surface perpendiculars be let fall on two conjugate directrix planes, the rectangle under those perpendiculars is to the square of the distance of the point from the corresponding focal centre in a constant ratio. But he does not observe the fact which gives the umbilicar generation its chief interest and value, namely, that the 'focal centre' may traverse the focal curve on which it lies, the 'directrix planes' changing along with it, while the generated surface remains unaltered. He then proceeds to state several properties of his 'focal centres' and 'directrix planes,' and among them I find those which I have marked (1), (2), (8), and (9). But these theorems are given by him only for his two 'focal centres' and his four 'directrix planes;' whereas they are really properties of every point on the umbilicar focal of the surface, and the directive planes corresponding to such point."

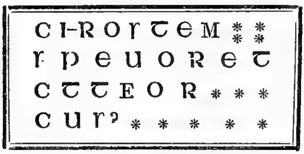
The following memorandum respecting some ancient Inscriptions in Scotland, by Mr. John Ramsay, of Heading Hill, Aberdeen, was read.

"Towards the end of January last, my attention was directed to an inscription on a portion of what was once the cross of St. Vigean, a parish of Forfarshire, contiguous to that of the town of Arbroath. Through the medium of a friend, I was permitted to inspect a handsome lithograph of this interesting monument of antiquity, executed, I understand, under the auspices of Patrick Chalmers, Esq., of Auldbar, a gentleman not less skilled than zealous in archæological pursuits. The cross referred to is thus mentioned in the *Statistical Account of the Parish of St. Vigean* (1845), written by the parochial clergyman, the Rev. John Muir: 'In the churchyard there formerly stood a large cross over the grave of some person of eminence, richly carved in hieroglyphical figures of the kind found on sepulchral stones in some other places of Scotland. The cross has been long ago demolished, but the stalk remains, *with characters at the base hitherto undeciphered*.'

"I entirely concur in the opinion of the reverend writer, that the cross in question was monumental. Such sepulchral monuments were common about the period to which the cross of St. Vigean seems to belong. A comparison of some of its ornaments with those of other crosses of the same kind, suggests that it was the production of the latter part of the tenth century. The peculiar and beautiful interlacery in the compartment immediately above the inscription, and on one of the *faces* of the cross, is of kindred character with that which is exhibited in similar monuments of the same era, sketches of which are given in Mr. Petrie's valuable Essay on the Ecclesiastical Architecture of Ireland. I observe that it is stated, in the *Account* of the parish already referred to, that St. Vigean lived in the latter part of the tenth century; and that he had his residence in the neighbourhood of the spot where the cross formerly stood. 'His original chapel and hermitage were at

Grange of Conan, where there are a small grove, and foundations of a chapel, and also a most copious fountain, which preserves his name. Three or four acres of land contiguous to these are by tradition held as belonging to the chapel.'

" May it not, then, be not unreasonably inferred, that this monument marked the place of St. Vigean's sepulture? This, of course, is merely a conjectural suggestion,—at all events the cross is evidently the monument of some person of distinction. Of the personal history of this saint I know nothing; but I think it not improbable, that he was of Irish origin or connexion. From the similarity to like monuments in Ireland, of the cross referred to, and of others in Forfarshire, and the adjoining districts, not to mention the round towers at Abernethy and Brechin, it is evident that *Irish* missionaries were intimately connected with those parts. The inscription, according to my copy of it is as follows:



" The above inscription appears to be partly in the old Irish, and partly in the Roman character. I take the alphabet of the former from *Armstrong's Gaelic Dictionary*. This mixed character of the inscription is quite common in monuments belonging to a period prior to the distinctive fixation of alphabets, established in later times, particularly after the introduction of printing. Supposing, as is not improbable, that the *aboriginal* alphabets of Britain and Ireland had been lost sight of in the darkness attendant on social convulsion, so remarkably coincident either with the extermination of the order, or the decay of the influence, of the pagan priesthood; a renewed acquaintance with the use of letters was only to be derived from *two* sources, either from the Romans, or from the early Christian missionaries.

" Hence, I believe, it comes to pass, that the *most ancient* native inscriptions in *Britain* (see *Borlase*) are in the Roman character. Subsequently, some letters were borrowed from the Greek, by the Christian missionaries, owing to their acquaintance with the original language of the New Testament. In all writings and inscriptions, then, of the earlier mediæval times, we may naturally expect a mixture of Roman and Greek characters. Hence, the strong similarity of the old Irish to the old Anglo-Saxon.

" This premised, I proceed further to observe, that the inscription above noted seems to be only *part* of that which originally belonged to the cross of St. Vigean. I conjecture, for reasons which will afterwards more clearly appear, that the *first* part must have been cut on the *top* of the cross, above the interlacery, which is now lost. It was not unusual to divide such inscriptions into two parts. An instance of such arrangement is to be found in Borlase's *Antiquities of Cornwall*, pp. 399, 400. Further, in monuments of the age to which the cross of St. Vigean belongs, the *beginning* of the inscription was usually prefixed with a small cross, either so (+), or so (⊕); but this is wanting in the portion of the inscription referred to. Taking all these circumstances into account, I venture to *restore* the inscription (for it has evidently *suffered*) as follows :

C h R O ꞅ Ꞇ ɛ M *P U*
ꞅ ꝺ ɛ U O R ɛ Ꞇ
ɛ Ꞇ Ꞇ E O R *P R O*
C U I^s *A N I M A*;

that is, using Roman capitals :

C H R O S. T E M *P U*
S. D E V O R E T
E T. T E. O R. *P R O*
C U I^s *A N I M A*.

" I do not pretend to give the *original letters* or *contractions*, which time or accident seems to have effaced from the inscription. It is impossible to determine what selection the stone-cutter may have made in his drafts on the Roman and Irish alphabets. At all events, he must have so managed matters, as to confine his work within the prescribed limits.

" I translate the above as follows:

" '*O! Cross! Time may destroy thee, too! Pray for his* (the person *named* in the *first* part of the inscription) *soul!*'

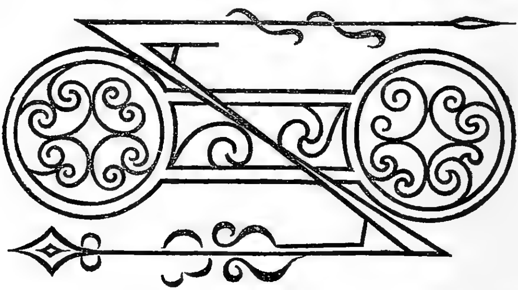
" Now, there is a singularity in this inscription: the first word (*Chros*) is *Gælic*, and the rest are *Latin*. How may this be accounted for? The *ancient* Gælic term for a cross is *cros*. The *vocative* is formed by *aspirating* the nominative into *chros*. To write the Latin *crux* with the *Irish* character was *impossible*. The alphabet has no *x*, and the sound of this letter is foreign to the Gælic language. Hence, instead of Sa*x*enach, we have *Sassenach*. Thus there was an obvious *necessity* for using the vocative of the Gælic word, *cros*.

" I conjecture that, as was usual in such cases, the first part of the inscription contained the name of the person to whose memory the cross was erected. Thus, the part above deciphered would be a very natural sequence. It is marked by all that touching simplicity which is characteristic of inscriptions on monuments of the same era, noticed by Mr. Petrie, whose accurate and tasteful researches have thrown so much light on some of the darkest and most interesting points of Gælic antiquities.

" Of the devices, animals, &c., on the back of the cross, I shall not here speak, as my present business is with the inscriptions. Suffice it to say, that I think I could *prove* that some of these devices are borrowed from monuments, still extant in Scotland, the age of which exceeds that of the cross by many centuries.

" The next inscription which I shall notice is that on an ancient monument in the Church of Fordun. Fordun is a parish of Kincardineshire, the county immediately north of Forfarshire. Kincardineshire is sometimes called the Mearns, and its people, 'the men of the Mearns.' In the old Irish Annals they are called '*Viri na Moerne.*' There are many interesting particulars connected with the parish of Fordun. John de Fordun, author of the *Scotichronicon*, was either a native of it, or resided there, when he wrote his History of Scotland. It was the native parish of George Wishart the Scottish martyr; of the eccentric Lord Monboddo; and of Beattie, author of 'the Minstrel.' Further, it was the *locale* of the famous shrine of St. Palladius. The remains of *Paldy* Chapel are still standing; there is still *Paldy*, or *Pady* Fair; and there is a well in the minister's garden, called St. Palladius' Well. Some will have it, that the famous Saint actually lived, died, and was buried here. I am not sufficiently acquainted with our early ecclesiastical history to give any opinion on the subject; but I am disposed to agree with those who think that Pady Chapel was built, not by the Saint, but by some of his Irish disciples, who came to this part of Scotland, probably with some of his relics. His mission certainly was to Ireland, '*ad Scotos in Christum credentes.*' The earlier Christian churches in this quarter were certainly *Columban;* but some may have been of *Ninian*, or *Palladian* origin. Even at the early period referred to, the spirit of ecclesiastical rivalry seems to have been at work. At all events the chapel of St. Palladius was always accounted the mother church of the Mearns.

"But to come to the matter in hand: the ancient monument to which I refer (some account of which was first given by the late Professor Stuart, of Marischal College, Aberdeen), was first observed upon taking down the old *church* of Fordun, some sixty years ago. 'It had been placed horizontally as a base for the pulpit to rest on, and was considered of so little consequence, as to be thrown aside for many years into the old chapel of St. Palladius, hard by.' This old church of Fordun was so old, that it was *new roofed* about 360 years ago. After lying neglected for a long time, the old stone attracted the attention of the parish minister, who had it cleaned, and a drawing of it taken. The material is a very coarse free-stone. The dimensions, five feet one inch in length, by two feet eleven inches broad, thickness fully four inches. It is carved on one side only. The emblematical devices are three figures on horseback, a greyhound, a wild boar, a serpent, or dragon ; and the peculiar *spectacle* device



like that on other old monuments in the north of Scotland. To these I do not refer at present; my business being with the inscription. Professor Stuart makes it probable that this monument commemorates the assassination of King Kenneth III., in the year 994. His Majesty is said, by our historians, to have been assassinated at the instigation of Finelè, "daughter," says the Professor, " of Cruchnè, Maormor of Angus." This should be *the Cruithne* (Pictish) Maormor of Angus. The royal residence was at Kincardine. In the neighbourhood are *Strath-Finella* and *Den-Finella*. In this case, history is confirmed by tradition and topographical etymology. A drawing of the fragmentary inscription will be found in the *Archæologia Scotica*, vol. ii. p. 315.

"There has been another line, if not more, above what remains, and I do not pretend to be able to decipher that with certainty; but it strikes me that it *looks* like *Kenkardin* or *Kinkardin*, the name of the royal residence. It is to be observed that the *costume* of the human figures on this monument is exactly the *same* as that of the only human figure on the cross of St. Vigean, belonging, as I conjecture, to the same period.

" The next inscription which I shall notice is that on one old monument which was found some years ago in the parish of Insch, Aberdeenshire. The dimensions of the stone are six feet by one foot eight inches. The inscription runs along the central *length* of the stone. It is—

This is evidently :

Orate pro Anima Radulphi Sacerdotis.

The characters shew the influence of Anglo-Saxonism at the period when the monument was executed. There are good grounds for believing, that it was placed over the grave of *Radulph*, Bishop of Aberdeen, who died in 1247.

" I have been induced to give the above specimens of ancient inscriptions in Scotland, in the hope that they may incite the able and zealous archæologists of Ireland to direct their attention to the subject. There are other inscriptions in this country of, perhaps, greater interest, to which I forbear to refer; partly because I confess my entire ignorance of their nature, and partly because I believe they have already attracted the notice of members of your Academy, from whom, if from any, the interpretation of those inscriptions may be expected.

" Between the antiquities of Ireland, and those of the north of Scotland, there are many points of interesting connexion. The aborigines of both countries belonged to the same great family of the human race ; both remained *almost* equally intact by the ambition of ancient Rome ; neither had to bow the neck to the yoke of the old Saxons ; both were harassed by the Danes ; and while the Picts were compelled, partially, to succumb to warriors of Irish descent, it was to missionaries of Irish origin that they owed their first acquaintance with the Gospel of Peace! In both countries are still to be found many memorials of aboriginal times, which had once their resemblances in England, but which have there disappeared under "the tramplings of three conquests," and the march of modern improvement. I refer, particularly, to those remote times when Druidism bore its mystic sway. Its usages yet linger in customs of popular superstition, although oblivion has long since fallen on the meaning attached to them by a crafty, powerful, and domineering hierarchy. Many an age has passed since its oracles became dumb; but the nomenclature of its religious creed is still employed to express, by the unwitting Gael of the present day, some of the mysteries of his purer faith! We have still the mysterious "temple," with its massive "cromlech," the poetry of the solitary moor, and seldom-trodden height,—many of which have been protected by our landed proprietors, with commendable feeling, disregarding not the protest against eviction of those *adscripta glebæ*, and refusing to abandon to

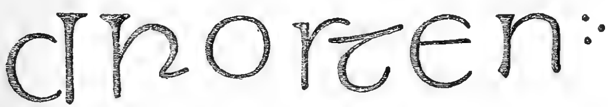
'Hands more rude than wintry winds,'

relics which have braved the buffetings of countless storms."

Mr. Petrie remarked, that he thought the Academy should feel great pleasure at every effort made by the Scottish antiquarians to illustrate their antiquities, which were so intimately connected with those of Ireland; and that they should be grateful to Mr. Ramsay for communicating to their Institution his very ingenious attempt to decipher and explain the remarkable inscription at St. Vigean's.

Mr. Petrie regretted, however, being obliged to state, that he could not, by any means, concur either in Mr. Ramsay's reading of this inscription, or his conclusions as to its age. He did not believe that there were any abbreviations of words, or varieties of language or alphabetic writing, in it, such as

Mr. Ramsay supposed; or that the inscription was in any way imperfect, or originally connected with another on the same cross, now destroyed. Mr. Petrie further stated, that, having been kindly supplied with two rubbings of this inscription, one from Mr. Chalmers of Auldbar, through his friend Mr. Worsaae, and the other from Mr. C. Innes, the able Secretary to the Royal Scottish Society of Antiquaries, he had given a good deal of attention to them, and had been so far successful as to read with certainty nearly one-half of it. As he still hoped, however, to be able to master the whole, and to present the results to the Academy, he would, on the present occasion, content himself with remarking, that the inscription was unquestionably one connected with the Pictish history; and that, as might be expected in a country where the literature had been, confessedly, entirely in the hands of Irish ecclesiastics, the letters of which it was composed were wholly of that description usually called *Irish*, though, in reality, only the corrupt form of the Roman alphabet, general in Europe during the fifth and some succeeding centuries. In proof of these conclusions he exhibited a tracing from the rubbing of the first line of the inscription (of which the following is a copy), and which plainly gives the name Drosten.



This, Mr. Petrie remarked, was peculiarly a Pictish name, and was equally connected with the ecclesiastical as with the regal history of Scotland. It was a diminutive of the name Drust, so common in the list of the Pictish Kings, and was that of a Pictish ecclesiastic who flourished in the sixth century, and who was spoken of in St. Adamnan's Life of St. Columba. Whether, however, the St. Vigean's monument or cross was erected to this Drosten, or one of the others of later date recorded in Pictish and Irish authorities, Mr. Petrie would not then offer an opinion, though he had no hesitation in stating that, from the forms of the letters, he had no doubt that the inscription was of an age some centuries earlier than that ascribed to it by Mr. Ramsay.

An extract was read from a letter addressed to Sir William Hamilton by Lieutenant Steevenson, on a mode of ascertaining the general state of the weather for any spring and summer from the general state of the preceding winter.

The writer is of opinion that the prevalence of westerly winds and rain, during our wet summers, arises from the presence in the Atlantic Ocean of a greater quantity than usual of ice, brought southwards by currents from the Arctic regions.

Robert Ball, Esq., Treasurer, read an abstract of the Accounts of the Academy, for the Year ending 31st of March, 1847, which was ordered to be printed in the Proceedings. (See *Appendix*, No. X.)

DONATIONS.

Comptes rendus Hebdomadaires des Séances de l'Académie des Sciences. From 8 Feb. to 8 March, 1847. Presented by the Academy.

Philosophical Transactions of the Royal Society of London, for 1846. Part IV.

Proceedings of the Royal Society (1846). No. LXVI.

List of Members of the Royal Society. Nov. 30, 1846. Presented by the Society.

Astronomical Observations made at the U. S. Naval Observatory, Washington, during the Year 1845. *By M. F. Maury, Lieut. U. S. Navy.* Presented by the Author.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 66.

May 10th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Edward Barnes, Esq., and Henry Freke, Esq., were elected Members of the Academy.

The Rev. Charles Graves read the following note on the development of a function in factorials of the variable upon which it depends.

The process of integration for factorials being simpler than that for powers, in the inverse calculus of finite differences, we sometimes have occasion to resolve a proposed function of x into a series of the form

$$A_0 + A_1 x + A_2 x (x-1) + A_3 x (x-1) (x-2) + \&c.;$$

and we may readily determine the coefficients A_0, A_1, A_2, A_3, &c., by making x successively equal to 0, 1, 2, 3, &c. In this way Sir John Herschel, in his *Collection of Examples of the Applications of the Calculus of finite Differences,* has solved the more general problem of developing a function $F(x)$ in a series of factorial terms of the form

$$A_0 + A_1 (x-f_1) + A_2 (x-f_1) (x-f_2) + \&c.$$

$F(x)$ being any function whatever of x, and f_1, f_2, &c., particular values of any other function $f(x)$, corresponding to the values 1, 2, &c., of x. The two methods of developing $F(x)$ in a series of factorials, which are here noticed, seem to have advantages over the method of indeterminate coefficients, in being more simple and direct, and in manifesting more clearly the law which the coefficients A_0, A_1, A_2, A_3, &c., follow. They furnish, at the same time, interesting examples of the use of separating symbols of operations from their operands; and it is for this latter reason, rather than on account of any novelty in the results arrived at, that they are now submitted to the notice of Members of the Academy.

I. Employing u* to denote the operation which changes $F(x)$ into $F(x+1)$ we are entitled to write

$$F(x+n) = u^n F(x) \text{ and } F(n) = u^n F(o).$$

But u is known to be equivalent to $1+\Delta$; we may therefore write

$$F(n) = (1+\Delta)^n F(o);$$

or, with the right-hand member of the equation developed,

$$F(n) = F(o) + \frac{\Delta F(o)}{1} n + \frac{\Delta^2 F(o)}{1.2} n(n-1) + \&c. \quad (1)$$

A particular case of this theorem is commonly given in treatises on the calculus of finite differences, viz.:

$$x^n = \frac{\Delta o^n}{1} x + \frac{\Delta^2 o^n}{1.2} x(x-1) + \&c.$$

And indeed the theorem itself may be derived from the fundamental expression for u_{x+n} by making $x = 0$.

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* Arbogast, in his *Calcul des Derivations*, has appropriated the letter E to this use, as being the initial of the word *Etat;* and in so doing he has been followed by recent writers. But against this usage it may be objected that the symbol E is now devoted to a different office in the theory of elliptic functions. And, on the other hand, there seems to be a peculiar fitness in denoting by u that operation which changes u_x into u_{x+1}.

II. If we take the differential coefficient of x^n, and multiply it by x, the result will be $x^n n$; that is to say,

$$x^n n = x \frac{d}{dx} x^n ;$$

and as a consequence of this equation, we shall likewise have,

$$x^n \,\mathrm{F}\,(n) = \mathrm{F}\left(x \frac{d}{dx}\right) x^n. \qquad (2)$$

In the right-hand member of this equation, let us put $1 + x - 1$ in place of x; and then expand by the binomial theorem; the result will be

$$x^n \mathrm{F}(n) = \mathrm{F}\left(x\frac{d}{dx}\right) x^0 + \frac{\mathrm{F}\left(x\frac{d}{dx}\right)(x-1)}{1} n + \frac{\mathrm{F}\left(x\frac{d}{dx}\right)(x-1)^2}{1.2} n(n-1)$$
$$+ \&c. \qquad (3)$$

The coefficient of $n\,(n-1)\ldots(n-m+1)$ in this development will be

$$\frac{1}{1.2..m}\left\{x^m \mathrm{F}(m) - m x^{m-1} \mathrm{F}(m-1) + \frac{m(m-1)}{1.2} x^{m-2} \mathrm{F}(m-2) - \&c.\right\}$$

and, if we now suppose $x = 1$, we shall have the development of F (n) in the desired form; the coefficient of the factorial $n\,(n-1)\ldots\ldots(n-m+1)$ being

$$\frac{1}{1.2\ldots m}\left\{\mathrm{F}\,(m) - m\,\mathrm{F}\,(m-1) + \frac{m\,(m-1)}{1.2}\,\mathrm{F}\,(m-2) - \&c.\right\}$$

Comparing the two expressions (1) and (3) we find, as we ought to do,

$$\Delta^m \,\mathrm{F}\,(o) = \mathrm{F}\,(m) - m\,\mathrm{F}\,(m-1) + \frac{m\,(m-1)}{1.2}\,\mathrm{F}(m-2) - \&c.,$$

a formula which might be obtained directly by making $x=0$ in the fundamental equation of the calculus of finite differences,

$$\Delta^m u_x = u_{x+m} - m u_{x+m-1} + \frac{m\,(m-1)}{1.2} u_{x+m-2} - \&c.$$

By the aid of the symbol $\left(x\frac{d}{dx}\right)$ we may obtain another interesting development. In virtue of the equation (2) we have

$$e^{hx\frac{d}{dx}}\, x^n = e^{hn}\, x^n = (e^h\, x)^n.$$

It is plain, then, that the symbol

$$e^{hx\frac{d}{dx}}$$

operates on any function of x by changing x into $e^h\, x$; that is to say,

$$\text{F}\,(e^h\, x) = e^{hx\frac{d}{dx}}\, \text{F}\,(x);$$

whence, developing the right hand member, we get

$$\text{F}\,(e^h\, x) = \text{F}\,(x) + \frac{\left(x\frac{d}{dx}\right)\text{F}\,(x)}{1}h + \frac{\left(x\frac{d}{dx}\right)^2\text{F}\,(x)}{1.2}h^2 + \&\text{c.} \quad (4)$$

As Taylor's theorem gives the altered state of F (x), after x has received an increment h, so the theorem just announced exhibits the new value of F (x) after x has been multiplied by a number whose logarithm is h; the series in both cases being arranged according to ascending powers of h.

In executing the operations indicated in the development (4) it must be remembered that $\left(x\frac{d}{dx}\right)^2$ is not equivalent to $x^2\frac{d^2}{dx^2}$ but to $x\frac{d}{dx}\, x\frac{d}{dx}$; and so on for the other powers of the symbol. Neglecting to make this distinction we should get the development of F $(x + xh)$ instead of F $(e^h x)$. The actual relation between the symbols $x^n\frac{d^n}{dx^n}$ and $x\frac{d}{dx}$ is obtained immediately from the equation (2) which gives us

$$x^n\frac{d^n}{dx^n} = \left(x\frac{d}{dx}\right)\left(x\frac{d}{dx} - 1\right)\ldots\ldots\left(x\frac{d}{dx} - n + 1\right).$$

Sir John Herschel has given the following theorem, which enables us to develope F (e^h) in a series of ascending powers of h when such a development is possible :—

$$\mathrm{F}(e^h) = \mathrm{F}(1) + \frac{h}{1}\mathrm{F}(1+\Delta)\,o + \frac{h^2}{1.2}\mathrm{F}(1+\Delta)\,o^2 + \&c.$$

Comparing this with the one given above, we obtain the following theorem :

$$\left(x\frac{d}{dx}\right)^n \mathrm{F}(x) = \mathrm{F}\,[x(1+\Delta)]\,o^n,$$

by the help of which we arrive at a still more general one,

$$f\left(x\frac{d}{dx}\right)\mathrm{F}(x) = \mathrm{F}\,[x(1+\Delta)]\,f(o).$$

Sir William R. Hamilton wished to be allowed to remind the Academy that he had communicated to them, in 1831, another extension of Herschel's Theorem, which was published in the seventeenth volume of the Transactions (page 236), namely, the following :

$$\nabla' f\psi(o') = f(1+\Delta)\,\nabla'\,(\psi(o'))^o;$$

where the accents in the first member might have been omitted, and where ∇' denoted any combination of differencings and differentiatings, performed with respect to o', and generally any operation with respect to that accented zero, of which the symbol might indifferently follow or precede $f(1+\Delta)$, as a symbolic factor. By making $\psi\,(o') = \varepsilon^{o'}$, and $\nabla' = \mathrm{D}'^x$, where $\mathrm{D}' = \frac{d}{do'}$, the theorem of Herschel is obtained. A much less general formula was cited as "Hamilton's theorem," in the last Number of the Cambridge and Dublin Mathematical Journal, namely, the following :

$$f(x) = f(1+\Delta)\,x^0;$$

which had, however, been also given in the same short paper of 1831.

The Rev. Charles Graves exhibited an ancient gold ornament, belonging to the Earl of Leitrim, of which the following description is given in Vallancey's Collectanea, Vol. V., p. 90:

"Mr. Burton Conyngham has now in his possession one of those double cupped patera, described and engraved in the 13th Number of the Collectanea. The instrument is of gold, was found in the county of Mayo, and weighs about six guineas. On the outside of one cup is an Ogham inscription; on the outside of the other an inscription in the Phœnician or Estrangelo character.—See Pl. III.,—where the cups are reversed to show the inscription. The Phœnician word is composed of the *Ain*, *Lamed*, *Tau*, *Aleph*, i. e. עלתא, i. e. *Alta* or *Olta*, signifying an holocaust. This confirms my former opinion, that these instruments were used in sacrifices. The Ogham characters are UOSER, *Uoser*, *Osir*, or *Usar*, the Sun, the principal deity of the pagan Irish. The names *Aesar*, *Aosar*, frequently occur in ancient Irish MSS., which are always translated God."

Mr. Graves stated that, whilst he recognised the gold ornament itself as being a genuine and a very fine specimen of the ancient manillæ,* of which many are preserved in the Museum of the Academy, he was forced, after a careful examination of it, to pronounce the inscription to be a forgery of comparatively recent date. For this conclusion he assigned the following reasons:

Faint tracings of all the characters scratched upon the surface, as if to serve as a pattern to be copied by the engraver, are still quite visible. There can hardly be a doubt but that casual attrition, and the action of the atmosphere or earth for a thousand years or more, would have effaced such marks.

The inscribed characters have a sharpness which is not to be seen in ancient work, even though executed in gold. All the original devices which appear on ancient gold articles

* See Sir William Betham's papers on Ring Money, in the Transactions of the Academy, vol. xvii.

possess a peculiar mellowness, from the action of the causes just alluded to.

These characters, moreover, have plainly been cut with a graver, such as is employed at the present day. But no traces appear, on genuine antique Irish ornaments, of the use of such an instrument. The lines and patterns on them seem to have been laboriously scratched with a point rather than cut in. The conclusiveness of these reasons is maintained by the judgment of Mr. West, the eminent jeweller, to whom Mr. Graves applied for his opinion on the subject. So many valuable relics of antiquity have passed through his hands, at different times, that his opinion on a point of this kind ought to be nearly decisive.

Mr. Graves further remarked, that the characters said by Vallancey to be "Phœnician or Estrangelo," are neither the one nor the other; and, what is more, in the scanty remains of Phœnician literature, which have been collected by Gesenius and Hammaker, we meet with no such word as *Olta*, meaning a holocaust. As for the word *Aesar*, which Vallancey professes to find, though somewhat deformed, in the Ogham inscription, Mr. Graves asserts that it does not frequently occur in ancient Irish MSS.; on the contrary, it is so rare that, with the aid of the most accomplished Irish scholars, Mr. Graves has not yet succeeded in finding a single instance of its use, except as it occurs in O'Reilly's and Shaw's Dictionaries. It certainly is an Etruscan word, meaning God, and it may have found its way into Irish glossaries, though not belonging to the Irish language.

In order to show how unsafe a guide Vallancey is in what relates to Ogham writing, or, it might be added, in any matter of Irish archæology or philology, Mr. Graves referred to a passage which occurs in the tract on Oghams, preserved in the Book of Ballymote. This passage stands thus in the original (*Book of Ballymote*, f. 168):

"Aṫair oġaim Oġma. Maṫair oġaim láṁ no scian Oġma. Isé so imorro in céḋna ni ro scriḃaḋ tri oġaim .i." &c.

"The father of Ogham was Ogma; the mother of Ogham was the hand or knife of Ogma. This indeed was the first thing that was written through Ogham, viz.," &c.

The meaning, as is quite plain from the context, being, that Ogma was the inventor of the Ogham character, and that the instrument with which he first executed it was his own hand or knife. Vallancey, in his Essay *on the Ogham Writing of the Ancient Irish* (*Collectanea*, vol. v. p. 79), gives the following reading and version of the same words:

"*Atair Ogaim, Ogma; mathar Ogaim, Lám, no Scian Ogma. Is sè Sóm in ceadna: sè ro scribtar tri Ogam," &c.*

"The father of Ogum was Ogma, his mother's name was Lám, or Scian Ogma (the helpmate of Ogam). The same is called Sóm: he wrote his own name in three Oghams," &c.

Here it will be seen that Vallancey has introduced two imaginary personages, Som and Lam, neither of whom were thought of by the Irish writer; and he expends a vast quantity of irrelevant erudition in making out this Som to be a Theban (Egyptian) Hercules, and Lam to be the daughter of Belus and Libya. "This helpmate" [of Ogam] he adds, "was named *Lám*, or *Lamia*, which signifies a horrid, dreadful monster; hence must have arisen the Grecian story of Hercules having begotten Scythes, the progenitor of the Scythians, on the body of a monster, half woman, half serpent. A fable which gained ground wherever the Scythians went,—from Scythia to Tartary, China, and Japan."

It ought to be added that, by tampering with two other passages in a like way, Vallancey has elsewhere educed the name of his Theban Som (*Collectanea*, vol. v. pp. 63, 69).

Mr. Graves referred to another instance in which, by a perverse ingenuity, additional darkness has been thrown upon the obscure subject of Ogham writing.

Mr. Beauford contributed to the first volume of the Transactions of the Academy a paper in which he describes twelve coins, on which he thinks he finds legends, in Ogham, Roman, and Runic characters intermixed ; and he gives readings of these, exhibiting various Irish names of persons and places.* Any person, the least conversant with numismatics, will at once recognise these coins as being all of them Hiberno-Danish. By the kindness of Dr. Aquilla Smith, Mr. Graves was enabled to exhibit to the Academy one of the actual coins figured by Mr. Beauford, viz., that marked No. 7 in the plate illustrating his paper.

This coin, now in Dr. Smith's collection, is appropriated by Mr. Lindsay, who has studied this class of coins with most attention, to Sihtric IV., King of Dublin, A. D. 1034. It may, however, belong to Sihtric III., A. D. 989.

Mr. Beauford's description of the coin is as follows:—

" Round the head, on the obverse, is the following inscription in Latin, Runic, and Ogham Croabh characters :

u mearc readon

or,

U mearc re a don, for *O More Re I dun.*

On the reverse, in one of the quarters of the cross, is a hand, with the following inscription in Latin, Runic, and Ogham Croabh characters :

mac ghealach ofutla

or,

Mac Ghealach O Futla, for *Magh Ghealach O Fodhla.*"

Subjoined is a figure of the coin in question, executed from

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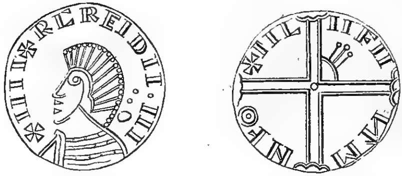
* Still more absurd misrepresentations respecting Ogham characters and writing may be seen in a paper by the same author, called *Druidism Revived*, which is inserted in the second volume of Vallancey's Collectanea.

a drawing by Dr. Smith, and also a fac simile of the (enlarged) figure given in Mr. Beauford's plate:



Mr. Graves concluded by apologising for having occupied the time of the Academy in the discussion of matters of so little intrinsic importance; but pleaded the necessity of breaking down the remnant of authority which still gives to the assertions of Vallancey and his adherents the power of leading students in Irish history and antiquities astray.

Practices like those now commented on once brought contempt upon Irish archæology; and philologists for a long while shrank from entering upon the rich field of inquiry which the study of the Celtic language and literature presents, through fear of sharing in the ill repute of former labourers. But these feelings are now happily dying away; and it is to be hoped that the Academy, encouraging such pursuits, when carried on in a scientific spirit, and vigilantly checking all attempts to mislead, will have the satisfaction of seeing permanently established amongst its members a sound and numerous school of antiquaries and scholars, really conversant with the language and antiquities of this country, and therefore able successfully to prosecute that work of illustrating its history, which a few, in recent times, have so well begun.

Sir William Betham exhibited two specimens of gold ring money, found at Chiusi and Perugia, in Italy. He also presented to the Academy an ancient brass basin, found in the King's County; and two antique metallic mirrors, found in Italy.

Sir William R. Hamilton stated and illustrated a theorem of anthodographic (or anthodic) isochronism, namely, that if two circular anthodes, having a common chord, which passes through or tends towards a common centre of force, be both cut perpendicularly by any third circle, the times of anthodically describing the intercepted arcs will be equal:—the *anthode* of a planet being the circular locus of the extremities of its vectors of slowness, or of straight lines representing, in length and in direction, the reciprocals of its velocities, and drawn from a common origin.

This theorem is intimately connected with the analogous theorem respecting hodographic isochronism (or synchronism), which was communicated to the Academy by Sir William Hamilton, in a note read at the Meeting in last March. He had been led to perceive that former theorem by combining the principles of his first paper on a General Method in Dynamics, published in the second part of the Philosophical Transactions for 1834, with those of his communication of last December, since published in the Proceedings of the Academy, respecting the Law of the Circular Hodograph. This *Hodograph* was, for a planet or comet, the circular locus of the extremities of its *vectors of velocity*, as the Anthode is the locus of the extremities of the vectors of slowness; so that the rectangular coordinates of the Hodograph are x', y', z', if

$$x' = \frac{dx}{dt},\ y' = \frac{dy}{dt},\ z' = \frac{dz}{dt};$$

while those of the Anthode may be denoted as follows:

$$x_{\prime} = -v^{-2}x', \; y_{\prime} = -v^{-2}y', \; z_{\prime} = -v^{-2}z';$$

where $v^2 = x'^2 + y'^2 + z'^2$.

He had effected the passage from the theorem respecting hodographic to that respecting anthodic isochronism, by the help of his calculus of quaternions; but had since been able to prove both theorems by means of certain elementary properties of the circle.

For a hyperbolic comet, the Anthode is a circular arc, *convex* to the sun; for a parabolic comet, the Anthode is a *straight line*. And for comets of this latter class the theorem of isochronism takes this curiously simple form: "Any two diameters of any one circle (or sphere) in space, are anthodically described in equal times, with reference to any one point, regarded as a common centre of force." By this last theorem, the general problem of determining the time of *orbital* description of a finite arc of a *parabola*, is reduced to that of determining the time of *anthodical* description of a finite *straight line* directed to the sun; and thus it is found that "the interval of time between any two positions of a parabolic comet, divided by the mass of the sun, is equal to the sixth part of the difference of the cubes of the sum and difference of the diagonals of the parallelogram, constructed with the initial and final vectors of slowness, as two adjacent sides." Another very simple expression for the time of description of a parabolic arc, to which Sir William Hamilton is conducted by his own method, but which he sees to admit of easy proof from known principles (though he does not remember meeting the expression itself), is given by the following formula:

$$t = \tfrac{1}{2}\mathrm{T} \tan\left(\theta - \tan^{-1} \tfrac{1}{2} \tan \tfrac{1}{2}\theta\right);$$

where θ is the true anomaly, and t is the time from perihelion, while T is the time of describing the first quadrant of true anomaly.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 67.

May 24th, 1847.

REV. WILLIAM HAMILTON DRUMMOND, D. D.,
Librarian, in the Chair.

M. Donovan, Esq., continued the reading of his paper on the Nature of the Agency which produces the Effects called Galvanic, Electro-magnetic, Magneto-electric, and Thermo-electric.

The next subject to which Mr. Donovan called the attention of the Academy was the instantaneous charge which a Leyden battery receives by a momentary contact with an extensive voltaic series. This has been always adduced as an argument in support of the affirmed enormous quantity of electricity which constitutes the voltaic current. Van Marum charged a Leyden battery of twenty-five jars by a momentary connexion with a pile consisting of silver coins and zinc discs, one inch and a half in diameter. The battery and pile were thus charged to the same intensity, so feebly, however, as to produce divergence in a gold leaf electrometer to the extent of five-eighths of an inch; but the shock from the battery was only equivalent to half that of the pile. Facts and calculations were adduced to show that the charge of electricity in this Leyden battery, when thus charged, could not have exceeded the quantity of two or three one-inch sparks. Sir H.

Davy charged a Leyden battery with 2000 pairs of zinc and copper plates, each plate exposing thirty-two superficial inches of metal to the exciting liquid; the total surface being 128,000 square inches. On making the proper connexions, with the Leyden battery, "either a shock or a spark could be *perceived*." Thus the shock was barely perceptible; and to this Mr. Donovan added his own testimony of the shock from a Leyden battery, charged by 1000 pairs of plates, which he represented as exceedingly feeble.

In support of the inference drawn of the trifling nature of the shock, and the inconsiderable quantity of electricity which a Leyden battery is capable of communicating, when charged by a voltaic series, Mr. Donovan detailed an experiment made by Professor E. Davy and himself, in which twenty Wedgwood ware troughs, each containing ten cells, were employed, with a total number of 200 pairs of plates excited by dilute acids. When charcoal points, fixed to the polar wires, were brought into contact, an instantaneous burst of light, of dazzling splendour, announced that the series was in high action. On attempting to charge a Leyden battery of twelve or thirteen square feet of coated surface with this voltaic series, neither shock nor spark could be obtained. Yet it was proved that the charge communicated to the Leyden battery by three turns of a very small electric machine was sufficient to enable the battery to give a spark visible in day-light, the three turns producing three weak sparks of one inch in length. Six turns of the cylinder, that is six one-inch sparks, enabled the Leyden battery to give a sensible shock. This failure was supposed to be explicable by the small size of the Leyden battery compared with that of Van Marum, the ratio being as $12\frac{1}{3}$ to $137\frac{1}{2}$.

So far as all the experiments, whether these last, or those of Van Marum or Davy, are concerned, there seems to be no evidence of great quantity of electricity in the voltaic series. But, even if there were, it was stated that none of the foregoing cases are applicable to the doctrine, and that no support can be derived from them. A series of arguments was then made use of, which cannot be abridged, to prove that a Leyden battery has never yet been really charged by the voltaic current; that the circumstances under which the charge has been supposed to have been communicated are such as to render the thing impossible; and that hence the so-called charge of a Leyden battery by a voltaic current does not prove the circulation of an enormous quantity of electricity in a voltaic series, but rather shows that the agent which gives the voltaic shock is not the same as that which acts in the phenomena of ordinary electricity. The shock of the coil apparatus was adduced as an evidence to the same effect; and, finally, it was observed that the sensation which constitutes the shock ought not to be received as evidence on either side of the question, since sensations depend more on the organ acted on than on the agent; and in support of this opinion a number of instances were adduced.

Whatever difference of opinion may exist relative to the nature of positive and negative electricity, it appears to be a position universally agreed to, that, when equal to each other, and at liberty to act, they mutually neutralize and destroy each other's properties; all symptoms of both disappear; a condition of absolute quiescence results; that of equilibrium is induced; and this state manifests no electrical properties. The poles of a voltaic battery, being in the positive and negative states, conform to the general law: when unconnected, they manifest their electrical condition; but as soon as they are connected by a good conductor, all symptoms of electricity vanish. This has been proved in a remarkable manner by Mr. Gassiott with a water battery consisting of 3520 pairs, exhibiting great power over even a *distant* gold-leaf electrometer while the poles were unconnected, but losing all energy when they were united.

Yet it is at the moment when the poles are united, and when all symptoms of electricity vanish, that the connecting wire of the voltaic series becomes magnetic. Is there not in this fact something repugnant to the idea that electricity is the agent. To admit that the two states of electricity, after having neutralized and virtually annihilated each other's properties, should at that moment be more active in calling into operation the magnetic power, would be to declare that in the natural state of the equilibrium of the electric fluid the magnetic influence must be perpetually active; that is, that all the bodies in nature are magnets. This objection applies to the opinion of those who maintain that electricity, considered as a simple element, is the cause of, or is identical with, or excites magnetism; but not, as Mr. Donovan conceived, to his own view, stated in the beginning of this Essay, relative to the supposed compound nature of the electric fluid.

The boldest of all the hypotheses of magnetism, and the most ingeniously supported, was described to be that of Ampere, who denies the existence of any magnetic agent called into action by electricity, but affirms the identity of both powers. Some experiments were described which cannot be here detailed, the object of which was to show that magnetism and electricity observe different laws, and that one may exist when the other is not present.

A word has of late years come into common use, which, while it explains nothing, conceals the solecism contained in the notion of neutralized electricities retaining their energies: the new term is the "current." The counter-current is thus kept out of view, which is the grand difficulty, because it must antagonize and destroy the current. This new current, consisting of both electricities, instead of being powerless, as was formerly the nature of such, is now said to be capable of exerting peculiar power; but it no longer harmonizes with those facts from which our knowledge of the true current was derived. Faraday's views of the current were examined, and the conclusion drawn that they are inconsistent with each other, and with facts. Mr. Donovan terminated this part of the inquiry with the following observations:

"I have thus freely expressed my opinions relative to the current, fearing that the old legitimate sense has been lost sight of; that many have understood it to mean something more than is warranted by *proved* properties; and that the universally admitted identity of the agent in electric and voltaic phenomena has emboldened philosophers to attribute qualities to the former which belong only to the latter. On the whole, I conceive that the current, in its modern acceptation, instead of explaining voltaic phenomena, is calculated to mislead; and that it is of no avail in obviating the difficulties which beset the alleged simultaneous operations of the two states of electricity, when present in a state of commixture, and which, instead of being at that moment in their condition of greatest energy, should be destitute of all sensible properties."

Sir Robert Kane read a communication from the Rev. Dr. Callan, Professor of Natural Philosophy in the College at Maynooth, on some improvements in the construction and use of the Galvanic Battery.

"Some time ago, whilst I was reflecting on the principle of action of Grove's and Bunsen's batteries, it occurred to me that lead might be substituted for the platina of Grove's and the carbon of Bunsen's. I put into the porous cell of a Grove's battery a plate of lead about one-sixteenth of an inch thick, two inches broad, and six inches long. I found that the voltaic current produced by the lead, excited by a mixture of concentrated nitric and sulphuric acid, was very powerful. I afterwards compared the power of the leaden battery with that of a Grove's battery of the same size, by sending at the same time, but in opposite directions, through the helix of a galvanometer, the current produced by the two batteries. Both batteries were charged with the same acids. The voltaic current from the platina battery destroyed the deflection of the needle produced by the current from the leaden one, and caused an opposite deflection, which indicated that the former current was nearly twice as strong as the latter. The two batteries were allowed to work for three hours and a half. At the end of that time the current from the lead was more than twice as strong as the current from the platina. The quantity of lead dissolved during these three hours and a half was very small.

" It struck me that, by diminishing the action of the acids on the lead, I might increase the power of the battery. I therefore covered a plate of lead with gold-leaf, and coated another of the same size with chloride of gold, in the same way in which sheet silver is platinized for Smee's battery. These two, and a platina plate of the same size, were put successively into a porous cell of a Grove's battery, and the voltaic current was sent through the helix of our large electro-magnet, in which the iron bar is about thirteen feet long, and two and a half inches thick, the copper wire is about 500 feet long, and one-sixth of an inch in diameter. The magnetic power given to the electro-magnet by the leaden plate coated with chloride of gold, appeared to be fully equal to that produced by the platina plate : the magnetic effect of the current from the leaden plate covered with gold-leaf was not so great. Platinized lead produced as strong a current as platina, or as lead coated with chloride of gold.

" On last Friday week, a leaden and platina battery of equal size were left working for four hours and a half. At the end of that time the plate of lead acted fully as well as the platina plate. When the nitric acid was so much exhausted that the lead was barely capable of magnetizing the large electro-magnet, so as to sustain a certain weight, the leaden plate was taken out of the porous cell, and a platina plate of the same size put in its place. The platina plate was not able to make the electro-magnet sustain the weight which the lead caused it to sustain.

" When a zinc plate is put into the porous cell, and a leaden one coated with gold powder is placed on each side of it, in the exterior cell, the effect of the voltaic current on the electro-magnet appears to be greater than that which is produced by a single platina and double zinc. The voltaic current from a leaden or platina battery is about fifteen times as great as that of a Wollaston battery of the same size. In all the experiments which I have described, the lead and platina plates were about two inches by six. The lead and platina were excited by a mixture of concentrated nitric and sulphuric acid, and the zinc by diluted sulphuric acid. The acids employed in some of the experiments had been previously used.

" When a platinized or gilded leaden plate is taken out of the cell, and afterwards immersed in the acid, it does not produce its full effect until it has acted for about a minute. The lead plate, after being used for a long time, requires to be again coated with platina or gold powder.

" Seeing that the concentrated acids, by dissolving a small quantity of the lead, gradually removed the powder of gold or of platina, and that the nitric acid was very expensive, I endeavoured to find a cheap substitute for it, which would not act on the lead. The first that occurred to me was common nitre. I dissolved about the eighth of an ounce of it in sulphuric acid, which I diluted with about an equal bulk of water. I poured the mixture into a porous cell of a Grove's battery; after putting into the cell a platina plate, I sent the current from the battery through the helix of the large electro-magnet. The magnetic power given to the electro-magnet appeared to be about equal to that which the same battery produced when the platina was excited by a mixture of equal parts of concentrated nitric and sulphuric acid. The platina plate was taken out of the cell, and a platinized leaden one of the same size put in its stead. The magnetic power given to the electro-magnet by this battery was greater than that which was given to it by the platina battery when the platina was excited either by the concentrated acids, or by the solution of nitre. The battery was kept working for nearly an hour. During that time there was very little diminution of its power, and apparently no action on the lead or platina powder.

" I afterwards tried the power of the leaden battery when the lead was excited first by the nitre dissolved in water without acid, and then by the diluted sulphuric acid without nitre. The voltaic current was feeble in both cases. When a few crystals of nitre were put into the cell containing the diluted sulphuric acid, the power of the battery increased as the nitre was dissolved. The power, then, must be ascribed to both: to the nitre and sulphuric acid.

" I have compared the power of platinized or gilded lead, excited by dilute sulphuric acid, with that of platinized silver, and have found the former fully equal to, and as constant as the latter.

" From the experiments which have been described, I infer, first, that a battery equal in power to that of Professor Grove may be made by exciting platina with a solution of nitre in dilute sulphuric acid, or by exciting gilded or platinized lead with concentrated nitro-sulphuric acid, or with common nitre dissolved in sulphuric acid diluted with about an equal bulk of water; secondly, that platinized or gilded lead may be substituted for platinized silver in Smee's battery.

" The advantages of what I may call the nitre platina battery over Professor Grove's nitric acid battery, are, first, that the expense of the nitre necessary for working the former for a given time, is only about the thirtieth part of the cost of the nitric acid required to work the latter: the diminution of expense in working the voltaic battery will contribute very much to make electro-magnetism a more economical prime mover than steam; secondly, that from the nitre battery there are no noxious or disagreeable fumes.

" The advantages of the nitre leaden battery over Professor Grove's platina battery, are, first, that the former is very cheap compared with the latter: a plate of platinized or gilded lead, one foot square, may be made for a shilling, but a platina plate of the same size will cost nearly three pounds; secondly, the expense of working the former for any time is much less than the expense of working the latter for the same time; thirdly, the nitre leaden battery does not emit fumes of nitrous gas.

"The leaden battery, when charged with nitro-sulphuric acid, appears not to exhaust the nitric acid so rapidly as the platina battery; probably, a good deal of the hydrogen, which would otherwise unite with the oxygen of the nitric acid, is dissipated by the gold or platina powder on the surface of the lead.

"The advantage of the platinized or gilded leaden battery over Smee's battery, is, that platinized or gilded lead costs far less than platinized silver. A plate of the former, six inches square, will not cost more than three-pence, whilst a plate of the latter of the same size, will cost about three shillings.

"I shall now mention a few more experiments which I have made with the leaden and platina batteries. I compared their power in magnetizing our large electro-magnet, and in producing heat. The magnetic power and heat produced by the platina, excited by concentrated nitric and sulphuric acid, was equal to that produced by the platinized lead excited by a mixture of nitre and sulphuric acid diluted with an equal bulk of water. Each of the batteries fused the thinner of the two wires which I enclose. In each battery there was but a single voltaic circle. The platina and leaden plates contained each about ten square inches; they were about five inches long and two broad. Neither of these batteries was able to fuse the thick wire; but when about a tea-spoonful of nitric acid was poured into the cell containing the leaden plate, and the current sent through the thick wire, it was fused. The platina battery only raised the thick wire to a white heat. Hence, I infer that platinized lead, excited by a mixture of dilute nitro-sulphuric acid and nitre, produces a more powerful voltaic current than platina does when excited by nitric and sulphuric acid. In consequence of the small quantity of acid contained in the lead cell, its power declined sooner than that of the platina. From the results of several experiments made with the platina and lead batteries, I have come to the conclusion that the expense of doing a given amount of work by the former, excited by nitric and sulphuric acid, would be about three times as great as if the work were done by the latter, excited by a mixture of nitre and sulphuric acid. I have tried a mixture containing one part of sulphuric acid and three parts of water, in which a little sulphate of soda and nitre was dissolved. When the platinized lead was excited by this mixture, the power of the battery was very great, but not so great as when the mixture contained as much sulphuric acid as water. I have not as yet tried any other sulphate as a substitute for the sulphuric acid. When the platinized or gilded lead is taken out of the cell, it should be rinsed in water, and dipped into a weak solution of chloride of gold or platina. By this means, and by amalgamating the lead plates with mercury, before they are gilded or platinized, the platina or gold powder may be kept on them for a long time.

"The reason why the platinized or gilded lead produces so powerful a voltaic current is, that the acting metals are not lead and zinc, but platina or gold powder and zinc. It appears to me that the current produced by zinc and platina, or gold powder, is more powerful than that which is produced by zinc and platina, or gold. Perhaps by depositing on lead a powder of some of the metals, such as tungsten, arsenic, &c., which are more negative, compared with zinc, than platina or gold is, a battery may be yet made which will be more powerful than the platina or platinized lead battery. I have tried antimony, but it did not answer."

The Rev. Dr. Todd gave an account of the formation of the following Catalogue, by Mr. Bindon, of MSS. in the Irish, English, French, and Latin languages, forming part of the Burgundian Library at Brussels, and serving as materials for Irish history.

The first volume having reference to Ireland, which will be found in the "Inventaire"* of this celebrated collection, is numbered 471. This MS. is a large vellum† folio in the Latin language, consisting of 205 leaves; it is entitled, "Dialogus Richardi Episcopi Armacani‡ contra Errores Armenorum." The initial letter (B) is about two inches in length, and beautifully illuminated in purple and gold; the initials commencing paragraphs are also ornamented, but they are small. At the conclusion of the text and the commencement of the index, or rather table of contents, which occupies six folios, there is a short note, stating that the MS. was finished upon the —— day of November, A. D. 1410. At the end of this table of contents there is another short note also, which is as follows: "Liber Mõsterii sñi Pauli ĩ Zonia seu Rubeævallis." This note, as well as the other alluded to, are in the same hand as the text. The monastery spoken of was that of the "Red

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* The "Inventaire" is the first volume of the printed catalogue. In it the MSS. are enumerated without reference to subject; the second volume, or "Repertoire," is a "Catalogue Methodique."

† All the MSS are upon paper, except when the contrary is stated.

‡ Richard Fitzrauph or Fitzralph. This individual was one of the most celebrated ecclesiastics of his age. He is said to have been born at Dundalk, and was advanced to the see of Armagh, A. D. 1347, by Clement VI. He died at Avignon in 1360; and ten years afterwards his body was removed to the place of his birth, by Stephen de Valle, Bishop of Meath, and a monument raised to his memory, which was remaining in 1624. This Prelate is said to have been the first who possessed an Irish translation of the New Testament made by himself. His canonization was proposed, but never effected. Some of his works, in manuscript, will be found in Trinity College Library, and others were printed at Paris so early as A. D. 1496.—See Ware's Bishops, p. 81; Ware's Writers, p. 84, and the authorities there quoted.

Cross," in the forest of Soigny. There are no notes in the Irish character or language, nor do any figures of animals, which are so commonly found in Irish MSS., form any part of the illuminations. The MS. is in good preservation, and the text commences thus: "Joannes qui ex literali." The name of the scribe has not been discovered in any part of the volume.

Vol. II. containing the Nos. 1160, 1161, 1162, and 1163, is a middle-sized folio also in the Latin language, bound in wood covered with calf-skin, and ornamented with brass clasps. This volume is entitled, in the classing of the old library, to which it belonged, as follows: "Navigatio S. Brendani ad varias Insulas cum aliis.—Beth. Louv. 48 F." At the commencement and also at the end of the MS. may be found the following note: "Pertinet Mõsterio Canõnor. Reglarm. in Bethleem p̃pe. Lovanium." The contents of the volume are given upon the first fly-leaf, and are as follows:

1160. It̃ navigatio S. Brendani abbatis ad diversas insulas.
1161. It̃ visio Tungdoli militis valde admirabilis.*
1162. It̃ epistola Presbiteri Joannis imperatoris majoris.
1163. It̃ itinerarium Joannis de Mandeville militis.

The initial letters of the four different pieces are about two inches in diameter, and ornamented in vermilion and pink colours, but not very neatly executed, and the same observation applies to the penmanship. The first leaf of this MS. is vellum, then follow four of paper, and then two of vellum, and so on until the eightieth, which is of vellum also, and which is the last in the volume. Mandeville and Tungdolus are in the text styled "milites." The date, or the name of the scribe, could not be found in the MS.; but, by comparison with MSS. of a known date, it appears as old as the "Inventaire" states, namely, the close of the fifteenth century.† No note in the

* There are three copies of this tract in the Library of Trinity College, Dublin. They are marked as follows: C. 4. 23.; E. 1. 29.; and E. 4. 12.

† Nos. 1161 and 1163 have short prologues; for other copies of these MSS. see Nos. 4531 et 7960.

Irish language or character appears; and the different pieces commence as follows:

1160. "Sanctus Brendanus filius Finlothæ."
1161. "Hibernia igitur insula est."
1162. "Presbiter Joannes potentia Dei."
1163. "Qui de Anglia Hibernia."

Vol. III. (2159–2167). This volume, which is a thick quarto, belonged to the "Domus probationis" of the Jesuits at Mechlin, and contains some interesting papers in reference to Irish history; among them I may mention "Magna supplicia et persecutiones in Hibernia," commencing thus: "Anno 1599, Guillielmus Drurius;" and No. 2167, which is a collectanea, entitled "De Rebus Hibernicis;" in the latter will be found a letter or memorandum in Latin, signed "Thomas Fleming," and dated 1612.

This compilation was written between the years 1602 and 1616, and is in the Latin language: the name of the scribe does not appear. The volume contains some short pieces relative to English history.

Vol. IV. (2324–2340). This is a thick quarto volume, divided into two parts, the first containing 105 and the second 246 pages, written upon both sides. The contents of the first part are given in an alphabetical table, which is at the beginning of the volume, and of which the following is an abstract:

In this abstract two or three entries are omitted, which are quite illegible in the original, in other respects the copy is substantially correct.

At the end of the volume is a table of contents for the second part, of which the following is an *exact* copy :

Vita S. Mædoci Fernensis. Hib., ... 60
Vita S. Colmani Elo. Hib., ... 111
Vita S. Senani. Hibernice, ... 118
Miracula S. Senani post mortem. Hib., ... 233
Poemata diversa S. Senani, S. Brendani, et aliorum. Hib., . 142
Poemata plur. S. Columbæ Kille et aliorum. Hib., . . . 154
Vita S. Caimini Metro-hibernice, ... 156
Vita S. Coemgheni. Hibernice, ... 166
Vita ejusdem aliter tradita. Metro-hibernice, ... 170
Vita S. Mochœvog. Hibernice, ... 179
Vita S. Callini. Hibernice, ... 195
Ejusdem poemata et Prophetia de statu Hib. metro-hibernice, nec non plura alia de diversis rebus, quæ in ejus vita et in ceteris usque ad finem, ... 244

The contents of this volume are entirely in the Irish character and language, except the list of the Franciscan Provincials, which is in Latin.

The paper upon which this MS. is written is very coarse, and some of the writing not well executed, in comparison with other MSS. written by Michael O'Clery, whose name appears at folio 75, and also at the end of the first part of the volume, where it is stated that the MS. was finished at Donegal, upon the 7th of August, 1631, by "Brother Michael O'Clery." The name of this individual appears in several places of the second part also, with various dates in the year 1629. See pp. 7, 22. From these dates it appears that the second portion of the work was written before the first; and from the appearance of the binding it may be concluded that the collection was bound after being written, as the paper of the first half of the volume does not correspond with that in the remainder.

Vol. V. (2542, 2543) is a thin, quarto volume, bound in vellum, and not entirely written upon. The first piece in this volume is an Irish poem, of 250 stanzas, with four verses to each stanza; and the following is its title: "Sliot̄ Daill Gulban mc̃ neıl;" it is accompanied with a gloss and notes. The second piece commences thus: "Tea ıngean luıdeach," and the penmanship of this is not so neatly executed as that of the other, than which nothing can be more beautiful. Upon the cover of this volume the following note, or rather title, will be found: "Genelaċ na naoṁ a ndán, Cum opusculis de uxoribus et matribus filiorum Milesii et successorum." This MS. was finished January, 1642, and although, judging by the handwriting, I am almost certain it was written by Michael O'Clery, I was unable to find his name in the volume.

Vol. VI. (2569, 2572). This is a small quarto, containing poems in the Irish character and language, some in the handwriting of Michael O'Clery, others,—the greater part, I may say,—in another hand, with the corrections, or rather annotations, of O'Clery, appearing here and there through them. The heading of some of the poems is as follows: "giolla na naom ua duın̄," "giolla caomaın c̃c." Fol. 23, "mc̃ darıg c̃c." At the conclusion of these poems is a continuous prose piece in Irish, commencing thus:

"Baı doc̃ide, ıongn̄ adbalmor ar er̄ vle."

This is in the handwriting of Michael O'Clery, and was finished by him at the convent of Donegal, in November, 1635.

This latter piece is styled in the "Inventaire" "Guerres des Irlandais et des Danois."

Vol. VII. (3195). This MS. is a large folio, containing 104 leaves, closely written on both sides. It is a History of the Franciscan Order in Ireland. Upon the fly-leaf is written the following note, first in red and again in black ink:

"Fr. Antonius Purcell compegit hunc librum
jussu Rdi. admodum P. Fratris Donah Mooney
ordinis minor. Regular. observantiæ
Ministri Provincialis, año domini 1617, die Novembris
2°. in collegio Fratrum Hybernorum Lovanii
pro quo pius Lector ex charitate
oret."

At the commencement of the volume is the following heading or title: "Tractatum sequentem de Provincia Hiberniæ concinnavit, R. adm. P. Donatus Monæus, dum esset Provincialis et huc ex Hibernia ad res hujus collegij S. Anthonii ordinandas advenisset." It may be remarked that this appears an interesting and valuable historical document, and, together with the papers before mentioned upon the Irish Franciscans, must be of great value in preparing a correct "Monasticon Hibernicum." Of course, their value is much increased when we remember that they were compiled under the directions of the heads of the Irish college of St. Anthony. The text of this history commences thus: "Provincia ordinis S. Francisci in Regno Hyberniæ."

Vol. VIII. (3201). This volume, which is a large folio, appears to be a "Collectanea" of the Lives and Acts of Saints for the month of March; probably it formed part of the Bollandist collection, although it has not their library mark. The above number is stated in the "Inventaire" to be "Vita S. Patricii," and under this title will be found four different lives of the Saint. The second life bears date 1641, and the third is extracted from Camden. There are also two copies of the "Purgatory;" the first, "ex MS. Hiber. Min. Lovanii;" and the other, "ex MS. Maximini Treveris;" besides these there is a life of St. Senan, "ex MS. Hibern. Seminar. Salamaticensis Soc. Jesu,* coll. cum MS. R. P. Ward;" and also a long poem and hymn to this Saint; some of the other lives appear to have been extracted "ex MS. F^{rs}. J. Colgani." This volume has been recently bound, and I was unable to discover any name or date, &c., indicating the compiler, or where it was written.

Vol. IX. (3824, 3825). The first part of this volume, which is a large, thin quarto, is occupied with the correspon-

* This MS. is now in the Burgundian Library. See vol. xxii. in this Catalogue.

dence of some Irish Jesuits and the heads of a college at Rome, called in the correspondence the "English College." Much of this correspondence is between Louis Newman, for some time a prisoner in Dublin Castle, and Mr. Thomas Roberts, of the aforesaid College, and bears various dates, from the years 1634 to 1639. There are also letters signed by Edward Blake and Robert Spreul, with memoirs of F. Slingisbey by the said Spreul, and also by William Moloney. The correspondence appears to be entirely about Slingisbey, and some of his letters are also given, signed, "F. H. Slingisbey, Kilkenny, this St. Joseph's day, 1640," and directed to Mr. John Thompson, of Rome. This person was also of the English College; one letter is signed "Franciscus Persæus,* alias Slingisbeus, propria manu, Rome, 14th February, 1639." The letters are in the Latin, Italian, and English languages.

At the end of the volume is a complete memoir of this person, entitled, "Relatio brevis de Vita et Moribus Rdi. P. Francisci Slingisbeii Societatis Jesu pro eo tempore quo vitam Sœcularem egit in Hibernia, postquam fuisset ad Fidem Catholicam conversus. P. Mauritij Wardœi Sacerdotis Hiberni." This latter is written upon duodecimo, in a very legible hand, and if it was printed the correspondence would make a necessary appendix.

Vol. X. (3944) is a very thin quarto volume, in the Latin language, bound in calf, and is the Obituary of the Irish College of St. Anthony of Padua, at Louvain, from 1614 to the year 1716. A couple of leaves are devoted to each month, and

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* In Mr. Burke's *Heraldic Dictionary of the Peerage and Baronetage of the British Empire*, vol. ii. p. 297, the following will be found: "Henry Piers, Esq., of Tristernagh. This gentleman conformed to the Church of Rome, and prevailed upon some of his children to embrace the same creed; of whom Thomas, his third son, became a Franciscan Friar; and Henry, his fourth son, left a son, John, who took orders in the Catholic Church." Very probably the "Franciscus Persæus," above alluded to, was a member of this family.

the names of the brothers are entered in the order of the calendar, opposite the dates upon which they died. The entries contain short notices of the labours, &c., of the fraternity, and indeed no Franciscan college has maintained with more zeal than this the character of the order, as expressed in their motto, "Doctrina et sanctitate." The obits of the benefactors of the college follow those of the brethren.

Vol. XI. (4190–4200) is a thick quarto, bound in vellum: the paper is very coarse, and the penmanship rudely executed. The contents of this volume are all in the Irish character and language, except one of the two lives of St. Molingus which is Latin.

1.	A life of St. Brigid,	fol.	1
2.	The Hymns, Life, and Vision of St. Adamnanus,	,,	—
3.	The Life of St. Molingus,	,,	43
4.	,, St. Berachus,	,,	66
5.	,, St. Grellanus,	,,	83
6.	,, St. Forsuianus,	,,	86
7.	,, St. Molassius Daimhinis,	,,	91
8.	,, St. Lassara Virgo,	,,	112
9.	,, St. Naulus,	,,	124
10.	,, St. Kieranus Saigir,	,,	139
11.	,, St. Kieranus Cluanensis,	,,	149
12.	,, St. Declanus,	,,	160
13.	,, St. Ruanus,	,,	186
14.	,, St. Finianus Cluainerard,	,,	196
15.	,, St. Benignus,	,,	203
16.	,, St. Aireranus seu Aileranus,	,,	212
17.	Sapientes precationes ejusdem,	,,	217
18.	The Life of St. Brendanus Cluanfert,	,,	259
19.	,, St. Mochudda seu Carthaci,	,,	265
20.	De St. Suanachi filiis et de St. Mochudda,	,,	268
21.	Fragment of the Life of St. Senan, and the poems of Dallanus upon that saint,	,,	—

Judging from the writing of this volume, one would not suppose it to be a compilation of Michael O'Clery's; however, his name appears as the scribe in several places; for instance, at folios 121, 131, and 183. The volume appears to have been written in the years 1628 and 1629; it contains 270 folios.

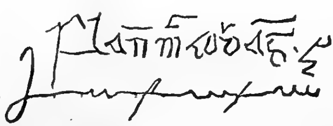
Vol. XII. (4241). This is a thick quarto, composed of different sized paper, and contains about 270 leaves, with short memoirs and notices of Irish saints in the Latin language; the arrangement is alphabetical, and the volume appears to be a note-book of some hagiographer. I was unable to discover a name or date in the book. At page 99 is a pedigree of "SS. Furseus, Foilanus, et Ultanus," the sons of Giltanus, "Rex Hiberniæ."

Vol. XIII. (4531) is a second copy, and I believe the oldest in the library, of the Visio, &c.; see Vol. II. (1162), and Vol. XXIV. (7960), in this catalogue. The MS. is written upon vellum, and beautifully executed; the sheet is small octavo, and there are eighteen of them occupied with the piece. It has no illumination, nor any trace of the Irish character, or the name of the scribe. It is attributed to the close of the thirteenth century.

Vol. XIV. (4639). This is a small duodecimo volume bound in calf; upon the fly-leaf is the following title:

"Martyrologium
Sanctorum Hiberniæ collegit et digessit
Michael O'Clerij
ord. S. Francisci
Duaii in Flandria Gallica 1629."

After this follow four pages in Irish, in the hand-writing of Michael O'Clery, and dated from Donegal, 1628. After this a short paragraph or testimonial in Irish, dated Nov. 1, 1636, and signed



This paragaph appears upon the next page also, and is signed "Connep Mac Ƃpaoᵹ," bearing date Nov. 11, 1636.

The testimonials, in the Latin language, of Thomas Fleming, Archbishop of Dublin and "Primate of Ireland," dated from Kildare, February 1, 1636, and from Father Roche of Kildare, dated from his convent, 8th January, 1637,* next follow, and then the Martyrology, first arranged according to the calendar, and then alphabetically. This MS. is in the Irish language, and can be read without any difficulty: it contains about 250 pages.

Vol. XV. (5057, 5058, 5059). This is a thin quarto, rudely stitched together, and in bad condition. It contains, first, a fragment of a catalogue of saints, then some poems by "Eogan mac an Bhaird," and by "Moel Patric," &c. &c., and ends with a fragment in prose, commencing "Gloriosus Episcopus Carthagus qui vulgo vocatur Mochuda." The contents are all in the Irish language, and I was unable to discover the name of the scribe or date of the compilation; however, I believe it belonged to the Louvain collection, and is justly attributed in the catalogue to the seventeenth century.

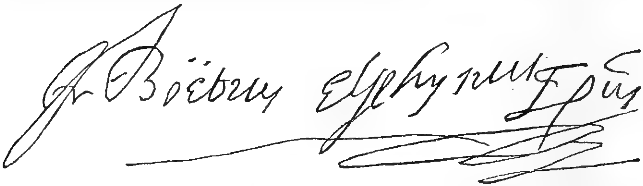
Vol. XVI. (5095, 5096) is a small, quarto volume, bound in vellum, in the Irish character and language; and is another copy† of the "Martyrologium Dungalense" of Michael O'Clery. It contains a preface by the author, in which he states that the work was finished at Donegal, 19th April, 1630, and then a short paragraph, signed Flan Mac Aodhagan as above.

Upon the following page this paragraph again occurs, signed "Conner M'Brady," Then follow the approbations, in Latin,

* Flann Mac Aodhagan, whose autograph is here given, was a famous antiquary, from the village of Bally Mac Aodhagan, in the county of Tipperary. It may be remarked that the individuals whose names are here given are the same who have given approbation to the Annals of the Four Masters; as may be seen in the autograph copy belonging to Trinity College Library. *Rerum Hib. Scriptores*, vol. iii. p. xv. The same observation applies to the other copy of this Martyrology. See vol. xvi. in this Catalogue.

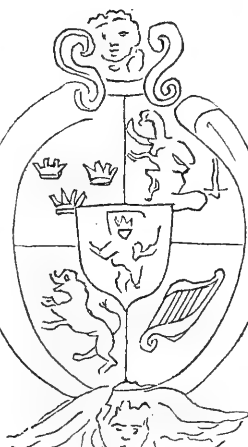
† See vol. xiv. No. 4639.

of Malachy, Archbishop of Tuam, dated from Galway, fifteenth of the kalends of December, 1636; that of



dated 27th December, 1636; that of F. Thomas Fleming, "Archiepiscopus Dubliniensis, Hib. Primas," dated from Kildare, February 6, 1636; also that of "Frater Rochus Kildarii," dated from his convent, 8th January, 1637, in which it is stated that "Fr. Florentius Keegan" and "D. Cornelius Bruodyn" examined the Martyrology.

The Martyrology is first arranged according to the calendar, and then alphabetically; the writing is clear and beautifully executed; some short notes in the Irish and Latin character are scattered through its pages.



At the commencement of the volume will be found a sketch of a shield with some armorial bearings as above.

Possibly the readers of the "Unkinde Deserter" may be able to conjecture the purpose of this composition, and assign some reason for finding the Lion of Belgium, the Harp, and crowns of Ireland quartered together.

Vol. XVII., containing Nos. 5101, 5102, 5103, and 5104, is a thin quarto volume; the first part is occupied with a collection of religious poems in the Irish language; some upon Columbanus, and others attributed to him and to St. Moling, also the rules of the Irish saints, commencing with St. Columbanus. This collection was finished in 1630, as may be seen at page 45, where it ends. Then commences "Festilogium S. Engussii Keledei," also in Irish, and beautifully written, unquestionably in the handwriting of Michael O'Clery; the accompanying gloss and notes are very full, and the "Festilogium" occupies fifty-one pages.

The next piece is entitled "Mariani Gormani Sancti de quibus dubito, an sint Hiberni an alij, quid non reperiantur in aliis Martyrologiis iis quibus denotantur diebus." It is in the Irish language, as well as a testimonial which precedes it, bearing date "18. aug. 1633," and signed Fearfeasa o maoilċonaire and Cucoiccriċe o Cleiriġ.

This Martyrology is in short metre, and contains 141 pages.

After this there is a third Martyrology, that of Tallaght, occupying about twenty-seven pages, and then the following testimonials :—

Nos infra scripti fidem facimus, et per presentes testamur has annexas duas copias transumptas fuisse per fratrem Michaelem Clery ex duobus codicibus manuscriptis, quibus a linguæ Hybernicæ peritioribus hucusque fides adhibebatur, uno nimirum vetustissimo codice pertinente ad Clann y Mulchoner, et altero qui est vera copia celeberrimi et vetustissimi codicis nunc Dublinij reservati; easdemque copias de verbo ad verbum sine styli ordinis aut substantiæ rerum inversione aut corruptione cum eisdem suis prothotypis per omnia concordare. In cujus rei fidem et testimonium his subscripsimus in deserto nostra mansionis die 29 Aprilis, 1636."

Fr. Bernardinus Clery.
Guardianus Dung.
Fr. Mauritius Ultanus.
Fr. Mauritius Ultanus.

Upon the opposite side will be seen a *fac simile* of this testimonial; I have thought it worth presenting to the reader, that it might be compared with the signatures of the same three individuals as attached to the testimonial upon vellum, in the Annals of the Four Masters, now in the Library of the Royal Irish Academy, as well as with their handwriting, which may be seen in another copy of the same work in Trinity College Library.

Then follows another testimonial to the same effect, signed "Joannes Culenanus. Epus Rapotensis," with his episcopal seal. There is also one in Irish, signed by Conell Mac Geoghegan, dated October, 1636, and some short historical pieces, finished at the Convent of Donegal, 28 April, 1636.

Upon the outside of this volume will be found the following note, in a hand apparently as old as that of the text or testimonials: "Continens Martyrologia Œngussei, Mariani Gormani et Tamlactense et Genealogias SS^{um} et plura alia opuscula." We may add, that the binding of this valuable volume is of vellum, with a piece of calf-skin rudely stitched upon its back.

Vol. XVIII., containing Nos. 5301 to 5320, inclusive, is a very thick quarto paper volume, bound, or rather stitched together, with a piece of calf-skin as a cover. Its contents are very varied, and, it is presumed, are of much historical interest; they are as follows:—

No. 5301. "Fragmenta tria Annalium Hiberniæ extracta ex codice membraneo Nehemiæ mac Ægan Senis, Hibernici juris peritissimi, in Ormonia, per Ferbissium ad usum R. D. Joannis Lynch. "Ab anno Xti circiter 571 ad annum plus minus," * * * * and commences thus: " Rt. caṫ ꝼſmın," "in quo victus est Colman," &c.

These extracts extend to seventy-one pages, and are well and closely written. At the end is an index, and an accompanying collation with the Annals of Donegal.

No. 5302 is a little fragment upon Irish hagiography, commencing " Notanda in opere fratris Jacobi," &c., consisting of four small leaves.

No. 5303 consists of sixty-five pages; the first twenty-six are entitled " Adversaria Rerum Hiberniæ excerpta ex mutila Historia D. Cantwelly," and commences thus: " Hoc anno ante diluvium." At page 25 commences " Annales Roscreenses." The initial line is " Patricius Archiep̄us in Hiberniam venit atque Scotos baptizare inchoat, nono anno Theodos. minoris," &c. These Annals, as well as the " Adversaria," are in Latin and Irish, and very badly written.

No. 5304 is a very long alphabetical Index of the Annals of Roscrea, made by " Frater Brendanus Conorus," accompanied by marginal references to the Annals of Donegal.

No. 5305 is " Familia, seu Monachi S. Fintani sive Munnæ Abbatis, numero 233, quos non uret ignis judicii: ex Mart. Taml. pergameno, 21 Octob." The names are alphabetically arranged, and this fragment is but four leaves in extent.

No. 5306 is very closely written, and is entitled, " De Multiplici Naturæ bono ad Hybernos in terrestri elemento istius regni," and commencing " Authores qui volunt regionem."

No. 5307. " Itenerarium in Hibernia," by " Frater Edmundus Mac Cana," and commencing " Monasterium de Kill Snabha."

No. 5308 is a " Descriptio Insulæ Sandæ," by the same author, and commences " Insula Sanda est."

No. 5309 is an extract from Joannes Major, " de historica Britannica antiqua," commencing " Ab Albina Regina."

No. 5310 is another extract, " de regibus Scotiæ," from

Hector Boetius, and commencing " Primus imperat Scotis ;" these extracts are only a few pages in length.

Nos. 5311, 5312, 5317, 5318, are, respectively, bulls by Popes Urban VIII. and Alexander VII.; and letters to the Irish bishops from Popes Zacharias and Gregory.

No. 5313 is "de Monasterio St. Jacobi Herbipolensis" (Wurtzburg); and commencing, " Circa hoc tempus multi in Scotia."

In 5314 is an extract from Marianus Scotus upon Ireland, very short, and commencing, " Hac quoque tempestate." There are also extracts from the " Annales Suevici" of Martinus Crucis concerning Ireland; and a collection upon the " Irish Apostles," with their labours in Belgium and Germany.

To the historical student wishing to pursue his researches in reference to the seminaries established on the Continent by these missionaries, the contents of the volume we are speaking of would be indeed an acquisition. In it he has the names of many authors with which, very possibly, he has hitherto been unacquainted, with references to their writings already arranged, which would give him considerable trouble even to make out.

Vol. XIX. (6131, 6132, 6133) is a large quarto, bound in vellum, containing a collection of Irish poems and pieces in prose, upon the O'Donnell family, and has been evidently left in an unfinished state; a good number of the poems are headed " Eoghan ruadh mc an bhaird *cc.*" Upon the outside of the volume its title is written in Irish and Latin as follows:

" Lebhar isirin í Domhnaill.
Liber poematum O'Donnellij."

Upon the inside of the cover I found a note scarcely legible. I was able to decipher with difficulty the following words at the conclusion: " O'Donnell a dall fall. . . . Brukelles, xiii. September, 1622." The writing in this volume is not well executed; and in the note I have just spoken of some of the words are in the Flemish language.

Vol. XX. containing No. 7299, is merely an extract from the theological works of Richard Archdekin, of Kilkenny, which are highly prized abroad, and have, consequently, gone through eleven editions.

Vol. XXI. containing Nos. 7658, 7659, 7660, 7661. This volume is a very large paper folio, bound in vellum; and appears by the class-mark $\left(\frac{\maltese.\text{Ms}}{158}\right)$ to have belonged to the Jesuits. The title or heading occupies half the first page, and has the following note written across it, in an old hand differing from the text: "Authore N. P. Stephano Vito, soc[is]. Jesu Hiberno, Clonmeliensi." The title is as follows:

"Vindiciæ Scotorum veterum, et Sanctorum indigenarum
Iberniæ oceani magnæ Insulæ, quæ olim ab immemora
bili tempore, passim per Europam usque ad annum
Christi saltem 1000 audiebat Scotia, deinde
vero per 200 et amplius annos dicebatur
Scotia major sive vetus, ad discrimen Scotiæ primoris et novæ,
quæ ante per plurima
secula audiebat Patria Pictorum Britanniæ.
In tres libros distributæ
ADVERSUS
Graves crebrosque errores novorum de rebus Scoticis his
toricorum Hectoris Boetii, Georgii Buccanani,
Georgij Tomsoni, Roberti Turneri sub nomine
Joannis Leslei, et asseclarum ipsorum qui Iber
norum nationem et patriam prisco nomine proprio
christianorum Scotorum et Scotiæ una cum
ingenti numero Sanctorum Iberniæ Scotorum
veterum immerito privant et transformant
in Neoscotos Britanniæ Insulæ posteros
priscæ Pictorum ac Dalreudinorum Gentis
Candido Lectori memorabilium antiquitatum amanti. S."

The preface, or rather the address to the reader, which commences thus: "Frons operis futuri Vindicias pro Patria præferat," &c. &c., occupies about six pages, and ends as follows: "si tibi lector ero. Vale, Fave, et fruere labore nostro." Next follow "Generales Censuræ variorum Authorum de fide Catholica et Libris Hectoris Boetii atque asseclarum ipsius;" including those of Polydore Virgil, Carolus Sigenijus, Thomas Boygus, Joannes Lelandus Londinensis, Luidyus, Camdenus, Petrus Berteyus Belgeus, Buccananus, Richard Stanihurst. After these *Censuræ* there follows something like a table of contents, divided into eleven chapters, and headed thus: "Referuntur capita potissima historicorum errorum qui in Vindiciis nostris refutantur."

The body of the work commences at page 14, and to it there is a short heading, thus: "Brevis descriptio Iberniæ Insulæ," &c. &c., and then a little prologue commencing: "Inter multos exteros nostri sæculi qui geographiam," &c., which occupies half a page; at the end of this, cap. i. commences, and is entitled: "Situs, magnitudo, cælum, solum, salubritas, fæcunditas aliaque commoda, ac dotes Iberniæ Insulæ." And the text of the work begins thus: "Vernaculo vocabulo Erin," which is then continued in 202 folios written upon both sides, when a different character of hand appears, which continues until page 309, written very closely, and also upon both sides.

After this a second part commences, with different paging, headed thus: "Ihesus Maria Deo gratias. Apologia pro Ibernia adversus Cambri calumnias, sive Fabularum et Famosorum libellorum Silvestri Gyraldi Cambrensis, sub vocabulis, Topographia sive de mirabilibus Iberniæ, et Historia Vaticinalis sive expugnationis ejusdem insulæ, refutatio." With an address to the reader commencing—"Benevo. lectori S. In contumeliam multorum cum Sanctorum." After this, and a table of contents, follows cap. i. headed thus: "Quid Stanihurst et alii senserint universim de intemperie et mordacitate Sylvestri Geraldi Cambrensis," and commencing, "Præter Hereticos tres." Cap. xix. is headed: "Fictitium aut surreptitium fuisse Diploma Adriani," &c. The last, or chapter xxvi., commences, "Audisti Lector Cambrensem," &c.*

At the end of the volume will be found a detached folio tract, entitled, upon the cover,

"Apologia pro Sanctis Scotiæ, sed Infirma videtur saltem si conferatur
cum Vindiciis P. Step. Viti pro Scotia an
tiqua Seu Hibernia."

The text of this is preceded by the following heading: "Apologia pro gente Scotica hodierna contra Hibernos;" and commences, "Dedimus in Lucem aliquando," &c. This also formed a part of the Jesuit Collection, and contains twenty folios written upon both sides; and it is accompanied by a few leaves of detached Irish MS. upon quarto paper, and very badly written, and of no importance.

The contents of this volume, that is to say, the part fastened within the binding, amount in all to about 1000 closely written pages, and it was evidently a copy, made perhaps for the author, by four, or, at all events, three different scribes. As for its contents as a valuable historical document, the writer has merely to mention that he has never seen a work upon Ireland, from which information appears to have been drawn from so many or such high authorities: one need only look at a single page of it, when he will at once perceive the immense amount of learning with which the author was gifted, and the facility of arrangement with which he has used it.†

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* Dr. Todd has just called my attention to a manuscript fragment in the Latin language, forming part of the Ussher Collection, in the Library of Trinity College, Dublin (E. 3. 19). Upon inspection, we have discovered that it is a part of this work of Stephen White, which is above described.

† Since the above was written, Mr. Charles P. Mac Donnell, M. R. I. A., has been kind enough to place in my hands the Rev. Dr. Oliver's "Collec-

Vol. XXII., containing Nos. 7672, 7673, and 7674, is perhaps one of the most valuable Irish MSS. in existence, being the second volume of a collection of lives of Irish saints, in the Latin language, and written in the fifteenth century. It consists of 177 large folios of parchment; the first is num-

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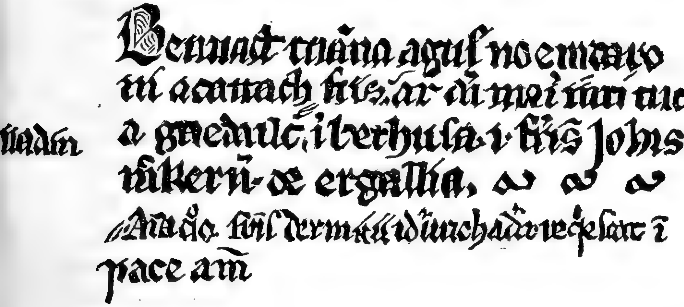
tions towards illustrating the Biography of the Scotch, English, and Irish Members of the Society of Jesus." Quoting from page 269, of this very interesting work, I find a confirmation of the opinions upon the merits of this MS. which I have just ventured to express.

"Stephen White. This Irish Father deserves a fuller eulogium than I am able to supply. He was the author of some historical pieces relating to Ireland, in confutation of the assertions of Giraldus Cambrensis. The Rev. John Lynch, who had the custody of this valuable MS., mentions it in chapters i. and xiv. of his *Cambrensis Eversus*, printed in 1662, and expresses his deep regret that a considerable part of it was lost during the civil wars. Archbishop Ussher, an excellent judge of these matters, in page 400 of his *Primordia*, gives F. White the character of being 'a man of exquisite knowledge in the antiquities, not only of Ireland, but also of other nations.' In a letter of Robert Nugent, superior of his brethren in Ireland, and addressed from Kilkenny, January 10, 1646, to F. Charles Sangri, I read what follows:

"'I have given the commission to four of our fathers diligently to examine the works of F. Stephen White, and to forward their judgment to your paternity, conformably to the directions you have recently sent us. His works are various, and as our fathers live in places very distant from each other; and notwithstanding the most Reverend Bishops (who are ready to defray the expenses of the printing), as also the supreme council, very earnestly insist, that a certain work of his "De Sanctis et Antiquitate Iberniæ," be instantly sent to the press, I find it difficult, and next to impossible, to resist their reasonable demand, since the MS. itself has been perused by several amongst them, and has been pronounced not only worthy of being printed, but highly necessary for the credit and advantage of this kingdom. Therefore, I have written again to the examiners, that each would privately report their opinions on this work as soon as possible to your paternity; though all in their letters to me greatly extol it, and declare it most worthy to issue from the press. But I am unwilling to allow any work to be printed that can give just cause of offence to any person; and yet here is less cause of apprehension in this case, as this book merely treats on the Saints and Antiquity of the Kingdom of Ireland.'"

Dr. Nicholson, in his Irish Historical Library, p. 90, calls Stephen White the friend of Archbishop Ussher; and the passage referred to in Cambrensis bered 48, consequently the forty-seven first leaves are wanting; and the MS. is written in double columns, of thirty-nine lines each, ornamented with small illuminated letters scattered through the volume. Between folios 128 and 129 is a semi-longitudinal page, upon which is found the following note in the Irish language :*



Eversus, alluding to White, is as follows: "Pater Stephanus Vitus e Societate Jesu, sacræ Theologiæ doctor, et professor emeritus, lucubrationem elaboravit accuratissimam, quæ infamiam Hiberniæ a Giraldo impactam luculenter amovit. Ejus operis exiguum fragmentum penes me habeo, quod reliquæ partis præstantiam, tanquam unguis leonem indicat; sed integrum alicui mutuo pridem traditum, proh dolor, in latebras aliquas reconditas abditum est, ut ex iis erui, ac in lucem, hominumque conspectum proferri exinde non potuerit."—*Cambrensis Eversus*, p. 1.

* [It would seem that these lines ought to be read and translated as follows:

> Bennaċt cuanna agus (na) noem caro-
> ui a cattach fris, ar ainmain in tí tuc
> a gaedailc illadin in bethusa .i. f̄ris Iohīs
> mac keruill. de ergallia.

> Anima quoque fratris Dermitii i Dhunchadha requiescat in
> pace Amen.

"The blessing of Cuanna, and (of the) saints who were in communion with him, be upon the soul of him who translated this life from the Irish into Latin, viz., Brother John Mac Keruill of Oriel."—Editor of the Proceedings.]

This is the only trace of the Irish language that I was enabled to find in the volume; the two last pages are also semi-pages, neither of which are written upon both sides, although all the others are; upon the back of the volume may be seen the following title: "MS. Salmantic. de SS. Hibernis II." A table of contents upon paper will be found in the commencement of the volume, apparently laid there by some person who had possession of the book, and of which the following is a copy:

"Codicem hunc Rector Collegij Salmanticensis Hibernici Soc̃tis Jesu dono dedit nostro Patri Œgidio de Smidt qui eundem donavit P. Heriberto Boswaydo."

INDEX VITARUM.

Upon the back of this table of contents is a portion of a letter in Latin, without any signature, referring the reader of it to Colgan and Speed for information as to the places mentioned in the book. The heading of the chapters is not in the same handwriting as the body of the work, but in the smaller hand which will be seen in the last couple of lines of the *fac simile.* The book itself is about the size of the Book of Ballymote, and has been cut down too closely in the binding, which is also of vellum. A map of Ireland, without any name, I think that of Speed, is laid into the volume, and I suppose was made by the individual who wrote the index, the paper of both being the same.

Vol. XXIII. (7806). The last piece in this volume is a copy of the "Purgatory of St. Patrick," commencing thus: "Patri suo peroptato in Christo." The volume itself is a large vellum folio, bound in wood, and attributed to the pen of "Henricus Saltereÿensis," and said to be written at the commencement of the fourteenth century. It has no illuminations, nor could I find any trace of the Irish character or language.

Vol. XXIV. In this volume will be found No. 7960, which is another copy of the "Visio cujusdam militis Hibr̃nensis ad edificationẽ multor̃ conscripta." It commences thus: "Incipit p̃logus Marci ad abbatissam quandam;" and after this prologue, commences, "Ibernia igitur Insula." The volume itself is a very ancient vellum folio MS., in double columns, attributed, and I believe justly, to the fourteenth century. The vision occupies twenty folios, written upon both sides. Upon the first folio of the volume may be found the following note about this piece:

> "Visio Tungdali militis Hiberni, an. 1148, auctore Marco in qua mentio fit SS. Patricii apli Hiberni Malachiæ ep. Dun. Ruadani Nenniæ ep. Cluan Cœlestini ep. Armach. Chaini ep. Lundinen."

This MS. is without any illuminations, nor could I find in it any trace of the Irish language.

Vol. XXV., containing Nos. 8530–8534, is a "collectanea" which belonged to some Jesuit library, and contains a tract entitled "De Sanctis Hiberniæ Item de Hiberniæ Historia et quod qui Scoti appellantur usque ad annum fere 900 patria Hiberni fuerunt."

The first part of this tract is a long alphabetical index to the Irish saints, with references to notices, &c., of them. The next part is a very interesting tract upon Irish history, being a collection of notices by the most ancient writers upon the country, arranged according to the subject. This tract is rather long. At the conclusion will be found a poem entitled "Sancti Iberniæ in Belgio," of fifty-two verses. The contents of this volume are entirely in Latin. The second part of the MS. is upon the Scotch saints. No date is to be found in this volume; possibly it formed part of the Bollandist collection.

Vol. XXVI. (8597) is a collectanea of hagiography, containing the offices of some Irish saints, written in the same hand as the last MSS. spoken of, and also belonging to some Jesuit library. No name or date appear in it.

Vol. XXVII. In this volume will be found No. 9035, which is a French translation of the Purgatory of St. Patrick, and begins, "Moult de fois allant demander." The commencement of this is ornamented with a drawing of the Purgatory, representing the souls in torment, the dimensions of which drawing are about four inches by two and a half: it is rather well executed, but presents nothing remarkable. The MS. is of the fifteenth century.

Vol. XXVIII. This volume contains, among other writings of St. Bernard, No. 9648, which is his sermon upon the death of St. Malachy, "Sermo in transitu Malachiæ," commencing, "De cælo nobis, dilectissimi." This MS. is attributed to the twelfth century, and is perhaps the oldest copy of the writings of this Father. The initial letter is ornamented with a portrait, said to be that of St. Malachy, and of which the following is a *fac simile.*

This little portrait is of interest, representing, as is supposed, the features of St. Malachy, and at all events exhibiting the episcopal costume of the times; with this view I thought it worthy of being presented to the reader, and, consequently, made an accurate copy.

Vol. XXIX. containing Nos. 11976–11980, which are, in the first place, " Memorabilis promotio Dni Joannis Scoti Canonici Dni Augustini prioris ecclesiæ de Busco dñi Isaac in cancellarium ordinis aurei velleris pr. fr. Hubertum Scotum." No. 11979 is " Oratio habita in Capitulo gen. Ord. Velleris Aur.," and bears date the 5th of the Kalends of Ap. 1532. They are both written upon paper of small quarto size; and at the end of the last piece (No. 11980) is the following epitaph:

" Epitaphium Dni Johannis Scoti
Prioris Ecclesiæ de Bosco Dmni Isaac"

" Quæ cameratus in hanc protexit lucem Joannis
Scoti sub saxo hoc ossa sepulta jacent
Si quis erat procerum clarorum insignia doctus
Artifices inter gloria prima fuit
Ordinis enituit proin Cancellarius Aurei

Velleris Augusto Cæsare sub Carolo
Hujus Cænobii tenuit moderamina pastor
Terdenis annis pervigil atque tribus."

This also bears date A. D. 1531.

Vol. XXX. In this volume, which is an historical collection upon the governors of the "Low Countries," will be found No. 16367, being the order of battle as given by Marshal Saxe before the celebrated battle of Fontenoy. It was printed at Gand, "par ordre," and is on official paper, something like a placard: it bears date 1st May, 1746.

It may be mentioned, that the order of battle, as presented in this paper, was not the one observed, as it differs materially from a plan of the engagement which is in the possession of the writer, and which was copied from that sent to the King of Prussia after the battle; a copy of this latter plan was also published at Lille in 1745, attached to Voltaire's Poem on Fontenoy, and had it not been correct it would not have been attached to the writings of a court poet. However, the discrepancy is easily accounted for; the Marshal may have changed his mind, and countermanded his orders as first issued.

Vol. XXXI. is a small portfolio, containing, among other numbers, 17056; this is entitled "Anecdotes a la vie de St. Dympna," and is merely a short notice of that saint, not differing from the ordinary memoir of her; it occupies but one sheet of foolscap.

Mr. Ingram read the following note on certain Properties of Curves and Surfaces of the Second Degree.

"The object of the present communication is to show that several large classes of the properties of curves and surfaces of the second degree, most of which, though of considerable interest for their generality and elegance, have hitherto been little studied by geometers, may, with advantage, be regarded as results of that simplest form of the polar transformation, in which a circle or a sphere (according as we are studying the geometry of two or of three dimensions) is used as our *auxiliary.*

"Beginning with curves of the second degree, we may assume the properties of the circle as known. Then,

"1. If we operate on the circle by the cyclo-polar method, we arrive at the modular generation of the conic sections, and are enabled to discover and prove almost all the properties relating to their foci and directrices. This mode of proceeding has been explained and practised by Poncelet, Gergonne, and others, and is familiar to all students of geometry.

"2. Let us next place the centre of our auxiliary circle at the centre of the conic, and perform a second transformation; and we shall thus deduce from the already proved focal properties a series of theorems relating to two remarkable lines, which may be termed the *secondary directrices* of the conic. These lines have already been the subject of some researches of Dr. Booth; but he has obscured the simplicity of their theory, by presenting their properties as results of a peculiar and somewhat intricate analytic method. I have already noticed, in another place,* the striking analogy which exists between these lines and the *cyclic arcs* in the spherical conics.

"3. But, instead of placing the origin of transformation at the centre of the conic, we may suppose it situated anywhere in the same plane; and in this way we shall be led to consider a conic section as the locus of a point which moves so that the square of its distance from a given point is constantly proportional to the rectangle under its distances from two fixed right lines. The transformation at the same time indicates a class of new properties, which might be called *quasi*-focal properties, inasmuch as in a particular case they reduce to the ordinary focal properties, and they are in all cases analogous to those of umbilicar foci in surfaces of the second degree. The generation of the conic sections with which these properties are connected

* Philosophical Magazine for September, 1844.

was originally given by Chasles, but the properties themselves to which I allude, have been overlooked both by him, and by Cauchy, who has since discussed analytically the generation in question in his well-known Report on M. Amyot's Memoir.

" Similar considerations will apply to surfaces of the second degree.

" We assume the properties of the sphere as known. Then,

" 1. From these the properties of the *non-modular* surfaces of revolution are deduced by means of a sphero-polar transformation. This conception has been completely elaborated by M. Chasles, in the Memoirs of the Royal Academy of Brussels. (*Nouv. Mem.* tom. v.)

" 2. Let us now take a non-modular surface of revolution, and make its centre the origin of a second transformation ; and from its properties, discovered in the way just mentioned, we shall infer a great number of the properties of the *modular* surfaces of revolution, especially those belonging to two planes connected with them by remarkable relations. This idea also M. Chasles has suggested, though he has not developed the results of it to any considerable extent. (See Liouville's *Journal de Math.* tom. i. p. 187.)†

" 3. If, instead of placing the origin of transformation at the centre of the non-modular surface, we assume it arbitrarily in space, and then operate on the surface as before, we shall light on the *umbilicar method,* and we shall be enabled to deduce, by a uniform process, all the properties of the directive planes and their poles, which arise out of that method. In this way we are led to the theorems which I stated to the Academy on the last night of meeting, as well as to many others of a similar kind.

" 4. It is natural now to inquire what results we should obtain by applying the sphero-polar transformation to the focal

* See also the *Memoire de Geometrie,* appended to the *Aperçu Historique.* § xxii.

properties of surfaces of the second degree with three unequal axes. And accordingly (in the Philosophical Magazine for September, 1844), I have examined what are the results of that process, when the centre of the surface is made the origin of transformation, and in this way I have been led to discover the properties of two cylinders, remarkably related to the surfaces of the second degree.

"5. But, more generally still, we may transform those focal properties, placing the centre of our auxiliary sphere at any point of space; and thus the following leading problem is suggested and solved:

"*Given any surface of the second degree, and a point anywhere situated, to trace on the surface all the plane curves, which, viewed from the given point, shall appear to be circles.*

"The brief indications given in the present note will, of course, require large developments; and these, with the permission of the Academy, I shall endeavour to supply on some future occasion. My object in what I have now said has been merely to direct attention in a general way to several considerable groups of the properties of curves and surfaces of the second degree, which may, with advantage, be studied as results of the Polar Transformation."

DONATIONS.

Mémoires présentés par divers Savants a l'Académie Royale des Sciences de l'Institut de France. Tome IX. (Sciences Mathématiques et Physiques.)

Mémoires de l'Académie Royale des Sciences de l'Institut de France. Tome XIX. Presented by the Institute.

On the Silurian Rocks and their Associates, in Parts of Sweden.

A brief Review of the Classification of the sedimentary Rocks of Cornwall. By Sir Roderick I. Murcheson. Presented by the Author.

Statistics of the Government Charitable Dispensaries of India.

Statistics of Civil and Criminal Justice in India.

Statistics of the Educational Institutions of the East India Company in India.

Mortality of the Madras Army, from official Records, and A Catalogue of Chinese Buddhistical Works. By Col. Sykes, H. M. R. I. A. Presented by the Author.

Memoirs of the Literary and Philosophical Society of Manchester. Vol. VII. Part 3. New Series. Presented by the Society.

Proceedings of the American Philosophical Society. Vol. IV. Nos. 34 and 35.

Transactions of the American Philosophical Society. Vol. IX. New Series. Part 3. Presented by the Society.

An old Fiddle; and an ancient Bronze Cheek-plate of a Bit, and a small Celt. Presented by the Earl of Enniskillen.

Proceedings of the Royal Astronomical Society. Vol. XII. Nos. 4–14.

Magnetical and Meteorological Observations made at the Royal Observatory, Greenwich, in the Year 1844.

Astronomical Observations made at the Royal Observatory, Greenwich, in the Year 1844. Presented by the Royal Astronomical Society.

Astronomical Observations made at the Royal Observatory, Edinburgh. Vol. VI. for the Year 1840. Printed by the Observatory.

Transactions of the Royal Society of Edinburgh. Vol. XVII. Part 2, containing the Makerstown Magnetical and Meteorological Observations for 1843. Presented by the Society.

A large Bronze Celt. Presented by the Rev. Dr. Sadleir, Provost, T. C. D.

A Review of the Sanatory Condition of Dublin. By John Aldridge, M. D. Presented by the Author.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 68.

June 14th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

Arthur Sidney Ormsby, Esq., and John C. Egan, Esq., were elected Members of the Academy.

It was Resolved,—That the Secretary of the Academy be requested to prepare a draft of an Address to His Excellency the Lord Lieutenant, to be submitted to the Academy at its next Meeting.

Sir William Rowan Hamilton made a communication respecting the application of the Calculus of Quaternions to the Theory of the Moon.

I. At two Meetings of the Royal Irish Academy, in the month of July, 1845, Sir William Rowan Hamilton had exhibited and illustrated the following general equation of motion of a system of bodies, with masses $m, m', \ldots$, and with vectors $a, a', \ldots$, and attracting each other according to Newton's law:

$$\frac{d^2 a}{dt^2} = \Sigma \frac{m'}{(a-a')\sqrt{\{-(a-a')^2\}}}. \qquad (1)$$

He had, at that time, deduced from this equation the known laws of the centre of gravity, of areas, and of living force, for any such multiple system; and had shown that the corresponding, but less general, equation of relative motion of a binary system, which (by changing $\alpha-\alpha'$ to α, and $m+m'$ to M) becomes

$$\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}=\frac{\mathrm{M}}{\alpha\sqrt{(-\alpha^2)}}, \qquad (2)$$

can be rigorously integrated by the processes of his new calculus of quaternions, so as to conduct, with facility, when the principles and plan have been caught, to the known laws of elliptic, parabolic, or hyperbolic motion of one of the two attracting bodies about the other. (See the Proceedings of July 14th and 21st, 1845, Appendix to Volume III., pp. xxxvii., &c.)

At a subsequent Meeting of the Academy, in December, 1845, Sir W. Hamilton had shown that the general differential equation (1) might be put under this other form:

$$0=\tfrac{1}{2}\Sigma.m\left(\delta\alpha\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}+\frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}\delta\alpha\right)+\delta\Sigma\frac{mm'}{\sqrt{\{-(\alpha-\alpha')^2\}}}; \qquad (3)$$

and that it might, theoretically, be integrated by an adaptation of that "General Method in Dynamics" which he had previously published in the Philosophical Transactions of the Royal Society of London, for the years 1834 and 1835; and which depended on a peculiar combination of the principles of variations and partial differentials, already illustrated by him, in earlier years, for the case of mathematical optics, in the Transactions of this Academy. (See Proceedings of December 8th, 1845, Appendix already cited, pp. lii., &c.)

At the same Meeting of December, 1845, Sir W. Hamilton assigned the two following rigorous differential equations for the internal motions of a system of *three* bodies, with masses m, m', m'', and with vectors α, $\beta+\alpha$, $\gamma+\alpha$,—that is, for the motions of the two latter of these three bodies (regarded as points) about the former,—as consequences of the general equation (1):

$$\frac{d^2\beta}{dt^2}=\frac{m+m'}{\beta\sqrt{(-\beta^2)}}+m''\left\{\frac{(\beta-\gamma)^{-1}}{\sqrt{\{-(\beta-\gamma)^2\}}}+\frac{\gamma^{-1}}{\sqrt{(-\gamma^2)}}\right\}; \quad (4)$$

$$\frac{d^2\gamma}{dt^2}=\frac{m+m''}{\gamma\sqrt{(-\gamma^2)}}+m'\left\{\frac{(\gamma-\beta)^{-1}}{\sqrt{\{-(\gamma-\beta)^2\}}}+\frac{\beta^{-1}}{\sqrt{(-\beta^2)}}\right\}. \quad (5)$$

It was remarked, that by regarding m, m', m'', as representing respectively the masses of the earth, moon, and sun, β and γ become the geocentric vectors of the two latter bodies; and that thus the laws of the disturbed motion of our satellite are contained in the two equations (4) and (5),—but especially in the first of those equations (the second serving chiefly to express the laws of the sun's relative motion).

The part of this equation (4), which is independent of the sun's mass m'', is of the form (2), and contains the laws of the undisturbed elliptic motion of the moon; the remainder is the disturbing part of the equation, and contains the laws of the chief lunar perturbations. A commencement was made of the development of this disturbing part, according to ascending powers of the vector of the moon, and descending powers of the vector of the sun; and an approximate expression was thereby obtained, which may be written thus:

$$m''\left\{\frac{(\beta-\gamma)^{-1}}{\sqrt{\{-(\beta-\gamma)^2\}}}+\frac{\gamma^{-1}}{\sqrt{(-\gamma^2)}}\right\}=m''\frac{(\beta+3\,\gamma^{-1}\beta\gamma)}{2\,(-\gamma^2)^{\frac{3}{2}}}. \quad (6)$$

There was also given a geometrical interpretation of this result, corresponding to a certain decomposition of the sun's disturbing force into two others, of which the greater is triple of the less, while the angle between them is bisected by the geocentric vector of the sun; and the lesser of these two component forces is in the direction of the moon's geocentric vector prolonged, so that it is an ablatitious force, which was shown to be one of nearly constant amount.

Although the foregoing formulæ may be found in the Appendices already cited, to the Proceedings of the above-mentioned dates, yet it is hoped that, in consideration of the importance and difficulty of the subject, and the novelty of the processes employed, the Academy will not be displeased at having had this brief recapitulation laid before them, as preparatory to a sketch of some additional developments and applications of the same general view, which have since been made by the author. It may, for the same reason, be not improper here to state again, what was stated on former occasions, that all expressions involving *vectors*, α, α', &c., such as are considered in this new sort of algebraical geometry, and enter into the foregoing equations, admit of being translated into others, which shall involve, instead of those vectors, three times as many rectangular *coordinates*, x, y, z, x', y', z', &c., by means of relations of the forms

$$\alpha = ix + jy + kz, \; \alpha' = ix' + jy' + kz', \; \&c.; \qquad (7)$$

where $i\,j\,k$ are the three original and coordinate vector units of Sir William Hamilton's theory of quaternions, and satisfy the fundamental equations

$$\left.\begin{array}{l} i^2 = j^2 = k^2 = 1\,; \\ ij = k,\; jk = i,\; ki = j\,; \\ ji = -k,\; kj = -i,\; ik = -j\,; \end{array}\right\} \qquad (8)$$

which were communicated to the Royal Irish Academy at the Meeting of the 13th November, 1843. (See the Proceedings of that date, and the author's First Series of Researches respecting Quaternions, which Series has lately been printed in the Transactions of the Academy, Vol. XXI. Part 2.)

II. It is evident, from inspection of the equations above recapitulated, that every transformation of the *vector function*,

$$\phi(\alpha) = \alpha^{-1}(-\alpha^2)^{-\frac{1}{2}} \qquad (9)$$

which represents, in direction and amount, the attraction exerted by one mass-unit, situated at the beginning of the vector

α, on another mass-unit situated at the end of that vector, must be important in the theory of the Moon; and generally in the investigation, by quaternions, of the mathematical consequences of the Newtonian Law of Attraction. The integration of the equation of motion (2) of a binary system was deduced, in the communication of July, 1845, from a transformation of that vector function, which may now be written thus:

$$\alpha^{-1}(-\alpha^2)^{-\frac{1}{2}} = \frac{2\mathrm{d} \,.\, \alpha(-\alpha^2)^{-\frac{1}{2}}}{\alpha \mathrm{d}\alpha - \mathrm{d}\alpha\, \alpha}; \tag{10}$$

where d is, as in former equations, the characteristic of differentiation. And the hodographic theory of the motion of a system of bodies, attracting each other according to the same Newtonian law, so far as it was symbolically stated to the Academy, at the meeting of the 14th of December, 1846, depends essentially on the same transformation. In fact, if we make

$$\mathrm{d}\alpha = \tau \mathrm{d}t, \quad \alpha = \int \tau \mathrm{d}t; \tag{11}$$

and if, by the use of notations explained in former communications, we employ the letters U and V as the characteristics of the operations of taking the versor and the vector of a quaternion, writing, therefore,

$$\mathrm{U}(\alpha) = \alpha(-\alpha^2)^{-\frac{1}{2}}; \qquad \mathrm{V}.\alpha\tau = -\mathrm{V}.\tau\alpha = \tfrac{1}{2}(\alpha\tau - \tau\alpha); \tag{12}$$

the equation (2) of the internal motion of a binary system becomes

$$\mathrm{d}\tau = \frac{-\mathrm{M}\mathrm{d}\mathrm{U}(\int \tau \mathrm{d}t)}{\mathrm{V}(\tau \int \tau \mathrm{d}t)}; \tag{13}$$

where the denominator in the second member is constant, by the law of the equable description of areas. Hence, this second member, like the first, is an exact differential; and an immediate integration, introducing an arbitrary but constant vector ε, coplanar with α and τ, gives the *law of the circular hodograph*, under the symbolical form

$$\tau = \frac{\mathrm{M}(\varepsilon - \mathrm{U}\int \tau \mathrm{d}t)}{\mathrm{V}.\tau \int \tau \mathrm{d}t}: \tag{14}$$

the constant part of this expression (14) for the vector of the velocity, τ, being the vector of the centre of the hodograph, drawn from that one of the two bodies which is regarded as the centre of force; while the variable part of the same expression for τ represents the variable radius of the same hodographic circle, or the vector of a point on its circumference, drawn from its own centre of figure as the origin.

Multiplying this integral equation (14) by $\int\tau dt$, taking the vector part of the product, dividing by M, and multiplying both members of the result into the constant denominator of the second member of (13) or of (14), we find, by the rules of the present calculus,

$$\frac{-(\mathrm{V}.\tau\int\tau d\tau)^2}{\mathrm{M}} = \mathrm{S}.\varepsilon\int\tau dt + \mathrm{T}.\int\tau dt; \tag{15}$$

where S and T are the characteristics of the operations of taking respectively the scalar and tensor of a quaternion, so that, as applied to the present question, they give the results,

$$\mathrm{T}.\int\tau dt = \mathrm{T}\alpha = \sqrt{(-\alpha^2)} = r; \tag{16}$$

and

$$\mathrm{S}.\varepsilon\int\tau dt = \tfrac{1}{2}(\varepsilon\alpha + \alpha\varepsilon) = er\cos v; \tag{17}$$

where

$$e = \mathrm{T}\varepsilon = \sqrt{(-\varepsilon^2)} = \text{const.}; \tag{18}$$

while v is the *angle* (of true anomaly) which the variable *vector* α of the orbit makes with the fixed vector $-\varepsilon$ in the plane of that orbit; and r denotes the *length* of α, or what is usually called (and may still in this theory be named) the *radius* vector of the relative orbit. The first member of the equation (15) is a positive and constant number, representing the quotient which is obtained when the square of the double areal velocity in the relative orbit is divided by the sum of the two masses; if then we denote, as usual, this constant quotient (or semiparameter) by p, and observe that the constant e is also numerical (expressing, as usual, the eccentricity of the orbit), we shall obtain again, by this process, as by that of July, 1845, the polar equation of the orbit, under the well-known form,

$$r = \frac{p}{1 + e\cos v}. \tag{19}$$

This sketch of a process for employing the general transformation (10) in the theory of a binary system, may make it easier, than it would otherwise be, to understand how the following equation for the motion of a multiple system,

$$\mathrm{d}\tau = \Sigma \frac{(m + \Delta m)\mathrm{d}\mathrm{U}(\textstyle\int \Delta\tau \mathrm{d}t)}{\mathrm{V}(\Delta\tau . \textstyle\int \Delta\tau \mathrm{d}t)}, \tag{20}$$

(where $m + \Delta m$, $\tau + \Delta\tau$, are the mass and the vector of velocity of an attracting body, as m, τ are those of an attracted one, which is analogous to, and includes, the equation (13) for the motion of a binary one, and which agrees with a formula communicated to the Academy in December, 1846), was obtained by the present author; and how it may hereafter be applied.

III. The vector function $\phi(\alpha)$ in (9) may be called the TRACTOR corresponding to the vector of position α, or simply the tractor of α; and another general transformation of this tractor, which is more intimately connected than the foregoing with the problem of *perturbation*, may be obtained by supposing the vector α to receive any small but finite increment β, representing a new but shorter vector, which begins, or is conceived to be drawn, in any arbitrary direction, from the point of space where the vector α ends; and, by then *developing*, in conformity with the rules of quaternions, the *new tractor* $\phi(\beta + \alpha)$, (answering to the new vector $\beta + \alpha$, which is drawn from the beginning of α to the end of β), according to the ascending powers of this added vector β. In this manner we find

$$\begin{aligned} \phi(\beta + \alpha) &= \{-(\beta + \alpha)^2\}^{-\frac{1}{2}} (\beta + \alpha)^{-1} = \\ &\quad \{-\alpha^2(1 + \alpha^{-1}\beta)(1 + \beta\alpha^{-1})\}^{-\frac{1}{2}} \{\alpha(1 + \alpha^{-1}\beta)\}^{-1} \\ &= (1 + \beta\alpha^{-1})^{-\frac{1}{2}} (1 + \alpha^{-1}\beta)^{-\frac{3}{2}} \alpha^{-1}(-\alpha^2)^{-\frac{1}{2}}; \end{aligned} \tag{21}$$

that is,

$$\phi(\beta + \alpha) = \Sigma_{n, n'} \phi_{n, n'}, \tag{22}$$

if we make, for abridgment,

$$\phi_{n,n'} = m_{n,n'} (\beta a)^n (a\beta)^{n'} a^{-1} (-a^2)^{-\frac{1}{2}-n-n'} \tag{23}$$

where

$$m_{n,n'} = \frac{1.3..(2n-1)}{2.4..(2n)} \times \frac{3.5..(2n'+1)}{2.4..(2n')}. \tag{24}$$

The attraction $\phi(\beta+a)$ which a mass-unit, situated at the beginning of the vector $\beta+a$, exerts on another mass-unit situated at the end of that vector, is thus decomposed into an infinite but convergent series of other forces, $\phi_{n,n'}$, of which the *intensities* are determined by the *tensors*, and of which the *directions* are determined by the *versors*, of the expressions included in the formula (23); or by the following expressions, which are derived from it by the rules of the calculus of quaternions:

$$\mathrm{T}\phi_{n,n'} = m_{n,n'} \left(\mathrm{T}\frac{\beta}{a}\right)^{n+n'} (\mathrm{T}a)^{-2}; \tag{25}$$

$$\mathrm{U}\phi_{n,n'} = (\mathrm{U}\,.\,\beta a)^{n-n'} (\mathrm{U}a)^{-1} = \left(\mathrm{U}\frac{\beta}{-a}\right)^{n-n'} \mathrm{U}(-a). \tag{26}$$

Let a, b, be the lengths (or tensors) of the vectors a, β, and let C be the angle between them, which angle we may conceive to express the amount of the positive rotation, in their common plane, from the direction of $-a$ to the direction of $+\beta$; then the positive or negative rotation in the same plane, from the same direction of $-a$, to the direction of the component force $\phi_{n,n'}$, is expressed as follows:

$$\textit{angle}, \text{ from } -a \text{ to force } \phi_{n,n'}, = (n-n')\mathrm{C}; \tag{27}$$

and

$$\textit{intensity} \text{ of same component force} = m_{n,n'} \left(\frac{b}{a}\right)^{n+n'} a^{-2}. \tag{28}$$

The case $n = o$, $n' = o$, answers to the old tractor $\phi(a)$, or to a force of which the intensity is represented by a^{-2}, while its direction is the same as that of $-a$.

IV. Thus, if the vector a be conceived to begin at a point B, and to end at the point C, while the vector β shall be conceived to begin at C, and to end at A; and if we conceive an unit-mass at B to attract two other masses, regarded as collected into points, and as situated respectively at C and at A; this attraction of B will *disturb* the relative motion of A about C, if A be supposed to be nearer than B is to C, by producing a *series of groups of smaller and smaller forces*, of which groups it may be sufficient here to consider the two following.

The first and principal group consists of the two disturbing forces $\phi_{1,0}$ and $\phi_{0,1}$, and of these the first is purely *ablatitious*, or is directed along the prolongation of the side of the triangle ABC, which is drawn from C to A, and it has its intensity denoted by the expression $\frac{1}{2}ba^{-3}$, since we have for this force, and for its tensor and versor, the expressions

$$\phi_{1,0}=\tfrac{1}{2}\beta(-\alpha^2)^{-\frac{3}{2}};\quad \mathrm{T}\phi_{1,0}=\tfrac{1}{2}ba^{-3};\quad \mathrm{U}\phi_{1,0}=\mathrm{U}\beta. \tag{29}$$

The second disturbing force, of this first group, has for expression

$$\phi_{0,1}=\tfrac{3}{2}\alpha\beta\alpha^{-1}(-\alpha^2)^{-\frac{3}{2}}=\tfrac{3}{2}\alpha\beta\alpha^{-1}a^{-3}; \tag{30}$$

its intensity is *exactly triple* of that of the former force, being represented by $\frac{3}{2}ba^{-3}$; and its direction is the same as that of a straight line drawn from C to A′, if A′ be a point such that the line AA′ is perpendicularly bisected by the line BC (prolonged through C if necessary). These two principal disturbing forces evidently correspond to those which were considered for the case of our own satellite in a communication above alluded to; the second force being the one which was described in that former communication as being directed to what was there called the "fictitious moon," and was conceived to be as far from the sun in the heavens on one side, as the actual moon is on the other side, but in the same great circle.

If we now extend that mode of speaking so far as to conceive a similar *reflexion* of the sun with respect to the moon, and to call the point in the heavens so found the "fictitious sun," the moon being thus imagined to be seen midway among the stars between the actual and the fictitious sun: and if we farther imagine a "second fictitious sun," so placed that the actual sun shall appear to be midway between this and the first fictitious sun; we shall then be able to describe in words the directions of the three disturbing forces of the second group, and to say that they tend respectively, for the case of our own satellite, to these three (real or fictitious) suns. For these three *forces* will have, for their respective expressions, the three corresponding *terms* of the development of the *tractor* (22), namely, the following:

$$\left.\begin{aligned} \phi_{2,0} &= \tfrac{3}{8}\beta\alpha\beta\,(-\alpha^2)^{-\frac{5}{2}}; \\ \phi_{1,1} &= \tfrac{3}{4}\beta^2\alpha\,(-\alpha^2)^{-\frac{5}{2}}; \\ \phi_{0,2} &= \tfrac{15}{8}\alpha\beta\alpha\beta\alpha^{-1}\,(-\alpha^2)^{-\frac{5}{2}}; \end{aligned}\right\} \qquad (31)$$

of which the *intensities* are respectively

$$\tfrac{3}{8}b^2a^{-4}; \quad \tfrac{3}{4}b^2a^{-4}; \quad \tfrac{15}{8}b^2a^{-4}; \qquad (32)$$

so that they are *exactly proportional to the three whole numbers*, 1, 2, 5; while they are *directed*, respectively, to the first fictitious sun, the actual sun, and the second fictitious sun. The *disturbing force of a superior planet*, exerted on an inferior one, may be developed or decomposed into a series of groups of lesser disturbing forces, the intensities of the several forces in each group being constantly proportional to whole numbers, in an exactly similar way; nor does the application of the principle and method of development thus employed terminate here. In the applications to the lunar theory, a and b, in the recent expressions, are to be regarded as denoting the variable distances of the sun and moon from the earth; and the expressions for the forces are to be multiplied by the mass of the sun. Nothing depends, so far, on any smallness of eccentricities or inclinations.

V. The lunar theory is, very approximately, contained in the differential equation (4), provided that we regard γ as the elliptic vector of the sun, drawn from the common centre of gravity of the earth and moon; and the laws of the sun's relative elliptic motion, with respect to that centre of gravity, are then contained in the following differential equation, which takes the place of the equation (5):

$$\frac{d^2\gamma}{dt^2} = \frac{m+m'+m''}{\gamma\sqrt{(-\gamma^2)}}. \qquad (33)$$

Indeed, when we come to consider the small disturbing forces which belong to the second group, and which depend on the inverse fourth power of the sun's distance, the corresponding terms of the development of the first member of the formula (6) are then, for greater accuracy, to be multiplied by the fraction $\frac{m-m'}{m+m'}$, which expresses the ratio of the difference to the sum of the masses of the earth and moon. But if we neglect, for the present, those small disturbing terms, we may regard that formula (6) as accurate, without as yet neglecting anything on account of smallness of eccentricities or of inclinations; and even without assuming any knowledge of the smallness of the moon's mass, as compared with the mass of the earth; γ still denoting, as just stated, the elliptic vector of the sun. And thus, if the moon's geocentric vector β be changed to the sum $\beta+\delta\beta$, where the term $\delta\beta$ is supposed to depend on the disturbing force, and to give a product which may be neglected when it is multiplied by or into the expression for that force, we shall have the following approximate differential equation, by developing the disturbed or *altered tractor* $\phi(\beta+\delta\beta)$, and confining ourselves to the first power of $\delta\beta$:

$$\frac{d^2\delta\beta}{dt^2} + \frac{d^2\beta}{dt^2} - \frac{m+m'}{\beta\sqrt{(-\beta^2)}} =$$

$$\frac{m+m'}{2(-\beta^2)^{\frac{3}{2}}}(\delta\beta + 3\beta^{-1}\delta\beta\,\beta) + \frac{m''}{2(-\gamma^2)^{\frac{3}{2}}}(\beta+3\gamma^{-1}\beta\gamma). \quad (34)$$

The *disturbance* $\delta\beta$ of the moon's geocentric *vector* is thus exhibited as giving rise to an *alteration* $\delta\phi(\beta)$ in the corresponding *tractor* $\phi(\beta)$, which alteration is *analogous to a dis-*

turbing force, and occasions the presence of the first of the two parts of the second member of the equation (34): which equation will be found to contain a considerable portion of the theory of the moon.

VI. The author will only mention here two very simple applications, which he has made of this equation (34), one to the Lunar Variation, and the other to the Regression of the Node. Treating *here* the sun's relative orbit as exactly circular, and the moon's as approximately such, neglecting the inclination, taking for units of their kinds the sum of the masses of the earth and moon, and the moon's mean distance and mean angular velocity, and employing, as usual, the letter m to denote (not now the earth's mass, but) the ratio of the sun's mean angular motion to the corresponding motion of the moon, the differential equation (34) becomes:

$$\frac{d^2\delta\beta}{dt^2} = \tfrac{1}{2}(\delta\beta + 3\beta^{-1}\delta\beta\,\beta) + \frac{m^2}{2}(\beta + 3\gamma^{-1}\beta\gamma); \qquad (35)$$

in which the laws of the circular revolutions of the vectors β and γ give

$$\frac{d^2\beta}{dt^2} = -\beta; \qquad \frac{d^2\gamma}{dt^2} = -m^2\gamma. \qquad (36)$$

Assuming, from some general indications of this theory, an expression for the perturbation of the moon's vector, which shall be of the form

$$\delta\beta = m^2(A\beta + B\gamma^{-1}\beta\gamma + C\beta^{-1}\gamma^{-1}\beta\gamma\beta), \qquad (37)$$

and neglecting all powers of m above the square, we find

$$\frac{d^2\delta\beta}{dt^2} = -m^2(A\beta + B\gamma^{-1}\beta\gamma + 3^2 C\beta^{-1}\gamma^{-1}\beta\gamma\beta); \qquad (38)$$

$$\beta^{-1}\delta\beta.\beta = m^2(A\beta + C\gamma^{-1}\beta\gamma + B\beta^{-1}\gamma^{-1}\beta\gamma\beta); \qquad (39)$$

so that the three numerical coefficients, A, B, C, must satisfy the three following equations of condition:

$$-A = 2A + \tfrac{1}{2}; \qquad \text{giving } A = -\tfrac{1}{6}; \qquad (40)$$

and

$$-B = \tfrac{1}{2}(B + 3C) + \tfrac{3}{2}; \qquad -9C = \tfrac{1}{2}(C + 3B); \qquad (41)$$

giving

$$B = -\tfrac{19}{16}; \qquad C = +\tfrac{3}{16}. \tag{42}$$

Thus, if we neglect eccentricities and inclinations, and confine ourselves to the first power of the disturbing force, or to the second power of m, the perturbation of the moon's vector, produced by the sun's attraction, is composed of the three following terms :

$$\delta\beta = -\frac{m^2}{6}\beta - \frac{19m^2}{16}\gamma^{-1}\beta\gamma + \frac{3m^2}{16}\beta^{-1}\gamma^{-1}\beta\gamma\beta. \tag{43}$$

The first of these three terms expresses that the sun's ablatitious force, by partially counteracting the earth's attractive force on the moon, allows our satellite to revolve in a somewhat smaller orbit than would otherwise be consistent with the observed periodic time : the ratio of the diminished to the undiminished radius of the orbit being that of $1 - \frac{m^2}{6}$ to 1. The second term expresses a displacement of the moon, through perturbation, from its diminished circular orbit, of which displacement the constant magnitude or length bears to the radius of the undiminished orbit the ratio of $\frac{19m^2}{16}$ to unity ; while the direction of this displacement is always *from* that fictitious moon, *to* which it has been seen that one of the two principal components of the sun's disturbing force is directed : an opposition of sign which may at first surprise, but which is exactly analogous to the *contraction* of the orbit produced by the *ablatitious* force (when the periodic time is given), and is to be explained upon similar principles. Finally, the third term of the formula (43) for $\delta\beta$, expresses that with the two foregoing displacements a third is to be combined, which is, like them, of constant amount, being equal to $\frac{3}{19}$ths of the second displacement, or bearing to the radius of the moon's orbit the ratio of $\frac{3m^2}{16}$ to unity ; but being always directed *to* what, by an extension of a recently employed phraseology, might be called the second fictitious moon, being so placed that the actual moon is midway in the heavens between *this* fictitious moon and the one which was before considered. These two latter terms of (43) contain the chief laws of the *Lunar Variation;* and are easily shown to give the known terms in the expressions of the moon's parallax and longitude,

$$\delta \frac{1}{r} = m^2 \cos 2(☽ - ☉); \quad \delta\theta = \frac{11m^2}{8} \sin 2(☽ - ☉). \qquad (44)$$

It may assist some readers to observe here, that when the inclination of the orbit is neglected, the longitudes of the first and second fictitious moons are, respectively,

$$2☉ - ☽, \text{ and } 3☽ - 2☉; \qquad (46)$$

while those of the first and second fictitious suns, mentioned in a former section of this abstract, are, under the same condition,

$$2☽ - ☉, \text{ and } 3☉ - 2☽. \qquad (47)$$

VII. The law and quantity of the regression of the Moon's Node may also be calculated on principles of the kind above stated, but we must content ourselves with writing here the formula for the angular velocity of a planet's node generally, considered as depending on the variable *vector of position* a, the *vector of velocity* $\frac{\mathrm{d}a}{\mathrm{d}t}$, and the *vector of acceleration* $\frac{\mathrm{d}^2a}{\mathrm{d}t^2}$, and also on a vector unit λ, supposed to be directed towards the north pole of a fixed ecliptic. The formula thus referred to is the following:

$$\mathrm{d}☊ = \frac{\mathrm{S}.a\lambda.\mathrm{S}.\mathrm{d}^2a\,\mathrm{d}a\;a}{(\mathrm{V}.\lambda\mathrm{V}.a\mathrm{d}a)^2}, \qquad (48)$$

where S and V are, as before, the characteristics of the operations of taking the scalar and vector of a quaternion. The author proposes to give a fuller account of his investigations on this class of dynamical questions, when the Third Series of his Researches respecting Quaternions shall come to be printed in the Transactions of the Academy: the Second Series being devoted to subjects more purely geometrical; as the First Series (already printed) relates chiefly to others which are of a more algebraical character.

Dr. Apjohn read a paper on the composition and optical properties of a variety of hyalite, from Mexico.

"This mineral," he observed, "came into my possession as Professor of Mineralogy to the University, having been presented, through Mr. Ball, to the College Museum, by Professor Radice. It occurs in detached mammillary and wrinkled masses, sometimes larger than a walnut, and pellucid in a high degree. It is harder than glass, but is scratched by steel, and, from an experiment very carefully made, was found to have the specific gravity of 2.1016. It is much more frangible than quartz, but when broken exhibits something of the conchoidal fracture.

"The different properties just enumerated belong also to other well-known varieties of hyalite; and, notwithstanding its occurring in detached glassy drops of unusual size, little hope was entertained that a chemical examination of it would conduct to any new result. Upon, however, subjecting it to experiment, this anticipation was not verified, for it was found to contain much less water than any variety of hyalite whose composition has been recorded. Thus, as the mean of four experiments scarcely differing from each other, it yielded,

Silex, ...	97.48
Water, ...	2.52

results which correspond accurately with the formula

$$15SO_3, 2HO.$$

"Beudant says, that in the case of the transparent opals, by which he, of course, means the *hyalites*, the loss by calcination amounted always, in his experiments, to from eight to ten per cent.; while, in the case of the semiopaque opals, the volatile matter expelled by heat varied from five to seven per cent. And in two analyses of hyalite, the details of which he gives, the water was in one 6.35, and in the other 8.68 parts in 100. The Mexican hyalite, therefore, is quite peculiar as respects the amount of its constituent water, this being not more than two-fifths of what has been found in the least hydrated variety of the mineral whose analysis has been published.

"The most remarkable circumstance connected with this mineral remains to be mentioned. The hyalites are, in all treatises on mineralogy, described as single refractors; but, upon transmitting through the Mexican hyalite a ray of plane-polarized light, and examining it in the usual manner, with an analyzing eye-piece, it was found to possess in a very marked degree the property of depolarizing the ray. The action, however, exerted by it, appeared of an anomalous kind; for, while in all doubly-refracting minerals, whether uniaxal or biaxal, there are two positions of the crystal, in which, if interposed (for example) between two tourmalines whose axes are crossed, the light continues excluded, there is no position into which a lamina of this hyalite can be brought, by revolving it in its own plane, in which the light will not be restored.

"The action, therefore, of this mineral on polarized light is not of the ordinary kind, and can, as far as my knowledge of this difficult subject extends, only be explained by supposing that, like rock crystal in one particular direction, and certain essential oils and aqueous solutions of sugar, dextrine, &c., it possesses the power of changing the position of the plane of polarization of a plane polarized ray, or of causing it to revolve through a greater or less angle. This view would seem to be corroborated by the following experiments.

"1. A ray of homogeneous red light was made to pass through a lamina of the hyalite, on which nearly parallel planes were cut and polished, this lamina being interposed between a pair of crossed tourmalines. The light was thus restored, and, by rotating the tourmaline next the eye to the *right*, it was again very nearly extinguished.

"2. The preceding experiment was repeated with yellow light, got from a spirit-lamp with salted wick, and with the same results.

"3. Upon substituting for the light of the spirit that transmitted through a window-blind of a yellowish colour, the phenomena of the last experiment were very distinctly reproduced.

"4. In operating, by aid of the apparatus used in the preceding experiments, with heterogeneous or solar light, and making the analyser revolve, a series of tints were produced, much feebler than those exhibited by quartz, but following apparently the same law.

"From these facts I think I am justified in concluding, that this hyalite exercises the power denominated rotatory polarization. This property, too, it possesses, no matter in what direction it is traversed by the plane polarized ray, a circumstance which would seem to identify it with that exerted by liquids, and distinguish it from the analogous influence of quartz, which is manifested only in the direction of the optic axis.

"Before concluding this paper I may be permitted to give expression to an opinion, which is, I believe, shared by most mineralogists, namely, that hyalite, opal, and calcedony, have all had a similar origin, or were originally silex in the gelatinous condition which it assumes when certain alkaline silicates are dissolved in dilute acid, and their solutions are slowly evaporated. All three include water, the hyalite in greatest, and the calcedony in smallest quantity; and in some specimens that I have seen, these minerals may be observed to pass by insensible gradations into each other. This is well illustrated by some lumps of the Mexican hyalite which have nodules of calcedony attached to them, the latter mineral differing in no respect from the former, save in being less transparent, and containing a smaller quantity of water,—its amount being about 0.53 per cent., according to a single experiment. The water, too, in all, is, I believe, present, not in a state of chemical, but of mechanical union, and gives to these minerals the different degrees of transparency which they exhibit, just as water confers the same property upon specimens of hydrophane or tabasheer, into whose pores it has been introduced by capillary absorption.

" From the analogy as to chemical constitution between hyalite and calcedony, one might be inclined to conclude that both would modify light transmitted through them in a similar manner. This idea, however, would appear to be inconsistent with experiment, for a plate of agate is said to polarize light in the ordinary manner, or in a single plane; and Sir David Brewster is of opinion that the action which it exerts is of the same nature with that which belongs to tourmaline. These statements are certainly not true of the mineral which I have this evening brought under the notice of the Academy.

" P. S.—Since the preceding remarks were put together, I have found that a ray of light polarized by reflexion, and made to disappear by the interposition of a Nicholl's prism, was restored when made to pass through a particle of hyalite from Frankfort on the Maine. *All* varieties, therefore, of this mineral may be concluded to possess the same optical properties.

"In respect to the experiments above detailed, I have found that the results are somewhat different, according to the part of the lamina of hyalite traversed by the light, the change in the position of the plane of polarization not having always the same direction or value, and being sometimes null. This fact would seem to point to a different explanation of the phenomena from that already suggested, and to identify the optical characters of hyalite with those of rock crystal, the differences being explicable upon the hypothesis of the former mineral being composed of a multitude of minute crystals of the latter, thrown together without any regular or symmetric arrangement."

Rev. T. R. Robinson, D. D., read a paper on the effect of Heat in lessening the Affinity of the Elements of Water.

The author referred to the experiments of Mr. Grove on the decomposition of water by the action of incandescent platinum; and, after noticing the objections which were urged against its being caused by heat, detailed results which he had obtained at a much lower temperature, and which appeared to him to accord with that hypothesis.

Proceeding on the theory of the voltaic circuit, which Ohm has given, he investigated the diminution of electric intensity, which is caused by placing in the circuit a cell where water is subjected to voltaic decomposition, and shewed that it is equal to the affinity of platina for oxygen *minus* twice that of hydrogen for oxygen, or

$$e = op - 2ho.$$

This quantity e can be measured by the instruments and processes described by Mr. Wheatstone in the Philosophical Transactions for 1843, with some modifications, which, in the author's opinion, increase their accuracy. After describing the apparatus he used, he finds for the value of e,

At the temperature 61° . . .	e = 598.9 . . .	12 obs.
135°.4	567.6 . . .	13
201°.3	531.0 . . .	12

These give, for an increase of temperature of 100°, the decrease of the affinity of the oxygen and hydrogen of water = 23.2. The author applies to this result the theory of probabilities, which has so much advanced astronomical and physical science; and finds the chances to be 10,000 to 1 that it is not all error of observation.

It might be objected, that this diminution of e is due to the expansion of the gases by heat enabling them to escape more freely from the electrodes. This was tested by placing the apparatus under the air-pump, and measuring the electrolytic resistance at a pressure of 1.1 inches. This gives

$$\text{at } 64^\circ.6 \; . \; . \; e = 589.6,$$

and it is shown, that the chances are 3 to 2 that the unexplained difference is mere error of observation. The mere escape of the gas, therefore, does not change e.

This change of temperature produces no alteration of metallic affinities, as is shown by the intensity of Daniell's cell being the same at 64° and 163°. The expression of this is $e = zo - 2cu.o$. That for a cell excited with dilute sulphuric acid $= zo - cu.o - ho$, and it is found to decrease 27.9 for 100°. The mean of all gives 25.1; and, if we might suppose this rate to be uniform through the thermometric scale, it would give 2386°, midway between the melting points of gold and cast-iron, for the temperature at which this affinity would cease.

The author concludes by expressing his doubts, that the combination of these gases is in any case produced by heat; and suggests that light is more probably the agent when the combustion is rapid, and the capillary force of the surfaces in contact with them, at lower temperatures, aided by some actinic influence extricated by the heat. Finally, he points out as a promising subject of mathematical research, the application of the undulatory theory to the phenomena of conducted and latent heat.

Sir William R. Hamilton read a paper by Professor Young, of Belfast, on an extension of a theorem of Euler.

The object of the author is to extend and generalize the theorem of Euler,—that the sum of four squares, multiplied by the sum of four squares, produces the sum of four squares. He commences by examining into the construction of the four-square formula, with the view of ascertaining whether any thing like a definite law or principle connects its component parts together; and from which a formula for a greater number of squares might be suggested. Such a principle is found to govern the generation of the four-square results, when these are arrived at by a peculiar process, which the author exhibits. The same process is then extended to the case of eight squares; and it is found that

$$
\begin{aligned}
&(s'^2 + t'^2 + u'^2 + v'^2 + w'^2 + x'^2 + y'^2 + z'^2)\times \\
&(s^2 + t^2 + u^2 + v^2 + w^2 + x^2 + y^2 + z^2) = \\
&\quad (ss' + tt' + uu' + vv' + ww' + xx' + yy' + zz')^2 \\
&+ (st' - ts' + uv' - vu' + wx' - xw' + yz' - zy')^2 \\
&+ (su' - us' + vt' - tv' + yw' - wy' + xz' - zx')^2 \\
&+ (sv' - vs' + tu' - ut' + wz' - zw' + xy' - yx')^2 \\
&+ (sw' - ws' + xt' - tx' + uy' - yu' + zv' - vz')^2 \\
&+ (sx' - xs' + tw' - wt' + yv' - vy' + zu' - uz')^2 \\
&+ (sy' - ys' + zt' - tz' + vx' - xv' + wu' - uw')^2 \\
&+ (sz' - zs' + ty' - yt' + vw' - wv' + ux' - xu')^2.
\end{aligned}
$$

These results are verified by the actual development of the several squares; which development, by the mutual cancelling of all the double products, reduces itself to the sixty-four squares furnished by the product of the proposed factors, when multiplied together in the ordinary way.

The author then enters into a more minute examination of the constitution of the preceding polynomial; and shows that the cancelling of the aforesaid double products is a necessary consequence of that constitution.

It is further shown that the product continues to be of the same form as each of the factors, when the coefficients a^0, a^1, a^2, a^3, &c., are introduced in order, in connexion with the squares entering those factors.

Sir William Rowan Hamilton stated also a theorem respecting products of sums of eight squares, which does not essentially differ from the foregoing, and was communicated to him by John T. Graves, Esq., about the end of the year 1843.

One form of the theorem is the following:

$$
\begin{aligned}
&(00' - 11' - 22' - 33' - 44' - 55' - 66' - 77')^2 \\
&+ (10' + 01' - 32' + 23' - 54' + 45' - 76' + 67')^2 \\
&+ (20' + 31' + 02' - 13' - 74' - 65' + 56' + 47')^2 \\
&+ (30' - 21' + 12' + 03' - 64' + 75' + 46' - 57')^2 \\
&+ (40' + 51' + 72' + 63' + 04' - 15' - 36' - 27')^2 \\
&+ (50' - 41' + 62' - 73' + 14' + 05' - 26' + 37')^2 \\
&+ (60' + 71' - 52' - 43' + 34' + 25' + 06' - 17')^2 \\
&+ (70' - 61' - 42' + 53' + 24' - 35' + 16' + 07')^2 \\
&= (0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2)\ (0'^2 + \\
&\qquad 1'^2 + 2'^2 + 3'^2 + 4'^2 + 5'^2 + 6'^2 + 7'^2).
\end{aligned}
$$

In a letter dated January 18th, 1844, Mr. Graves communicated to Sir William Hamilton his theorem respecting sums of eight squares under the form:

$$
\begin{aligned}
&(a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2)\times \\
&(a'^2+b'^2+c'^2+d'^2+e'^2+f'^2+g'^2+h'^2) = \\
&a''^2+b''^2+c''^2+d''^2+e''^2+f''^2+g''^2+h''^2;
\end{aligned}
$$

where $a'' \,.\,.\, h''$ denoted the following expressions;

$$
\begin{aligned}
a'' &= aa' - bb' - cc' - dd' - ee' - ff' - gg' - hh'; \\
b'' &= ba' + ab' - dc' + cd' - fe' + ef' + hg' - gh'; \\
c'' &= ca' + db' + ac' - bd' - ge' - hf' + eg' + fh'; \\
d'' &= da' - cb' + bc' + ad' - he' + gf' - fg' + eh'; \\
e'' &= ea' + fb' + gc' + hd' + ae' - bf' - cg' - dh'; \\
f'' &= fa' - eb' + hc' - gd' + be' + af' + dg' - ch'; \\
g'' &= ga' - hb' - ec' + fd' + ce' - df' + ag' + bh'; \\
h'' &= ha' + gb' - fc' - ed' + de' + cf' - bg' + ah'.
\end{aligned}
$$

In a letter of somewhat earlier date, but evidently written in haste, upon a journey, and dated December 26th, 1843, analogous expressions had been given, containing, however, some errors in the signs, which were soon afterwards corrected as above. That earlier letter also indicated an expectation that a theory of *octaves*, including a new and extended system of imaginaries, which had thus been suggested to the writer (J. T. Graves, Esq.) by Sir William R. Hamilton's theory of *quaternions*, might itself be extended so as to form a theory of what Mr. Graves at the time proposed to call 2^m-*ions*: but in a letter written shortly afterwards, doubts were expressed respecting the possibility of this additional extension, from octaves to sets of sixteen.

The special thanks of the Academy were given to Lord Farnham, for his Donation to the Museum of the following Collection of Antiquities:

Two Gun Barrels, one Gun Lock, three Cannon Balls, several fragments of Cannon Balls and Shells, four Leaden Bullets, and one Sword, from Cloughoughter Castle.

Curious ancient Keys, from Newtownbarry, and Ballyjamesduff.

Fifty-five Flint Arrow-Heads.

Six Flint Knives.

Twenty-four Bronze Celts, of different patterns.

An ancient Hand-mill, perfect; the top Stone of another; and a round Grind Stone.

An old Glass Bottle, from Cavan.

A Sling Stone, from Dunshaughlin.

One perfect Bronze Sword, found at Corchor, Co. Cavan, and fragments of another.

Five Blades of Bronze Daggers.

Two Bronze Spears.

Four Bronze Pins, with Rings.

Two Bronze Bosses.

A Brass Spoon.

A Brass Spur, found at Shaneloon, Co. Cavan.

A Brass Cross.

A Silver Cross.

A Brass Pistol.

A Snuff Box made from a Cannon used at the Siege of Derry.

A Snuff Box found on the battle ground at Vinegar Hill.

A fragment of an ancient illuminated Breviary, found in a bog at Myshal, Co. Carlow.

Two large BrassPots, and an Iron one, from the Lake of Virginia.

An ancient Bronze Bucket-shaped Pot.

Three Methers.

A small Wooden Cup.

Four Grey-beard Jars.

One beautiful cinerary Urn, found at Killinagh, "lying in the centre of six large stones, placed perpendicularly, and opposite to each other, three at either side. Near the spot was a large, rough flag, supposed to have been originally covering the stones. The urn was buried about two feet under ground, and when found was nearly full of ashes, and the place about bore evident marks of fire; no lid was found, nor any other marks of its being a place of burial."—*Extracted from the Rev. C. S. Montgomery's Note to Lord Farnham.*

Two Shoes found in a bog.

Ten Spindle Stones, and two Weights.

Two Stone Hammers.

One square perforated Stone.

One triangular Jet Ornament.

Two Flint and eighteen Stone Celts.

One Sharpening Stone.

A Stone shaped like a Cow's Foot.

Two perfect Bronze Trumpets, and one imperfect, found at Coracanway, Co. Cavan.

Three Iron Stirrups.

A fragment of shed Horn of the Irish Elk, found at Lisduff, Co. Cavan.

A Letter, found in the pocket of the Rev. Mr. Murphy, who was killed at Arklow, on the 9th of June, 1798.

PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

1846–7. No. 69.

June 28th, 1847.

REV. HUMPHREY LLOYD, D. D., President, in the Chair.

His Grace the Archbishop of Dublin was elected a Member of Council, on the Committee of Polite Literature.

It was Resolved,—That the sum of £50 be placed at the disposal of the Council, for the purchase of antiquities.

The Secretary read the draft of an Address to His Excellency the Lord Lieutenant, which had been ordered at the last Meeting.

Resolved,—That the Address now read be adopted by the Academy.

John Anster, LL.D., read a paper by the Rev. James Wills, in continuation of his former papers on "the Association of Ideas."

The author commenced by stating, that, according to the view to which he had been led, the subject might be divided under three heads of inquiry:—the class of associations formed by habits of action and perception, as described in his first Essay; those from accidental association, explained in the second; and those next to be considered, originating in the mind itself. From the first he had endeavoured to trace the main stock of human ideas and capabilities of action; to the second he had traced the process of memory; the third he would show to be mainly instrumental in invention, and in various ways operative in art and literature, as also upon human character and conduct, and, lastly, upon the operations of judgment and reasoning.

The formation of new ideas by the mind might, he observed, be effected by means, not directly to be described as single operations of thought; of this nature were purely artistic ideas, which might be framed by rules according to certain models, and then become ideas of association or not, according to circumstances. Such results were excluded from the author's inquiry, and were only mentioned to guard against any misconception, and for the sake of a distinction, which would be available in his illustrations.

The process at present to be considered by the author is mainly distinguishable from that examined in his first Essay, by the fact that, while the first class of associations were framed gradually from the immediate repetition of acts or perceptions, those now to be explained were instantaneously put together from general *analogies*, which were, however, themselves framed from habitual experience, like the former. These analogies are insensibly contracted through life, and are the nearest approach to universal ideas, consisting of characters, forms, colours, proportions, and properties, which are variously combined throughout all known existence. Such are the elements of conceptual power, or the faculty of spontaneous association, of which the action and exercise could be variously determined by the habits and character of each individual.

The author briefly exemplified the mode of operation; and went on to say that he would pursue the subject in relation to literary composition and art,—to moral sentiment,—and, lastly, to the operations of reasoning.

From this the author gave an explanation shewing the justice of Mr. Locke's distinction between wit and judgment.

The first head of this division might be regarded as coincident with those mental operations commonly included under the term *imagination*, which he would occasionally use, as a convenient term, and of familiar use.

The author next proceeded to show that, as the ideas he had described were essentially those of sensible properties, it must be a consequence that their combination must be sensible associations, and therefore affecting the mind, in whatever degree, in the same manner as the sensible presence of such objects, had they any real or external existence. Such effects would be very indistinct in some (probably in most) minds, and very intense in others. It would be apparent from these considerations, that in one class of writers, or artists, the mind constructs a combination by mere rules, and in another from a distinct and sensible conception; and further, that in the analysis of writings or works of art, some indications might be discoverable of these two different modes of operation.

There was also another consideration, which, though seemingly leading to a difficulty, would very much tend to aid in the clear exemplification of this process. Its nature being to produce effects similar to the known effects of present reality, may be traced more clearly by comparison with them. The author would, he said, avail himself of this inference as a means of illustration.

He then proceeded to cite various examples from standard poets, in which he traced the indications of distinct presence, or conception of presence, which he severally contrasted with conjectural cases of the opposite mode of artistic construction, without those conceptions.

The author next proceeded to notice other kinds of examples in the conception of characters, events, and in translation of the thoughts of others, in which he showed that the difference of language could frequently be only remedied by equivalents supplied by the aid of the original conception.

The Secretary read the following communication from Sir William Betham, on a leaden seal of Herbert de la Mara, now preserved in the Museum of the Academy:

" This is a singular matrix, inasmuch as lead is a very unusual material for a seal of any age, but especially of the antiquity I take this to be.

" Considering the material of which it is made, it is very perfect. Herbert is in armour on horseback, in full gallop, as princes and great nobles of that day were represented; on his left arm is the triangular shield of the twelfth and thirteenth centuries; in his right, which is extended, is something like a short club or mace. The bit of the bridle is very long and severe. Round the circle is the following inscription:

+ SIGILLVM: HEREEBERTI DELAMARA.

" When Mr. Underwood first produced this seal to me, I considered that it was the seal of Herbert Delamare, who lived in the reign of Edward III.; at the same time, I thought the shield indicated a higher antiquity, and further investigation convinced me it was of the end of the twelfth or beginning of the thirteenth century.

" In a MS. in my possession I find the following notes respecting the Delamares:

" ' Richard Delamara was witness to a charter of William Rufus to the monastery of Bermondsey.

" ' Henry Delamare was huntsman to the King, 5 Stephen.

" ' Richard Delamare, said to be son of Henry, was sheriff of Oxfordshire, 34 Hen. II., and of Oxford and Bucks, 1 & 2 Ric. I.

" ' Geoffrey Delamare had lands in Berkshire, 10 John.

" ' John Delamare, his grandson, was knighted 34 Edw. I., and summoned to the Parliament of England 6th February, 34 Edw. I. (1305), as Baron Delamare.'

" The name is evidently French, but as I do not find it in Battle Abbey Roll, or in Brampton's List, I should suppose the first who came to England must have followed the Conqueror after he was settled as King of England, or in the reign of William Rufus.

" In another MS. in my possession I find the following passage, which I consider refers to the possessor of this seal:

" ' Herbert Delamare, of Norman French extraction, and a leading man, came to Ireland on the first invasion thereof by the English (with Hugh de Lacy, who obtained the grant of the kingdom of Meath from King Henry II., to be held in as ample a manner as O'Melaghlin held it; in fact the King granted him an honour, or Palatinate, as ample as it could be granted: the words are "*quod ibi habeo vel illi dare possum.*") He obtained ample grants of great possessions in the western parts of Meath, of which he was made governor, and from him all the Delamers of Ireland were descended. From him this family were called by the Irish *Mac Erbert.*

" ' William Delamer, son of Herbert, lived in the reign of Henry the Third, and founded and endowed the Abbey of Multifernam.

" ' John Delamer, a powerful nobleman, built the strong castle of Maghbreacy, in the country of Annaly, now called the county of Longford, and made it his chief seat in 1294, which it continued until the family were deprived of their estates by Cromwell. In that year he joined John Fitzgerald, Baron of Ophaley, against Richard De Burgo, Earl of Ulster, and took him prisoner, and confined him in the Castle of Leix. This John was slain by the O'Ferralls of Annaly.

" ' Sir William Delamare, of Herbertstown, in the County of Meath, was living in 1322, and was married to a lady named Margery, and was father of

" ' Herbert Delamare, who entered into recognizance for the fealty of all his clan, 1347; his son was

" 'John Delamare, of Herbertstown, was living in 1354, whose eldest son, William, died before his father, leaving a son,

" 'John Delamare, of Herbertstown, who succeeded his grandfather, and was living 11 Richard II., 1387.

" 'Herbert Delamare, of Herbertstown, was living 1419, and was father of two sons,—Meyler, who left a son, Herbert, who left a son, Herbert, who died 10th November, 1580, without issue;—and Walter, who was father of Theobald, whose son, Walter, succeeded to the estate on the death of Herbert, and died 1613, and was succeeded by his grandson, Richard, who was twenty-six years of age, in 1613.' "

A note by the Rev. Charles Graves was read, "on the Solution of linear differential Equations, and other Equations of the same kind, by the Separation of Symbols."

Mr. Graves gives two distinct developments, which furnish solutions of the linear differential equations of the nth order. The terms of one of these developments are obtained by successive quadratures; and the solution so arrived at is the general solution of the proposed equation, containing n arbitrary constants.

All the terms of the other development are obtained by mere differentiation; but the solution itself is only a particular integral.

The solution of the linear differential equation of the second order may be obtained in a sufficiently simple form by this method, which applies itself with peculiar facility to the treatment of equations of differences.

Rev. George Salmon read a note "on the Generation of Surfaces of the Second Degree."

" Mr. Ingram having, at a late Meeting of the Academy, alluded to a mode of generating surfaces of the second degree which I obtained a few years ago, I wish to state the method by which I arrived at it, as it involves a principle of very useful application in the theory of reciprocal polars.

" Given a point and a plane,—if we take the reciprocal plane and point with regard to any sphere, the ratio of the distances of the first point from the centre of the sphere, and from the first plane, is equal to the ratio of the distances of the second point from the centre of the sphere, and from the second plane.

" Hence, since in a sphere the distance of any tangent plane from the centre is constant, the reciprocal of a sphere is a surface such that the distance of any point from a fixed point is to its distance from a fixed plane in a given ratio.

" Again, since in an ellipsoid of revolution round the axis major the product of the distances of any tangent plane from the foci is constant, the reciprocal of such an ellipsoid is a surface such, that the square of the distance of any point from a fixed point is in a constant ratio to the product of its distances from two fixed planes. Or, more generally, if a surface be such, that the product of the distances of a tangent plane from n fixed points is constant, the reciprocal surface will be such, that the ratio of the nth power of the distance of any point from a fixed point, to the product of its distances from n fixed planes, will be constant.

I add one or two instances of transformations of plane curves by the same principle.

In a conic section the product of the distances of any point on the curve from two fixed tangents, is in a constant ratio to the square of the line joining their points of contact. Hence, the square of the distance of any tangent to a conic section from a fixed point, is in a constant ratio to the product of its distances from the two points of contact of tangents drawn from the fixed point.

" As a particular case of this, we derive the well-known property,—any tangent to a conic will intercept, on two fixed parallel tangents, segments whose rectangle will be constant.

" This theorem may be extended as follows: If a curve be such that only three tangents can be drawn parallel to a given line, any tangent will intercept, on three such tangents, segments whose product will bear a given ratio to the segment intercepted on another fixed parallel."

The Secretary read a communication from Mr. Bindon, and presented a tracing from the original credentials which Father Hugo de Burgo carried with him to Belgium in 1644.

" With the view of making my search as useful and complete as possible, I thought it advisable, after having examined the Burgundian Library, to visit the *Archives du Royaume* of the Belgian government.

" In that collection the original document, a *fac simile* of which is now presented to the reader, will be found. It is the credential which Father Hugh de Burgo, a Franciscan friar, carried with him when visiting Belgium, as the representative of the Irish Catholic Confederates assembled at Kilkenny in the year 1644.

" It is presumed that this document is of some historical interest, presenting as it does the signatures of several leading members of the " Supreme Council," including those of its President, Richard Butler Viscount Mountgarret, and the secretary, Sir Richard Belling. The other signatures are those of Thomas Preston, the brother of Lord Gormanstown, and commander of the confederate forces in Leinster; Thomas Walsh, Archbishop of Cashel; Malachy Quelly or O'Kelly, Archbishop of Tuam, who was killed leading an attack upon Sligo; Sir Daniel O'Bryen, one of the Protestants who had joined the Catholic Confederates; Robert Lynch, and Terence O'Shaugnessy. The ninth name I have not been able to decipher.

" This mission of Father de Burgo to the Low Countries has not been noticed, as far I can discover, by any historians treating of the wars of 1641–1652. Carte, in his Life of the Duke of Ormond, states that " Father Hugh Burke" was sent to Spain: and in the "Unkinde Deserter" we read of a Bishop being sent to the Duke of Lorraine, in company with other persons, in the year 1651: but in these authorities there is no mention whatever of the embassy which this paper helps to disclose. It is believed that Father de Burgo was afterwards Bishop of "Duacensis in Anglia."—*Hibernia Dominicana*, p. 490.

"Besides this document, a large collection of papers relative to the Stapleton foundation in the University of Louvain will be found among the Archives. They are not of general interest, being principally the pedigrees of adverse claimants to the many bourses which were bequeathed by the founder to the Irish pastoral college of Louvain; and as these bourses are yet in existence, the documents may be of much use to the different members of Dr. Stapleton's family in proving their respective claims.

"A few letters giving an official account of the reception of the Duke of Cumberland, after his landing in the Low Countries on the eve of the battle of Fontenoy, and a short memoir of the Irish College of Lille, complete the collection as far as Irish history is concerned."

The special thanks of the Academy were given to William R. Wilde, Esq., for his Donation of a large collection of the remains of oxen, sheep, goats, dogs, &c., found at Dunshaughlin, described by him to the Academy, April 27, 1840,* and since deposited for inspection in the Museum of the Academy. Mr. Wilde also presented an ox head of the ancient short-horned variety, from Navan, similar to those found at Dunshaughlin; and eleven specimens of Bronze Celts, and one Brass Spur.

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* See Proceedings, Vol. I. p. 420.

DONATIONS.

A Treatise on Atmospheric Phenomena. By Edward Joseph Lowe, Esq. Presented by the Author.

A Reply to Mr. Babbage's Letter to the Times, on the Planet Neptune, and the Royal Astronomical Society's Medal. By the Rev. R. Sheepshanks, V. P. R. A. S., &c. Presented by the Author.

A Cannon Ball, found in the Townland of Baltrasna, Mullingar. Presented by W. R. Wilde, Esq.

Comptes rendus Hebdomadaires des Séances de l'Académie des Sciences. From 5th to 7th June, 1847. Presented by the Academy.

Report on Lunatic Asylums in Ireland, 1846. Presented by Dr. White.

Address to the Geological Society. By Robert Mallet, Esq., President. Presented by the Society.

Journal of the Geological Society of Dublin. Vol. III. Parts 3 and 4, 1847. Presented by the Society.

The Traveller in the East. By Godfrey Levinge, Esq. Presented by the Author.

Inventaire des Manuscrits de l'ancienne Bibliothèque Royale des Ducs de Bourgogne. Nos. 1 to 18,000. Presented by S. H. Bindon, Esq.

A Stone Celt, from the vicinity of the Tumulus at Killiney. Presented by the Rev. W. Wildbore.

Two Ivory Figures found at Balasore, East Indies, by Frederick Furnell, Esq. Presented by W. Furnell, Esq.

A Silver (*cancelled*) *Seal of the Board of First Fruits in Ireland; a Seal of the Right Rev. Nathaniel Alexander, D. D., Bishop of Meath* (1823) *; and a Seal of the Right Rev. Nathaniel Alexander, D. D., Bishop of Down and Connor* (1840).

An ancient Shoe, made of carved Leather. This shoe was found in a turf bog at Ballymacomb, county of Derry, at the depth of seven feet. It was discovered lying between the branch and trunk of a fir tree. Presented by Miss Alexander.

Die Ueberbleibsel der Altägyptischen Menschenrace. Von Dr. Franz Pruner. Presented by the Author.

Abhandlungen der Mathematisch-Physikalischen classe der Koeniglich Bayerischen Akademie der Wissenschaften. Part 4, Vol. III. of this class, or nineteenth Volume of the whole series.

Gelehrte Anzeigen. Vols. XVI.—XXIII.

Bulletin der Königl. Akademie der Wissenschaften. For the Year 1846. Nos. VI.—LXXVII. Presented by the Royal Academy of Munich.

A Copy of the Census of Ireland for 1841. Presented by W. R. Wilde, Esq.

Ecclesiastical Antiquities of Down, Connor, and Dromore, with Notes and Illustrations. By the Rev. William Reeves, M. B., M. R. I. A., &c. Presented by the Author.

Journal of the Franklin Institute. Vol. XII. Third series. Presented by the Institute.

Patent Rolls of James I. and Henry VIII. Presented by Sir William Betham.

An Ancient Key. Presented by Aquilla Smith, M. D.

APPENDIX.

No. I.

CORRESPONDENCE,

READ TO THE ACADEMY, JANUARY 27TH, AND FEBRUARY 24TH, 1845,

FROM

THE MINUTES OF COUNCIL.

I.

"*Dublin Castle, 18th Jan.*, 1845.

"GENTLEMEN,—I am directed by the Lord Lieutenant to transmit the accompanying statement, which has been received from Sir William Betham, for any observations the Council of the Royal Irish Academy may wish to offer for his Excellency's information.

"I am, Gentlemen, your obedient Servant,

"ELIOT.

"*The Secretaries to the*
Royal Irish Academy, &c. &c."

"*Dublin Castle, 15th Jan.*, 1845.

"MY LORD,—I take the liberty of drawing your lordship's attention, and that of Her Majesty's Government, to the following brief statement of facts.

"In the year 1830, the Council of the Royal Irish Academy put an advertisement into the public journals, offering apremium of fifty pounds, with the Gold Medal, for the best Essay on the Origin and Use of the Round Towers of Ireland.

"On the 17th December, 1832, they awarded the money and Medal to Mr. George Petrie, one of the members of the Council, for his Essay, as the best of those sent in. The money was paid accordingly, and the medal was delivered. Mr. Petrie was then permitted to take away the manuscript, to prepare it for the Press; but he never returned it to the Council for publication, though frequently urged to do so. In the year 1840, eight years after the delivery and payment of this medal and prize, myself and other members of the Academy called upon the Council to account why this Essay, which had excited so much interest, had not as yet appeared in the Transactions. In reply, we were assured that it then was, or very shortly would be, in the Press.

"In July, 1844, we again inquired when the Essay was to appear, and were told it would be published by the 1st January, 1845. I, therefore, gave notice that I would move at the next meeting of the Academy for certain returns respecting the proceedings of the Council. On the 30th November these returns were laid on the table of the Academy, in compliance with my notice, without motion. By these returns it appears that the Council had permitted Mr. Petrie to enlarge his Essay (which, when read and adjudged, consisted of about fifty pages), so much as to occupy an entire volume of the Transactions (about 500 pages), and had expended £144 in wood-cut engravings, to illustrate it. They also appointed a committee to confer with Mr. Petrie (himself a member of Council), relative to the publication of his Essay on the Round Towers, who, on the 29th June, 1840, reported the following proposition from Mr. Petrie:

"'*22nd June,* 1840

"'I propose to supply the Academy with 400 or 450 copies of my Essay on the Round Towers, at thirty shillings per copy, printed in the form of the Transactions.

"'Signed, GEORGE PETRIE.'

And the Committee recommended the Council to adopt the above proposal, and to request Mr. Petrie to send the work to Press immediately.

"The Council decided that the proposal was not sufficiently explicit, and 'Resolved, that Mr. Petrie be allowed to substitute the following:

"'*29th June*, 1840.

"'I propose to publish, at my own expense, my Essay on the Round Towers, as the twentieth volume of the Transactions, on condition that the Academy take from me 450 copies, at the rate of thirty shillings per copy; the expense already incurred by the Academy for engravings to be deducted from the £675 to be paid for the 450 copies. It is, of course, understood, that the blocks for woodcuts are my property.

"'Signed, George Petrie.'

'Resolved, that the proposal in the latter form be adopted.'

"From this it appears, that the Council, in 1840, had alienated to Mr. Petrie the copyright of the Essay for which £50 had been paid, and a Gold Medal adjudged, in 1832! And further, had allowed that gentleman to print the work for his own benefit, virtually at the expense of the Academy; for by the acceptance of the proposal of 29th June, 1840, the Council agreed to pay thirty shillings a copy for 450 copies of the volume, that being the selling price of the volumes of the Transactions, which amounts to £675, and includes the bookseller's profit, which may be estimated at least at thirty-five per cent., thus making Mr. Petrie a present of £234 13*s.* 4*d.* of the Academy's money, which would pay for a considerable edition for his own benefit, and which they might have saved, by printing the work themselves. It should here be observed, first, that it is contrary to the laws of the Academy to return the manuscripts even of unsuccessful competitors for prizes and medals, while they claim as property all Essays read at their ordinary meetings, and ordered to be printed; and secondly, that the Council have no power to expend more than twenty pounds, without the consent of the Academy at large.

"In March, 1832, the Council inserted another advertisement in the public journals, offering a premium of £50, and a Gold Medal, for the best Essay on the Remains of Ancient Military Architecture in Ireland.

"In March, 1834, the Council awarded and paid the Gold Medal and premium aforesaid to the same Mr. George Petrie, himself a member of the Council, and the proposer of the question, as he was also of that of the Round Towers; and, as in that case, he took away the manuscript, to prepare it for the Press; but, from 1834, to the present hour, no step appears to have been taken by Mr. Petrie for its publication.

"In the year 1838, Mr. Petrie read an Essay on Ancient Bells before the Academy, which was referred to the Council for publication; for the embellishment of which, the sum of £31 10*s*. appears to have been paid to one artist, and £44 to another, for engraving copperplates to illustrate it; which Essay has not appeared in the Transactions, nor does it appear that the Council have taken any steps for its publication.

"In the year , another Gold Medal was awarded to the same Mr. Petrie, for what was called an Essay on Tarah, which was read before the Academy, as was alleged, by permission of Colonel Colby, it having been prepared, under the direction of the Ordnance Survey, by the persons employed thereon, of whom, I believe, Mr. Petrie was *one*. This work was certainly the production of several.

"I have thought it right to lay these facts before your Lordship, in consequence of having failed in prevailing on the Council and the Academy to correct these deviations, as I consider them, from the correct mode of conducting the affairs, and disposing of the funds of the Academy intrusted to the Council. Your Lordship's letter to the Academy of the 23rd November, 1843, calling for an account of 'how far the objects of the Academy have been attained, and the circumstances under which the public grant to the Academy has been made and continued to the Institution,' renders it necessary that Government should be in possession of correct information.

"The returns made to the Academy by the Council of 30th November, 1844, also contain statements of the assets, debts, and engagements of the Academy.

"I have the honour to be your Lordship's
"Obedient Servant,
"W. Betham, M. R. I. A.

"*To Right Honourable the
Lord Eliot, &c. &c.*"

II.

"*Royal Irish Academy,*
"*23rd Jan.*, 1845.

"My Lord,—In compliance with the desire of the Lord Lieutenant, as communicated to the Council of the Royal Irish Academy, in your Lordship's letter of the 18th of January, I am directed by the Council to offer the following observations for the information of His Excellency, with regard to the representations made by Sir William Betham to Her Majesty's Government, concerning the mode in which the business of the Academy has been conducted.

"The Council, in the first place, beg to say, that nothing would give them greater pleasure than an inquiry, on the part of the Government, into the management of the affairs of the Academy; that body, as is very well known, having for some years back displayed a degree of energy and efficiency, which has gained for it the entire confidence of the public. But an inquiry of such a kind, on the part of Sir W. Betham, is not a thing to be encouraged by the Council; and, that it is not a thing which the Academy are disposed to countenance, Sir W. Betham was given to understand, by a vote of the Academy, on the 13th of the present month, when he attempted to bring forward there those charges against the Council, which he has since thought proper to carry before the Government.

"In the next place, the Council have to express their great regret that the different Essays of Mr. Petrie, about which Sir W. Betham shows so much concern, have not yet been published; but they beg to observe, that the peculiar circumstances of the case have thrown entirely into the author's hands the publication of the Essay on the Round Towers of Ireland,—and that, while he is employed upon this, which is a very large work, it would be useless to press upon him the publication of the others. Mr. Petrie, however, has addressed to the Council a statement of the causes which have retarded the appearance of his Essays, and this statement is herewith transmitted to your Lordship.

"The Council also take leave to transmit two Estimates—one from the Publishers of Mr. Petrie's Essay on the Round Towers, the other from the Printer of that work. By the first of these it appears that the publishing price of the volume which Mr. Petrie has engaged to give to the Academy for thirty shillings (and the printing of which is now greatly advanced), cannot be less than two guineas, and is more likely to be 2½ guineas. By the second estimate it appears that if the work were printed by the Academy, as a volume of its Transactions, the impression being limited, as usual, to 500 copies, each copy would cost the Academy above three pounds; that is, more than double the price for which Mr. Petrie has undertaken to supply it. Yet Sir W. Betham has not hesitated to state, that the Council have allowed Mr. Petrie 'to print the work for his own benefit, virtually at the expense of the Academy;' and that, by agreeing to pay him thirty shillings a copy for 450 copies, they have given him 'booksellers' profit'—and have 'made him a present of £234 13*s*. 4*d*. of the Academy's money, which would pay for a considerable edition for his own benefit, and which they might have saved by printing the work themselves.'

"To show the illegal nature of the agreement entered into with Mr. Petrie, Sir W. Betham observes, 'that the Council have no power to expend more than £20, without the consent of the Academy at large.' But the Council have power to contract liabilities to any amount that may be necessary for bringing out the volumes of the Transactions. This is part of their office. They are not even obliged to bring such engagements under the notice of the Academy, until a grant of money is wanted to discharge them; but it so happens, that the agreement made with Mr. Petrie was laid before the Academy, along with other matters, at a stated meeting on the 16th of March, 1841, in a report which was then adopted, and ordered to be entered on the minutes. In this case no grant of money has been called for, as Mr. Petrie's part of the engagement is not yet fulfilled; nor has any sum, large or small, been expended in connexion with the Essay on the Round Towers, except what had been paid for engravings previously to the aforesaid agreement; which sum, by the terms of the agreement, is to be taken into account in the final settlement.

"Sir W. Betham further states, that 'it is contrary to the laws of the Academy, to return the manuscripts even of unsuccessful candidates for prizes and medals, while they claim as property all Essays read at their ordinary meetings and ordered to be printed.' It is true that it is part of a law of the Academy, that 'all communications shall be deemed the property of the Academy' (Chap. VII. Sect. 5, of the By-laws); but it is part of the same law, 'that the author of any communication may, by petition to Council, reclaim such communication, which shall be restored to him on said petition being granted.' And this law applies to every sort of communication.

" Sir W. Betham charges the Council with having ' alienated to Mr. Petrie the copyright of the Essay on the Round Towers.' But it is apparent, from the foregoing statements, that the most prudent course was to allow Mr. Petrie to publish the work at his own risk. Besides, it is not usual, nor does it seem very proper, in societies such as the Academy, to raise questions about copyright.

" The Council do not find any other charges brought by Sir W. Betham, of a kind proper to be noticed officially. On the tone and manner of Sir W. Betham's statements they refrain altogether from making any comment.

" I have the honour to be, my Lord,
" Your obedient Servant,
" J. Mac Cullagh,
" *Secretary of the Academy.*

" *To the Right Honorable*
the Chief Secretary for Ireland."

III.

" *Dublin Castle*, 3rd *Feb.*, 1845.

" Sir,—I am desired by the Lord Lieutenant to acknowledge the receipt of your letter of the 23rd ult., containing observations with regard to representations made by Sir William Betham, respecting the mode in which the business of the Royal Irish Academy has been conducted.

" I am, Sir,
" Your most obedient humble Servant,
" C. Lucas.

" *To James Mac Cullagh, Esq.*,
" *Secretary Royal Irish Academy.*"

DOCUMENTS REFERRED TO IN THE REPLY OF COUNCIL.

1.

[Extract.]

" *To the President and Council of the Royal Irish Academy.*

" Gentlemen,—I have the honour to address you in compliance with the request of the Secretaries of the Academy, that I would supply the Council with a statement of the circumstances which have caused so much delay in the publication of some Essays read by me at the Academy, as well as to explain other matters charged against me, in a letter addressed to Lord Eliot, Chief Secretary for Ireland, in order that it may be appended to the answer of the Council to that letter.

" In the first place, then, I beg to acknowledge, that whatever blame may be attached to the delay in the publication of these papers, it should fall alone on me; for the Academy has no power, either moral or legal, to force authors to print papers in its Transactions, if it be contrary to their wish or convenience to do so. The Academy, like all other institutions of the kind, has been chartered chiefly for the purpose of fostering science and literature, by giving facility to the publication of Essays considered valuable, but which, from their abstract or archæological nature, could not be given to the world without great probability, if not certainty, of pecuniary loss to their authors, by publishing them at their own risk; and hence it becomes a high honour to an author to be permitted to publish in its Transactions, if it be his wish or convenience to do so, but not an obligation on him if otherwise, or that he should prefer publishing on his own account. Such at least has been my impression on this matter, and such also has been the opinion of Sir William Betham, as often expressed while in friendship with me, and urged with a view to persuade me to do as he said he was determined to do with his own papers, whenever they were of a sufficiently popular character to be likely to sell, namely, to publish on my own account and at my own risk; and in accordance with which, he sent in his Essay on the Gael and Cimbri to the Academy, as an anonymous competition paper for the prize and Gold Medal, printed as an octavo volume, as it afterwards appeared, with only the addition of a title-page, and, as I believe, a preface and index. With this advice, however, being anxious to sustain and advance, if in my power, the character of the Academy by my labours, I never had any intention of complying; and the delay which has occurred in the printing of my Essays has been entirely caused by circumstances over which I had no sufficient control.

"And first, as regards my Essay on the Round Towers. This Essay did certainly, as Sir William Betham states, receive the reward of the Academy at the close of the year 1832; and I immediately afterwards applied myself to its preparation for publication by improving its matter and increasing its necessary illustrations, by every means in the power of a man who had to sustain a large family solely by the daily practice of his profession as an artist. But the labour was a great and a tedious one, and having soon after, perhaps imprudently for my own interests, accepted an employment as director under Captain Larcom of the orthographical and historical department of the Ordnance Survey of Ireland, formed in part with a view to the publication of memoirs to illustrate the map, its duties so entirely abstracted me as to put it wholly out of my power, while thus employed, to make the great number of drawings necessary to the illustration of the work; nor was it possible to get them done by others. The chagrin which this circumstance necessarily caused me, was, however, considerably lessened by the circumstance, that, in consequence of the Council having, at the suggestion of Sir William Betham, and on the score of economy, transferred the printing of its Transactions to another printer, their succeeding volume was such as to render it impossible for me, having a due regard to the appearance of works so extensively illustrated, to put any of my Essays into the volume of Transactions which followed.

"It is also true that I received the prize and Gold Medal of the Academy, for an Essay on Irish Military Architecture, in March, 1834; and I believe it true that I, a member of the Council myself, was the proposer of this question, as well as that on the origin of the Round Towers, as Sir William Betham states with an obvious object. As questions for prize essays must originate with the Council, I had as much right to suggest them for approbation as another, and, in point of fact, I did, at the request of the Council, draw up a list of themes for dissertations on Irish History and Antiquities, from which they might select, similar to those relative to the history of Scotland, suggested to the learned of that country by the celebrated John Pinkerton, who remarks, that 'Scotland is certainly that country in Europe, if we except Ireland, in which national history and antiquities are most neglected.' But amongst those questions there were many indeed, which I should never have thought of treating of,—as, for instance, the question, 'who were the Scoti?' which Sir William Betham induced the Council to propose, and in competition for the prize for which Sir William Betham sent in his printed volume, entitled, 'the Gael and Cimbri.'

"I may also remark, that though I may have suggested to the Council the question on 'Irish Military Architecture,' I never seriously contemplated competing for the prize offered, my time being occupied on the Ordnance Survey, till one week precisely before the day appointed for delivering in the papers, when, at the earnest solicitation of my friend, Captain Larcom, R. E., I was induced to write an Essay, as that gentleman will, I have no doubt, be ready to testify. And with respect to the delay in the publication of this Essay, I should state, that I was at all times ready to undertake it, after the printing of the Transactions of the Academy repassed into the original hands, not only because it was an Essay of smaller size, and requiring fewer illustrations, but that I was even anxious to do so, as it would have been the proper precursor to my larger work, which relates to a later class of antiquities. With this desire of mine, however, the Council showed the strongest indisposition to comply ; and, consequently, I applied my mind, as much as circumstances would permit, to the preparation for the Press, of my Essay on the Round Towers. But being still employed on the Ordnance Survey, the time at my disposal was very limited, and it was not till the department in which I was employed, was finally broken up at the close of the year 1842, that I was enabled to give nearly my whole time and attention to the work, and commenced its printing. Since that period, I may say, I have done little else than labour at it. I have allowed myself no leisure for enjoyment or for exercise. In my devotion to it, I have reduced myself to poverty, and injured my constitution, perhaps irretrievably. And yet, all these efforts have only enabled me to see through the Press a volume of it, now on the eve of publication ; for the subjects treated of, in the copious manner which I conceived that they merited, have run the work to the extent of two volumes.

"It is true, that many may say that they did not require a work of so elaborate a character; but surely an author himself should be considered the best judge of what was necessary to his subject, at least till, by the publication of his work, he has enabled others to prove that he was in error.

"With respect to the charge against the Council, for having permitted me to enlarge my Essay, I have only to say, that I am sure they did so from an anxious desire to promote, to the best of their ability, one of the primary objects for which the Academy was instituted. And with respect to their agreement to take from me four hundred and fifty copies of my Essay, at thirty shillings per copy, I can safely state my conviction, that it was made solely with a view to the financial interests of the Academy, and without any regard to mine.

"I must also state, that it is my conviction that they were driven to make this agreement with me, chiefly by Sir William Betham himself. When I commenced making the drawings in wood to illustrate the work, it was on the understanding with Council, that as the Academy did not publish with a view to gain, they would allow me the use of the wood-cuts, after the publication of their volume, to publish an edition of my Essay for my own advantage. But Sir William Betham's assertions before the Academy, that the publication of this work would reduce the institution to a state of bankruptcy, and his threats that he would, by an application to the Government, stop the payments to the wood-cutter, had the effect of inducing some members of the Council to propose, contrary to the practice of the Academy and the spirit of its institution, to publish such unlimited number of copies of my work as would repay the Academy, by their sale, the expense of its publication. To this proposal, which was nothing less than to resign for ever nearly the labour of a whole life, and which would still take me years of undivided attention to accomplish, I could not bring myself to agree, either as an individual whose interests were concerned, or as a member of the Academy, who had its honour at heart. Hence I made the proposal to supply the Academy with such a number of copies as the Council desired, at the rate of thirty shillings a copy; I, of course, obtaining thereby the copyright of my work, and the management of its publication; and by this agreement the Academy will get the work at considerably less than half what it would cost them to bring out an edition of it of their usual number of five hundred copies, and are moreover saved from the expense of the paper and printing of fifty copies, always given to authors who publish Essays in the Transactions.

"With respect to the delay in the publication of my Essay on the Ancient Irish Bells, which I wrote in compliance with the wish of the Academy, as conveyed to me from the Chair, by their former illustrious President, Bishop Brinkley, I have only to state, that the cause of that delay is solely attributable, since the plates were furnished, to my being wholly employed on the publication of my Essay on the Round Towers.

"And lastly, with respect to the charge that my paper on the Antiquities of Tara Hill, was read by the permission of Colonel Colby, it having been prepared under his direction, for the Ordnance Survey, by the persons employed thereon, I have only to remark, that it is certainly true that I wrote and read the paper by permission of Colonel Colby, because, being employed in the Ordnance Survey at the time, I could not, with propriety, have written or read it without such permission; but that it was my own work, assisted, as I have constantly been, by more competent Irish scholars than myself, and written expressly for the Transactions of the Academy, I have already given the most unquestionable evidence, namely, that of Colonel Colby and Captain Larcom, the directors of the Survey.

. . . .

"I have the honour to be, Gentlemen,
"Your obedient Servant,
"GEORGE PETRIE.

"21, *Great Charles-st.*,
"*Jan. 23rd*, 1845."

2.

"104, *Grafton-street*,
"*20th January*, 1845."

"Hodges and Smith present their respects to the Secretary of the Royal Irish Academy, and in reply to his note relative to the probable price of Mr. Petrie's work on the Round Towers, they beg to say, that it cannot, under any circumstances, be less than 2 guineas, but they think it more likely to be $2\frac{1}{2}$ guineas."

3.

" *University Press Office,*
" *Dublin, 20th January,* 1845.

" SIR,—In answer to your question relative to the expense of Mr. Petrie's 'Round Towers,' I beg in reply to state it is my opinion, that if the Royal Irish Academy had to pay the expense of the Drawings, and the Engravings from them of the numerous and finely-executed wood-cuts, together with the cost of printing and paper, and confined themselves to the printing of an edition of 500 copies (their usual number), each copy of the work would stand them in a sum above three pounds.

" I am, Sir, respectfully,
" Your very obedient and humble Servant,
" M. H. GILL.

"*James Mac Cullagh, Esq.*"

No. II.

METEOROLOGICAL JOURNAL,

COMMENCING

1st JANUARY, 1844, AND ENDING 31st DECEMBER, 1844,

BY

GEORGE YEATES.

The instruments employed, and the general circumstances of the mode of observing, have been described in the preliminary observations to the Tables of the year 1843, in the 2nd volume of the Proceedings of the Academy, Appendix V.

JANUARY.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	50°	31°	29.630	. . .	W.
2	36	29	29.630	. . .	W.
3	35	26	29.730	. . .	N. E.
4	48	30	29.732	. . .	W.
5	54	47	29.650	.100	E. by S.
6	43	40	29.250	. . .	W.
7	47	35	29.600	.004	W.
8	42	31	29.450	.006	W.
9	47	36	30.000	.480	E. S. E.
10	51	38	30.300	.145	W. S. W.
11	46	36	30.350	. . .	W. by N.
12	47	39	30.020	.125	W.
13	45	35	30.260	.010	W. by N.
14	42	35	30.400	. . .	E. N. E.
15	40	29	30.400	. . .	W. by N.
16	37	29	30.350	. . .	N. W.
17	43	36	30.350	. . .	N. W.
18	49	44	30.300	. . .	N. W.
19	45	41	30.100	.004	W. N. W.
20	44	40	30.150	.002	W. N. W.
21	45	42	30.000	. . .	N. W.
22	45	41	29.900	.005	E. N. E.
23	46	32	29.900	. . .	W. N. W.
24	57	53	30.400	. . .	W. S. W.
25	50	41	30.420	. . .	W.
26	52	45	30.350	. . .	W.
27	49	41	30.412	. . .	W.
28	50	45	30.000	.045	W. by N.
29	49	41	29.820	.010	W. S. W.
30	52	41	29.900	. . .	W.
31	47	35	30.000	.095	W. N. W.

FEBRUARY.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	43°	36°	30.070	. . .	W. S. W.
2	41	35	29.650	.330	W. N. W.
3	35	30	30.100	.045	W. N. W.
4	42	35	29.700	.090	W. N. W.
5	43	34	29.512	.046	N. W.
6	42	31	29.500	. . .	W.
7	43	36	29.160	.045	W. N. W.
8	42	31	29.260	. . .	W. S. W.
9	40	30	29.260	. . .	N. N. W.
10	41	30	29.720	. . .	W. N. W.
11	39	29	30.018	. . .	N. W.
12	43	31	29.954	.010	N.
13	44	36	30.054	. . .	S. W.
14	47	40	29.912	. . .	W. S. W.
15	57	45	29.912	. . .	W. S. W.
16	48	39	30.214	. . .	W. S. W.
17	47	39	29.750	. . .	S. W.
18	50	44	29.530	. . .	S. S. W.
19	51	38	29.450	.250	W. N. W.
20	44	37	29.900	. . .	W. N. W.
21	39	30	29.250	.200	E.
22	37	33	29.600	.290	N. E.
23	38	26	29.350	·045	S. W.
24	39	30	29.240	.456	S. W.
25	45	34	29.150	.120	W. S. W.
26	48	35	28.800	.045	W. N. W.
27	38	26	29.550	.260	W. N. W.
28	40	36	29.450	.046	S. E.
29	45	34	29.400	.030	W. S. W.

MARCH.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	48°	37°	29.270	.170	W. S. W.
2	47	35	29.250	.050	W. S. W.
3	47	34	29.160	.050	W.
4	48	33	29.500	.025	N. W.
5	43	27	29.850	.218	S. W.
6	42	29	29.950	. . .	N. E.
7	42	29	30.250	.010	W.
8	50	38	30.050	.010	S. W.
9	53	46	29.920	.005	S. W.
10	53	38	29.900	.095	N. W.
11	49	43	29.400	.305	W. S. W.
12	49	35	29.700	.200	W. N. W.
13	47	33	29.060	.025	W. N. W.
14	46	40	29.700	.060	S. E.
15	48	34	29.420	.050	S. W.*
16	46	32	29.510	.119	E. N. E.†
17	38	34	30.120	. . .	E. N. E.‡
18	45	31	30.200	.500	E. N. E.
19	45	31	30.166	. . .	N. N. W.
20	48	39	29.750	.060	N. W.
21	49	31	30.000	.007	S. W.
22	57	39	29.550	.015	N. N. W.
23	57	38	29.660	.018	N. N. W.
24	49	41	29.450	. . .	W. N. W.
25	55	42	29.360	.030	S. W.
26	55	45	29.708	.020	W.
27	56	49	29.780	. . .	W. S. W.
28	58	37	30.400	. . .	S. W.§
29	58	38	30.420	. . .	E.‖
30	57	30	30.400	. . .	E.¶
31	56	40	30.150	. . .	E. S. E.

* Stormy. † Very stormy. ‡ Stormy.

§ Fine. ‖ Very fine. ¶ Very fine.

APRIL.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	59°	42°	29.200	. . .	W.
2	53	41	29.914	. . .	S. W.
3	61	40	29.716	.265	S. W.
4	50	42	29.552	. . .	S. S. E.
5	51	42	29.572	. . .	S. E.
6	50	44	29.930	.010	W.
7	50	37	30.150	. . .	W. N. W.
8	61	46	30.350	. . .	S. W.
9	70	51	30.500	.010	S. W.
10	71	41	30.250	. . .	S. W.
11	64	42	29.900	.020	W. S. W.
12	56	42	29.750	. . .	S. W.
13	54	43	29.900	. . .	W. N. W.
14	59	46	29.950	.115	W.
15	59	50	29.900	.030	E. S. E.
16	57	40	30.116	.050	S. W.
17	61	45	30.100	. . .	S. W.
18	64	42	30.300	. . .	N. W.*
19	58	46	30.400	. . .	W.
20	59	45	30.216	. . .	S. W.
21	60	51	30.250	.025	W.
22	61	49	30.200	.020	S. W.
23	58	44	30.150	. . .	S. W.
24	58	45	30.200	. . .	N. W.
25	62	44	30.150	. . .	E.
26	65	47	30.100	. . .	W.
27	56	39	30.250	. . .	W. by N.
28	59	41	30.370	. . .	N. E.
29	62	43	30.278	. . .	E. by N.
30	64	46	30.300	. . .	E. by N.

* Fine weather.

MAY.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	65°	45°	30.410	. . .	E. by N.
2	66	46	30.500	. . .	E. by N.*
3	67	50	30.350	. . .	N. E.
4	68	56	30.350	. . .	E. N. E.
5	63	44	30.178	. . .	E. by N.
6	67	45	29.900	. . .	E. by S.
7	67	44	29.976	. . .	W. by S.
8	64	43	30.000	. . .	E. by S.
9	64	46	29.950	.005	E.
10	61	42	30.100	.205	N. W.
11	61	44	30.150	. . .	W. by N.
12	64	47	30.350	. . .	W. by N.
13	71	50	30.460	. . .	W. by N.
14	68	57	30.462	. . .	N. W.
15	68	43	30.462	. . .	E.
16	65	43	30.250	.106	N. W.†
17	57	44	30.200	. . .	N. N. E.‡
18	56	32	30.200	. . .	N. N. E.§
19	55	39	30.260	. . .	N. N. E.
20	58	48	30.250	. . .	N.
21	63	46	30.250	. . .	N.
22	65	50	30.290	. . .	N.
23	69	48	30.300	. . .	N. E.
24	70	50	30.180	. . .	N. E.
25	68	55	30.200	. . .	N. E.
26	64	47	30.450	. . .	N. E.
27	60	42	30.400	. . .	N. E.
28	63	43	30.250	. . .	N. N. E.
29	64	44	30.112	. . .	N. E.
30	66	42	30.150	. . .	N. E.
31	60	44	30.160	. . .	N. E.

* Fine. † Cold. ‡ Dry. § Very dry.

JUNE.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	60°	44°	30.150	. . .	N. E.
2	63	46	30.060	. . .	N. E.
3	60	51	30.150	. . .	E. S. E.
4	63	52	30.100	. . .	S. W.
5	68	55	29.700	.120	S. W.
6	69	54	29.620	.038	W.
7	68	55	29.540	. . .	S. W.
8	63	55	29.700	. . .	W. S. W.
9	69	53	29.900	.105	S.
10	63	47	30.000	.120	W.
11	69	50	30.270	. . .	S.
12	70	51	30.068	. . .	S. W.
13	71	55	29.840	.006	W. S. W.
14	68	51	30.060	. . .	W.
15	69	48	30.150	. . .	W. by N.
16	67	45	30.150	.010	W. by N.
17	64	50	30.050	.080	S. E.
18	58	52	29.860	.340	N. by W.
19	60	45	30.100	.080	N. by W.
20	67	52	29.900	.215	W. by S.
21	70	58	29.818	.015	S. by W.
22	68	56	29.680	.090	S.
23	70	57	29.700	.110	S.
24	69	56	29.600	.130	E. by S.
25	71	53	29.750	. . .	S. by E.
26	73	57	29.850	.080	E.
27	60	53	29.950	. . .	E. by N.
28	67	47	30.100	. . .	W. by N.
29	70	50	30.150	. . .	W. by N.
30	70	50	30.050	. . .	E. S. E.

JULY.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	66°	49°	29.950	. . .	E. S. E.
2	58	47	30.000	. . .	W. by N.
3	66	49	29.900	. . .	E.
4	68	52	29.700	.190	E. by N.
5	63	53	29.800	.002	N.
6	65	52	30.000	. . .	W. by N.
7	56	44	30.050	. . .	N. by W.
8	68	55	30.000	. . .	W. by N.
9	68	54	30.050	. . .	N. by W.
10	68	53	29.950	. . .	W. by N.
11	67	54	29.950	.003	W. by N.
12	67	53	30.000	. . .	S. by W.
13	67	53	29.650	.280	S. by E.
14	63	54	29.660	.070	N. W.
15	58	52	29.900	.110	N. W.
16	69	48	30.100	. . .	W.
17	71	47	29.820	. . .	W.
18	62	49	29.800	.095	W.
19	61	48	29.900	.310	W. by N.
20	62	47	30.160	.040	E.
21	70	56	30.200	.170	S. by E.
22	77	54	30.124	. . .	E.
23	81	57	29.759	.110	E. by N.
24	81	61	30.000	.100	E. by N.
25	76	54	29.750	.030	E.
26	68	56	30.200	.250	W. by S.
27	73	54	30.300	.004	W.
28	74	60	30.160	.020	S. E.
29	69	51	30.150	.074	S. E.
30	71	55	29.450	.090	W. by N.
31	67	53	29.760	.200	W. by N.

AUGUST.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	62°	48°	29.850	.210	N. W.
2	67	49	29.820	.027	S. W.
3	70	56	29.260	.250	N. E.
4	68	48	29.750	.130	W. by N.
5	66	46	29.796	. . .	W. by S.
6	61	55	29.980	1.320	N. E.*
7	62	58	29.490	.600	W.
8	66	52	29.550	.100	W. by N.
9	66	52	29.700	.010	W. by N.
10	65	51	29.600	.095	E. by N.
11	65	52	29.670	.012	S. W.
12	65	55	29.590	.120	E. by N.
13	64	54	29.550	.095	E.
14	66	52	29.320	.210	W. by N.
15	65	53	29.760	.129	W. by N.
16	63	43	29.870	. . .	E. by N.
17	68	53	29.830	.045	W. by N.
18	65	42	30.190	.020	W. by N.
19	66	44	30.170	. . .	S. W.
20	70	56	29.890	. . .	W. by N.
21	65	46	29.830	. . .	W. by N.
22	63	51	29.720	. . .	W. by N.
23	60	48	29.620	.100	W. by S.
24	62	52	29.700	. . .	N. by W.
25	67	50	29.916	. . .	W. by N.
26	65	53	30.100	. . .	W. by N.
27	60	48	30.160	. . .	W. by N.
28	66	45	30.130	. . .	W.
29	66	50	30.050	. . .	E. by S.
30	68	48	30.000	. . .	E. by S.
31	76	48	30.430	. . .	E.

* Great rain.

SEPTEMBER.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	74°	50°	30.300	. . .	E. by S.
2	76	53	30.230	. . .	E. by S.
3	72	46	30.160	. . .	E. by S.
4	68	58	30.130	. . .	N. E.
5	69	60	29.990	. . .	N. E.
6	67	61	29.850	.060	S. E.
7	70	58	29.730	.200	E.
8	69	55	29.860	.070	W.
9	66	50	29.860	.142	W. by N.
10	58	50	29.840	. . .	W. by N.
11	63	55	30.000	. . .	S. by W.
12	61	53	30.040	.020	W. by S.
13	64	51	30.100	.090	W. by S.
14	62	56	29.770	. . .	W. by S.
15	70	61	29.640	.720	E. by S.
16	66	55	29.750	.090	W. by S.
17	66	55	29.700	.093	N. E.
18	57	46	29.870	.050	E.
19	62	45	30.150	. . .	E. by S.
20	60	51	30.090	. . .	N. E.
21	50	45	30.220	.210	N. E.
22	59	42	30.190	. . .	N. E.
23	58	45	29.890	. . .	N. E.
24	58	48	30.070	. . .	W. by N.
25	58	38	30.200	. . .	W. by N.
26	66	42	30.240	. . .	W. by S.
27	63	52	30.180	. . .	W. by S.
28	64	57	29.890	. . .	W. by S.
29	60	41	30.190	.100	W. by N.
30	59	30	30.260	. . .	S. by W.

OCTOBER.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	60°	52°	30.000	.020	W.
2	60	49	29.780	. . .	W. by S.
3	61	55	29.680	.040	W. by S.
4	63	46	29.900	. . .	W. by S.
5	63	48	29.690	.100	W. by S.
6	61	43	29.840	.005	W. by S.
7	59	38	29.960	. . .	W. by N.
8	55	38	29.140	.025	S. E.
9	57	51	28.830	1.200	E. by N.*
10	59	49	28.860	.620	W. by S.
11	59	46	29.500	.120	S. W.
12	59	49	29.400	.100	S. E.
13	60	52	29.060	.100	S. E.
14	60	48	29.000	.050	S. W.
15	59	39	28.890	. . .	N. W.
16	58	36	29.080	. . .	W.
17	55	43	29.456	. . .	S. W.
18	53	33	29.790	. . .	S. W.
19	51	31	29.480	.050	W. by N.
20	52	48	29.140	. . .	S. W.
21	52	39	29.530	. . .	N. W.
22	53	31	29.870	. . .	N. W
23	55	35	29.730	. . .	S. W.
24	57	44	29.790	. . .	N. W.
25	55	34	30.000	. . .	N. W.
26	51	31	30.070	. . .	N.
27	54	43	30.170	. . .	S. E.
28	56	46	30.170	. . .	E.
29	55	46	29.880	. . .	S. E.
30	55	46	29.600	.100	S. E.
31	57	50	29.540	. . .	E.

* Storm, 12 o'clock, 28.500.

NOVEMBER.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	55°	47°	29.260	.290	S. E.*
2	54	42	29.180	.480	S. E.
3	52	42	29.590	.300	N. E.
4	52	42	29.550	.020	N. E.
5	53	40	29.400	.180	N. E.
6	52	48	29.360	.200	N. E.
7	50	49	29.300	.900	S. E.
8	51	43	29.050	.070	N. E.
9	52	47	28.940	.200	W.
10	51	40	29.030	.018	W.
11	49	36	29.420	.150	W.
12	47	37	29.270	.100	S. E.
13	56	42	29.660	.130	W. by S.
14	55	40	29.940	. . .	E.†
15	58	43	29.790	.200	W.
16	57	41	30.140	. . .	N. E.
17	52	45	30.180	. . .	S. W.
18	63	45	30.100	. . .	S. W.
19	59	50	30.040	.024	W. by S.
20	55	49	30.040	.120	S. W.
21	55	29	30.280	. . .	N. W.
22	51	30	30.120	.005	E. by S.
23	53	45	29.990	. . .	W.
24	54	29	29.940	. . .	S. E.
25	51	30	29.960	. . .	S. E.
26	49	35	30.190	. . .	S. W.
27	54	41	30.060	.130	S. E.
28	59	48	29.760	. . .	W. S. W.
29	56	48	29.900	.225	E. by S.
30	54	42	30.090	. . .	S. E.

* Storm. † Stormy.

DECEMBER.

	Thermometer.		Barometer.	Rain.	Wind.
	Max.	Min.			
1	49°	40°	30.160	.050	S. E.
2	43	39	29.870	. . .	E. by S.
3	47	39	30.160	. . .	S. E.
4	48	28	30.270	. . .	S. E.
5	45	36	30.090	. . .	W.
6	44	30	30.120	. . .	N. W.
7	46	31	30.190	. . .	S. E.
8	49	30	30.160	. . .	E. by S.
9	43	29	30.120	. . .	S. E.
10	43	30	30.170	. . .	S. W.
11	44	30	29.850	. . .	E.
12	44	34	29.780	.100	N. E.
13	44	30	29.300	.086	N. E.
14	44	30	29.760	.145	N. E.
15	45	34	29.400	.007	N. E.
16	47	40	29.240	. . .	E. S. E.
17	49	44	29.230	.620	E. S. E.
18	50	44	29.480	.880	E. by N.
19	50	42	30.100	.100	E. by S.
20	48	35	30.350	. . .	S. E.
21	43	37	30.340	. . .	E. by S.
22	50	42	30.390	. . .	S. E.
23	44	34	30.180	. . .	E.
24	43	32	30.170	. . .	S. E.
25	45	32	30.080	. . .	S. E.
26	48	36	29.940	. . .	S. E.
27	48	41	29.800	.160	S. E.
28	51	43	29.700	.014	E. by S.
29	51	35	29.800	.030	N. E.
30	50	35	29.990	. . .	E. by N.
31	51	40	30.100	. . .	W.

GENERAL RESULTS.

	Amount of Rain. Inches.	Mean Temp.
January,	1.031	42°
February,	2.308	34
March,	2.042	38
April,	.645	$41\frac{1}{3}$
May,	.316	$61\frac{1}{2}$
June,	1.579	$67\frac{1}{4}$
July,	2.148	68
August,	3.473	70
September,	1.845	67
October,	2.530	56
November,	3.742	49
December,	2.192	51
	23.711	

BAROMETER.

Greatest pressure, August 27, ... 30.660
Lowest point, January 13, ... 28.100

THERMOMETER.

Maximum, September 8, ... 80°
Minimum, February 16, ... 21

2, *Grafton-street, Dublin.*

No. III.

The Abstract of Sir William R. Hamilton's Memoir on Quaternions, read on February 10, 1845, and referred to in page 64, has not been received for insertion in this Appendix.

Nor have the Abstracts of Sir William R. Hamilton's Memoirs on the New Imaginaries, read on July 14 and 21, 1845, and referred to in pages 111, 112, been received.

No. III.

February 10, 1845. (*See* page 64.)

The spirit of Sir William Hamilton's communication, which was designed as a further illustration from geometry of the author's theory of algebraic quaternions, consisted in regarding operations on such quaternions as admitting of being ultimately interpreted as *operations on straight lines;* each line being considered as having not only a determinate *length*, but also a determinate *direction*. The *quotient* $\frac{b}{a}$ obtained by the division of one such line (b) by another (a), is, generally, in the author's view, a *quaternion;* it depends, in general, on *four* distinct elements, of which one, namely, the *modulus*, is a positive or absolute number expressing the *relative magnitude* of the dividend and divisor lines, while the three other elements serve jointly to express the *relative direction* of those two lines. Of the three latter, one is the *amplitude*, and marks the *inclination* of one line to the other, or the magnitude of the angle which they include; while the two others determine the *plane* of that angle, and are what have been called, in a former communication, the *directional coordinate*, such as the *longitude* and *colatitude* of the quaternion. In this comparatively geometrical view, as in the more algebraical view which was formerly stated to the Academy, the consideration of these four elements, modulus, amplitude, longitude, and colatitude, presents itself, therefore, naturally. We may also speak of the *axis* of a quaternion, meaning thereby the axis perpendicular to the plane of the two straight lines of which that quaternion is a quotient; and may say, that such an axis is itself *positive* or *negative*, or that it is taken in the positive or in the negative direction, according as it is the axis of a positive or a negative *rotation*, from the divisor to the dividend line. Quaternions may be said to be *coaxal* when their axes coincide, or only differ in sign. A quaternion is not altered in value when the two lines of which it is the quotient are transferred, without altering their directions, to any other positions in space; or when their lengths are both changed together in any common ratio; or when they are both made to revolve together, through any common amount of rotation, round the axis of the quaternion, without ceasing to be still in (or parallel to) the same common plane as before. It is, therefore, always possible to prepare any two proposed quaternions, or geometrical quotients or fractions of the kind above described, so as to have one *common denominator* or divisor line; and then the *addition* or *subtraction* of those two quaternions is effected, by retaining that common line as the denominator or divisor of the new quaternion, and by adding or subtracting the numerator lines, in order to obtain the new numerator of the same new quaternion, that is to say, of the sum or difference of the two old quaternions; addition and subtraction of straight *lines* (when those lines are supposed to have not only lengths but also *directions*) being performed according to the rules which have already been proposed by several writers, and which correspond to compositions and decompositions of rectilinear *motions* (or of forces). *Multiplication* of two quaternions may be effected by preparing them so, that the denominator (b) of the multiplier, may be equal to, or the same line with, the numerator (b) of the multiplicand (lines being *equal* when their directions as well as their lengths are the same), and by then treating the numerator (c) of the multiplier as the numerator of the product, and the denominator (a) of the multiplicand as the denominator of the product: and *division* may be regarded as the return to the multiplier, from a given product and multiplicand.

With this view of multiplication, it is evident that the product of the moduli of the two factors is equal to the modulus of the product. It is clear also, that if we construct a spherical triangle ABC, of which the three corners, or the radii drawn to them from the centre of the sphere, represent the directions of the three lines a, b, c, then the arc, or side of the triangle, AB, will represent the amplitude of the multiplicand quaternion; the arc or side BC will represent the amplitude of the multiplier; and the remaining arc or side AC the amplitude of the product, so that the spherical triangle will be constructed with these three amplitudes for its three sides. And we see that in the triangle thus constructed, the spherical angles at A and C, which are respectively opposite to the amplitudes of the multiplier and multiplicand, are equal to the respective inclinations of the axes of the multiplicand and multiplier to the axis of the product of the quaternions; and that the remaining spherical angle at B, which is opposite to the amplitude of the product, is equal to the supplement of the inclination of the axes of the factors to each other: a form almost the same with that under which the fundamental connexion of quaternions with spherical trigonometry was stated by Sir William Hamilton, in his first letter on the subject, to John T. Graves, Esq., which was written in October, 1843, and has been printed in the Supplementary number of the *Philosophical Magazine* for December, 1844. The other form of the same fundamental connexion, which was communicated to the Academy in November, 1843, may be deduced from the foregoing, by the consideration of that polar or supplementary triangle, of which the corners mark the directions of the *axes* of the factors and the product, and were then called the *representative* points of the three quaternions compared. If the order of the factors be changed, the (positive) axis of the product falls to the other side of the plane of the axes of the factors, being always so situated that the rotation round the axis of the multiplier from the axis of the multiplicand to that of the product is positive; *multiplication of qua-*

ternions is therefore seen, in this as in other ways, to be *not in general a commutative operation*, or the result depends, in general, essentially on the *order* in which the factors are taken.

The same remarkable conclusion follows from the comparison of the lately mentioned spherical triangle ABC with another triangle C′BA′, vertically opposite and equal thereto, and such that the common corner B bisects each of the two arcs C′C, A′A, joining the two pairs of corresponding corners; which other triangle may represent the directions of three lines c′, b, a′, related to the system of the three former lines c, b, a, by the two following equations between geometrical quotients, or quaternions,

$$\frac{a'}{b} = \frac{b}{a}, \quad \frac{b}{c'} = \frac{c}{b};$$

for then, by the definition of multiplication of such quotients here proposed, we have the two different results,

$$\frac{c}{b} \times \frac{b}{a} = \frac{c}{a}; \quad \frac{b}{a} \times \frac{c}{b} = \frac{a'}{c'};$$

and although these two resulting quaternion products have equal moduli and equal amplitudes, yet they have in general *different axes*, because the arcs AC and A′C′, though equally long, are parts of different great circles, and are therefore situated in different planes. However, in that particular but useful and often occurring case, where the two factors have one common axis, the order of those factors becomes indifferent; and if attention be paid to positive and negative signs, it may be said that *coaxal quaternions* may be multiplied together, in either order, by adding their amplitudes, multiplying their moduli, and retaining their common axis. In general, it may be proved, from the views here given of multiplication and addition, that, although the *commutative* property of ordinary multiplication does not usually extend to operations on quaternions, yet the *distributive* and *associative* properties of that operation do always so extend; and that the commutative and associative properties of addition hold good in like manner for quaternions: results which were indeed stated to the Academy in November, 1843, as consequences from the algebraical definitions of a quaternion, and of operations performed thereon, but have now been mentioned again, as following from more geometrical definitions also.

Comparing the view here proposed with that which was submitted to the Academy in November 1844, a quaternion may be said to reduce itself to a *scalar* (or ordinary real number), when the two straight lines, of which it is the quotient, are *parallel;* the scalar being positive when those lines are similar, but negative when they are opposite in direction. And, on the other hand, the scalar part vanishes, and the quaternion becomes a pure *vector*, when it is a quotient of two *rectangular* lines: and, in this last case, it may be conveniently constructed by a *third line* perpendicular to both of them, namely, by one drawn in the direction of the positive axis of the quaternion, with a length which bears to an assumed unit of length the ratio marked by the modulus. This third line, which thus represents or constructs the quotient of two other lines perpendicular to it and to each other, may, by a suitable choice of those two lines, receive any proposed length, and any proposed direction; and *every straight line having length and direction in space* may, in this view, be regarded as a particular quaternion, namely, as one of the class above called *vectors.* It is easy to *prove* that when lines are thus treated as quotients, they have the same sums, differences, and quotients, as those obtained by the processes or conceptions above described or alluded to: and hence it would be natural to *define*, as we should be at liberty to do, that the *product* of two lines is also in general a quaternion, obtained by multiplying two vector factors together, according to the rules of multiplication of quaternions. We should then be able to establish, in this new way, all the rules, already communicated to the Academy, for the *multiplication of straight lines in space;*

and especially should be conducted anew to those *two rules*, or principles, which presented themselves to the author in his earliest researches on quaternions (as described in the printed letter already referred to), and which he still regards as fundamental in their theory: namely, first, that *the product of two straight lines, which agree in direction, is to be considered as a negative number*, namely, as the product of their two lengths taken negatively; and, secondly, that *the product of two rectangular lines is to be regarded as a third line perpendicular to both*, of which the length represents the product of their lengths, and *to* which the rotation, *from* the multiplicand line, *round* the multiplier line, is positive. The paradoxical, or, at least, unusual appearance of these two fundamental rules, combined with the variety of the applications of which the author has found them susceptible, induce him to hope that he shall be pardoned for thus offering new confirmations or new illustrations of them, derived from considerations of the manner in which they present themselves from various points of view.

July 14 and 21, 1845. (*See* pages 110 and 111.)

The following is the substance of the communications made to the Academy by Sir William Hamilton, on the application of the method of Quaternions to some dynamical questions:

The author stated that, during a visit which he had lately made to England, Sir John Herschel suggested to him that the *internal* character (if it may be so called) of the method of quaternions, or of vectors, as applied to algebraical geometry,—that character by which it is independent of any foreign and arbitrary axes of coordinates,—might make it useful in researches respecting the attractions of a system of bodies. A beginning of such a research had been made by Sir William Hamilton in October, 1844, which went so far, but only so far, as the deducing of the constancy of the plane of an orbit, and the equable description of areas, under one common formula, namely, the following:

$$\rho \frac{d\rho}{dt} - \frac{d\rho}{dt} \rho = \text{const},$$

from the general expression of a central force, namely, from the equation

$$\rho \frac{d^2\rho}{dt^2} - \frac{d^2\rho}{dt^2} \rho = 0,$$

which asserts merely the *coaxality* of the vector ρ and the force $\frac{d^2\rho}{dt^2}$, or the existence of one common line along which this vector and this force are (similarly or oppositely) directed.

Since the suggestion above acknowledged was made, Sir William Hamilton has proposed to himself to express by an equation, on the principles of the method of vectors, the problem of any number of bodies attracting according to Newton's law: and has arrived at the formula

$$\frac{d^2 a}{dt^2} = \Sigma \frac{m + \Delta m}{- \Delta a \sqrt{(- \Delta a^2)}}; \qquad \text{(A)}$$

which may also be thus written,

$$\frac{d^2 a}{dt^2} = \Sigma \frac{m'}{(a - a') \sqrt{\{-(a - a')^2\}}}; \qquad \text{(B)}$$

and from which he has deduced anew the known laws of the centre of gravity, of areas, and of the *vis viva*, under the forms:

$$\left(\frac{d}{dt}\right)^2 \Sigma . ma = 0; \qquad \text{(C)}$$

$$\frac{d}{dt} \Sigma . m \left(a \frac{da}{dt} - \frac{da}{dt} a\right) = 0; \qquad \text{(D)}$$

$$\frac{d}{dt} \Sigma . \frac{m}{2} \left(\frac{da}{dt}\right)^2 + \frac{d}{dt} \Sigma . \frac{mm'}{\sqrt{\{-(a - a')^2\}}} = 0. \qquad \text{(E)}$$

a is the vector and m the mass of one body; a' and m' of another;

Σ sums for the system; t is the time, d the characteristic of differentiation; Δ (where used) is the mark of finite differencing.

To illustrate the method of treating equations of such forms as these, let us consider briefly the problem of *two* bodies, or of *one* body, as it presents itself, in the method of quaternions, with Newton's law of attraction, coordinates being not employed. The differential equation may be thus written,

$$\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} = \frac{\mathrm{M}}{a\sqrt{(-a^2)}}; \qquad (1)$$

a being the vector of the attracted body, drawn from the attracting one; t the time; d the mark of differentiation; and M the attracting mass, or the sum of the two such masses. This equation gives

$$a\frac{\mathrm{d}^2 a}{\mathrm{d}t^2} - \frac{\mathrm{d}^2 a}{\mathrm{d}t^2}a = 0; \qquad (2)$$

which expresses merely that the force is central; and gives by integration a result already alluded to (as independent of that function of the distance which enters into the law of attraction), namely,

$$\frac{a}{2}\frac{\mathrm{d}a}{\mathrm{d}t} - \frac{\mathrm{d}a}{\mathrm{d}t}\frac{a}{2} = \beta; \ \mathrm{d}\beta = 0; \qquad (3)$$

the constant β being a new vector, perpendicular in direction to the plane of the orbit, and in magnitude representing the double of the areal velocity, which velocity is thus seen to be constant, as also is the plane. For we have at once, by (3),

$$a\beta + \beta a = 0, \qquad (4)$$

implying that the variable vector a is perpendicular to the constant vector β; and also

$$\int(a \,.\, \mathrm{d}a - \mathrm{d}a \,.\, a) = 2\beta(t - t_0), \qquad (5)$$

if t_0 be the value of t at the commencement of the integral.

Make now, to distinguish between the length and direction of the vector,

$$a = r\iota, \quad r = \sqrt{(-a^2)}, \quad \iota^2 = -1; \tag{6}$$

we shall have

$$\mathrm{d}a = r.\mathrm{d}\iota + \mathrm{d}r.\iota, \tag{7}$$

and because r and $\mathrm{d}r$ are scalar (or real) quantities,

$$a.\mathrm{d}a = r^2\iota.\mathrm{d}\iota - r.\mathrm{d}r, \quad \mathrm{d}a.a = r^2\mathrm{d}\iota.\iota - r.\mathrm{d}r; \tag{8}$$

therefore

$$\beta.\mathrm{d}t = \tfrac{1}{2}(a.\mathrm{d}a - \mathrm{d}a.a) = \frac{r^2}{2}(\iota.\mathrm{d}\iota - \mathrm{d}\iota.\iota) = r^2\iota.\mathrm{d}\iota, \tag{9}$$

observing that the equation

$$\iota^2 = -1 \text{ gives } \iota.\mathrm{d}\iota + \mathrm{d}\iota.\iota = 0. \tag{10}$$

The fundamental equation (1) of the problem becomes, by (6) and (9),

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{\mathrm{M}}{r^2\iota} = \frac{\mathrm{d}\iota}{\mathrm{d}t}\frac{\mathrm{M}}{\beta}, \tag{11}$$

(in which last member the order of the factors is not indifferent), and therefore gives, by integration, since β as well as M is constant,

$$\frac{\mathrm{d}a}{\mathrm{d}t} - \iota\frac{\mathrm{M}}{\beta} = \text{const.}; \tag{12}$$

or, as we may also write it,

$$\iota - \frac{\mathrm{d}a}{\mathrm{d}t}\frac{\beta}{\mathrm{M}} = \varepsilon, \quad \mathrm{d}\varepsilon = 0. \tag{13}$$

We have, consequently, by (6) and (4),

$$a\varepsilon = -r - a\frac{\mathrm{d}a}{\mathrm{d}t}\frac{\beta}{\mathrm{M}}, \quad \varepsilon a = -r + \frac{\mathrm{d}a}{\mathrm{d}t}a\frac{\beta}{\mathrm{M}}; \tag{14}$$

and finally, by (3),

$$a\varepsilon + \varepsilon a + 2r = 2p, \ (15) \qquad \text{if } p = \frac{-\beta^2}{\mathrm{M}}, \tag{16}$$

the constant p being here not only a scalar but an essentially positive quantity, because the force is supposed to be attractive, or $\text{M} > 0$, while $\beta^2 < 0$. The equation (15) thus obtained, contains the law of elliptic, parabolic, or hyperbolic motion. For if we make (by way of comparison with known results),

$$\sqrt{(-\varepsilon^2)} = e, \quad (17) \qquad \text{and } (\alpha, -\varepsilon) = v, \quad (18)$$

$(\alpha, -\varepsilon)$ denoting here the angle between the directions of α and ε, we have (by the formula (a) of the abstract of last November),

$$\alpha\varepsilon + \varepsilon\alpha = 2er \cos v; \qquad (19)$$

and therefore, by (15),

$$r = \frac{p}{1 + e \cos v}, \qquad (20)$$

which is the known equation of a conic section, referred to a focus. The Greek letters, throughout, represent vectors: and the Italics, scalar quantities.

Supposing that we had no previous knowledge of the properties of cosines or of conics, we might have proceeded thus to investigate the nature of the locus represented by the equation (15). This locus is a *surface of revolution* round the line ε; because the differential of its equation being

$$\mathrm{d}\alpha \,.\, \varepsilon + \varepsilon \,.\, \mathrm{d}\alpha + 2\mathrm{d}r = 0 \qquad (21)$$

if we cut it by a series of concentric spheres round the origin of vectors, the sections are contained in a series of planes perpendicular to ε; since

$$\mathrm{d}r = 0, \qquad (22)$$

which is the differential equation of the first series, gives, by (21),

$$\mathrm{d}\alpha\varepsilon + \varepsilon\mathrm{d}\alpha = 0, \qquad (23)$$

which is the differential equation of the second series. To study more closely this surface of revolution (15), make

$$\alpha = \gamma + \alpha', \qquad (24)$$

γ being an arbitrary constant, and a' a variable vector; and since it must evidently give simpler and more symmetric results to suppose the vector γ co-axal with ε, than to make the contrary supposition, since we shall thus place the origin of the new vectors a' upon the axis of revolution of the surface, let

$$\varepsilon\gamma - \gamma\varepsilon = 0, \text{ or } \gamma = g\varepsilon, \qquad (25)$$

g being an arbitrary scalar, to be disposed of according to convenience. Equations (24) and (25), combined with (6) and (17), will give, for every point of space,

$$-a'^2 = -(a-\gamma)^2 = r^2 + g^2 e^2 + g(a\varepsilon + \varepsilon a); \qquad (26)$$

and therefore, for every point of the locus (15),

$$-a'^2 = r^2 - 2gr + g^2 e^2 + 2gp. \qquad (27)$$

The second member of this last equation may be made an exact square, by assuming

$$g^2 e^2 + 2gp = g^2, \text{ that is, } g = \frac{2p}{1-e^2} = 2a; \qquad (28)$$

the scalar quotient

$$\frac{p}{1-e^2} = a, \text{ or the transformation } p = a(1-e^2), \qquad (29)$$

being thus suggested to our attention; and with this value of g we shall have, by (27),

$$-a'^2 = (2a - r)^2, \qquad (30)$$

that is,

$$2a = \sqrt{(-a^2)} \pm \sqrt{(-a'^2)}; \qquad (31)$$

so that either the sum or the difference of the distances of any point of the locus (15) from the two *foci* of which the vectors are respectively 0 and $2a\varepsilon$, is equal to the constant $2a$. It is not difficult to prove that the upper or the lower sign is to be taken, in the formula (31), according as e^2 is $<$ or $>$ 1. For the case $e^2 = 1$, the recent transformation fails.

Again, to find whether the locus has a *centre*, we may make

$$a = \gamma' + \delta = g'\varepsilon + \delta, \qquad (32)$$

g' being a new disposable scalar, and δ a new variable vector; and, after having cleared the equation (15) of the radical r or $\sqrt{(-a^2)}$, by writing it as follows,

$$a^2 + \left(p - \frac{\alpha\varepsilon + \varepsilon\alpha}{2}\right)^2 = 0, \tag{33}$$

we get

$$0 = (g'\varepsilon + \delta)^2 + \left(p + g'e^2 - \frac{\delta\varepsilon + \varepsilon\delta}{2}\right)^2 \tag{34}$$

$$= \delta^2 + \left(\frac{\delta\varepsilon + \varepsilon\delta}{2}\right)^2 + g''(\delta\varepsilon + \varepsilon\delta) + (p + g'e^2)^2 - g'^2e^2;$$

if we make for abridgment

$$g'' = g' - (p + g'e^2). \tag{35}$$

If γ' or $g'\varepsilon$ is to be the constant vector of the centre of the locus, it is necessary that to every variable vector, δ, which satisfies the equation (34), should correspond another vector $-\delta$, equal in length but opposite in direction, and satisfying the same equation; therefore the terms $g''(\delta\varepsilon + \varepsilon\delta)$ must disappear, and we must have

$$g'' = 0,\ g' = \frac{p}{1 - e^2} = a, \tag{36}$$

the constant a being thus suggested by the search after a centre, as well as by the search after a second focus. Making then $g' = a$ in (34), we find the following equation of the surface, when referred to its centre,

$$0 = \delta^2 + \left(\frac{\delta\varepsilon + \varepsilon\delta}{2}\right)^2 + ap; \tag{37}$$

in which

$$ap = a^2(1 - e^2) = a^2(1 + \varepsilon^2). \tag{38}$$

And because in general, for any two vectors δ, ε, the following relation holds good,

$$\left(\frac{\delta\varepsilon + \varepsilon\delta}{2}\right)^2 = \left(\frac{\delta\varepsilon - \varepsilon\delta}{2}\right)^2 + \delta^2\varepsilon^2, \tag{39}$$

we may write the equation (37) under the form

$$0 = (1 + \varepsilon^2)(\delta^2 + a^2) + \left(\frac{\delta\varepsilon - \varepsilon\delta}{2}\right)^2. \qquad (40)$$

This last equation shows that

$$\text{when } \delta\varepsilon - \varepsilon\delta = 0, \text{ then } \delta^2 + a^2 = 0\,; \qquad (41)$$

that is to say, when δ is co-axal with, or parallel to ε, or, in other words, when the vector from the centre coincides (in either direction) with the axis of revolution of the surface, its length is $= \pm\, a$, according as a is $>$ or $<$ 0.

The equation (37) shows that

$$\text{when } \delta\varepsilon + \varepsilon\delta = 0, \text{ then } \delta^2 + a^2(1 + \varepsilon^2) = 0\,; \quad (42)$$

if therefore ε^2 be > -1, that is, if $e^2 < 1$, the length of every vector drawn from the centre perpendicularly to the axis of revolution will be

$$\sqrt{(-\delta^2)} = a\sqrt{(1 - e^2)} = b, \qquad (43)$$

b being a new scalar quantity; but if $e^2 > 1$, $\varepsilon^2 < -1$, $1 + \varepsilon^2 < 0$, then we shall have, by (42), *the absurd result of a* VECTOR δ *appearing to have a* POSITIVE SQUARE: whereas it is a first principle of the present method of calculation, *that the square of every vector* is to be regarded as a *negative number: which symbolical contradiction indicates the* GEOMETRICAL IMPOSSIBILITY *of drawing from the centre to any point of the locus, a straight line which shall be perpendicular to the axis of revolution, in the case where* $e^2 > 1$. The locus has, in this case, two infinite branches enclosed within the two branches of the *asymptotic cone* which has for its equation

$$\delta^2 + \left(\frac{\delta\varepsilon + \varepsilon\delta}{2}\right)^2 = 0\,; \qquad (44)$$

and nowhere penetrates within that *inscribed spheric surface,* which has for its equation

$$\delta^2 + a^2 = 0, \qquad (45)$$

though it touches this last surface at the two points where it meets the axis of revolution. On the other hand, when $e^2 < 1$, the locus is entirely contained *within* the spheric surface (45), touching it, however, in like manner in two points upon the axis of revolution. A *finite* surface of revolution (the *ellipsoid*) might thus have been discovered, of which each point has a constant *sum* of distances from two fixed foci; and an *infinite* surface (the *hyperboloid*), with two separate sheets, of which each point has a constant *difference* of distances from two such foci: and all the other properties of these two surfaces of revolution might have been found, and may be proved anew, by pursuing this sort of analysis. A third distinct surface of the same class, but infinite in *one* direction only (the *paraboloid*), might have been suggested by the observation that the reduction to a centre fails in the case $e^2 = 1$, $\varepsilon^2 = -1$. Its equation may be put under the form

$$(\varepsilon a'' - a''\varepsilon)^2 = 4p(\varepsilon a'' + a''\varepsilon), \tag{46}$$

by making

$$a = a'' - \frac{p\varepsilon}{2}, \tag{47}$$

so that a'' is the vector from the vertex: and it lies entirely on one side of the plane which touches it at the vertex, namely, the plane

$$\varepsilon a'' + a''\varepsilon = 0. \tag{48}$$

In general whatever e or ε may be, and therefore for all the three surfaces, the length of the focal vector perpendicular to the axis is p; for, by (33), if we make

$$\varepsilon a + a\varepsilon = 0, \tag{49}$$

we get

$$a^2 + p^2 = 0. \tag{50}$$

Indeed (15) then gives $r = p$.

Since

$$a^2 + r^2 = 0, \qquad a \,.\, \mathrm{d}a + \mathrm{d}a \,.\, a + 2r\,\mathrm{d}r = 0, \tag{51}$$

the differential equation (21) of the locus (15) may be put under the form

$$(r\varepsilon - a)\,\mathrm{d}a + \mathrm{d}a\,(r\varepsilon - a) = 0\,; \tag{52}$$

thus shewing that the vector $r\varepsilon - a$ is perpendicular to the differential d of the focal vector a, or that it is parallel to the normal to the locus, at the extremity of that focal vector. That normal, therefore, intersects the axis of revolution in a point, of which the focal vector is $r\varepsilon$; the position of the normal is, therefore, entirely known, and every thing that depends upon it may be found, for the particular surfaces of revolution which have been here considered. For example, in the ellipsoid, the vector of the second focus, drawn from the first, has been seen to be $2a\varepsilon$; if, then, we make

$$2a - r = r', \tag{53}$$

so that r' denotes the length of the second focal vector, drawn to the same point as the first focal vector, of which the length is r, we have $-\,r'\varepsilon$ for the second focal vector of the intersection of the normal with the axis; the normal, therefore, cuts (internally) the interval between the two foci, into segments proportional to the two conterminous focal distances of the point upon the ellipsoid, and consequently bisects the angle between those focal distances. Again, if we divide the expression $r\varepsilon - a$ by the scalar quantity r, and multiply the quotient by a, we find that $\varepsilon - \iota$ and $a\varepsilon - a\iota$ are also expressions for vectors in the normal direction; and because $a\varepsilon$ is the focal vector of the centre, while $-\,a\iota$ is a radius of the circumscribed sphere, opposite in direction to the focal vector of the point upon the ellipsoid, we see that if the focal vector of the extremity of this radius of the sphere be prolonged through the focus, it will cut perpendicularly the tangent plane to the ellipsoid. Again, the expression

$$\tau = a(\varepsilon + \iota) - a = (a - r)\,\iota + a\varepsilon, \tag{54}$$

is easily seen to denote here a vector perpendicular to $a - r\varepsilon$, and therefore to the normal, because

$$a\tau + \tau a = r(\varepsilon\tau + \tau\varepsilon) = 2(ap - rr'); \qquad (55)$$

but τ is also in the same plane with a and ε, and therefore is a vector parallel to the tangent to the elliptic section of the locus made by a plane passing through the axis of revolution; $a\iota$ is therefore the central vector of a point upon this tangent, because $a - a\varepsilon$ is the central vector of the point of contact; and the central vector of the second focus being $a\varepsilon$, we have $a\iota - a\varepsilon$ as an expression for the second focal vector of the same point upon the tangent; this second focal vector is therefore parallel to the normal, because $\iota - \varepsilon$ is parallel thereto, and, consequently, it is the perpendicular let fall from the second focus on the tangent line or plane: and the foot of this perpendicular is thus seen to be at the extremity of that radius of the circumscribed circle or sphere, which is drawn in a direction similar (and not, as lately, opposite) to the direction of the first focal vector of the point on the ellipse or ellipsoid. We see, at the same time, that $-\tau$ is a symbol for the projection of the second focal vector upon the tangent line or plane; from which we may infer, by (55), that the product of the lengths of the two projections of the two focal vectors on the tangent is $= rr' - ap$, and therefore that it is less than the product rr' of the lengths of those two vectors by the constant quantity ap, or b^2, which constant must thus be equal to the product of the lengths of the projections of the same two vectors on the normal, so that we may write the equation

$$\mathrm{PP}' = ap = b^2, \qquad (56)$$

if P and P' denote the lengths of the perpendiculars let fall from the two foci on the tangent, while b is the axis minor of the ellipse. Analogous reasoning may be applied to the hyperbola, or to the surface formed by its revolution round its transverse axis. Most of the foregoing geometrical results are well known, and probably all of them are so: but it may be considered worth while to have briefly indicated the manner in which they reproduce themselves in these new processes of calculation.

The vector drawn from the focus first considered to any arbitrary point upon the normal, may be represented by the expression

$$\nu = (1 - n)\, a + nr\varepsilon, \tag{57}$$

in which n is an arbitrary scalar; and if this normal intersect another normal infinitely near it, then we may write, as the expression of this relation,

$$0 = \mathrm{d}\nu = (1 - n)\, \mathrm{d}a + n\varepsilon \mathrm{d}r + (r\varepsilon - a)\, \mathrm{d}n: \tag{58}$$

comparing which differential equation with the forms (52) and (21) of the differential equation of the surface of revolution (15), we can eliminate the scalar differential $\mathrm{d}n$, and deduce for n itself the expression

$$n = \frac{\mathrm{d}a^2}{\mathrm{d}r^2 + \mathrm{d}a^2}. \tag{59}$$

One way of satisfying these conditions is to suppose

$$n = 1, \qquad \mathrm{d}r = 0, \qquad \nu = r\varepsilon; \tag{60}$$

which comes to considering the intersection of the given normal with the axis, and therefore with the other normals from points of the same generating circle of the surface of revolution: and this intersection is accordingly one centre of curvature of that surface. The only other way of obtaining an intersection of two normals infinitely near, is to suppose, by (58), the element $\mathrm{d}a$ coplanar with a and ε, or to pass to consecutive normals contained in the same plane drawn through the axis; that is to say, the other centre of curvature of the surface is the centre of curvature of its meridian. The length of the element of this meridian, that is the length of $\mathrm{d}a$, is denoted by the radical $\sqrt{(-\mathrm{d}a^2)}$, because the differential $\mathrm{d}a$ is a vector; and the length of the projection of this element on the focal vector is $\pm\, \mathrm{d}r = \sqrt{(+\mathrm{d}r^2)}$, because $\mathrm{d}r$ is a scalar differential; therefore the length of the projection of the same element on a line perpendicular to the focal vector, and drawn in the plane through the axis, is denoted by this other radical, $\sqrt{(-\mathrm{d}a^2 - \mathrm{d}r^2)}$; but the length of this last projection is evidently to the length of the element itself, as the length P of the perpendicular let fall from the focus on the tangent is to the length r of the focal vector of the point of contact; such, therefore, is, by (59), the ratio of $n^{-\frac{1}{2}}$ to 1, if the scalar n, in the *equation of the normal* (57), receive the value which corresponds to the centre of curvature of the meridian; therefore we have

$$\frac{\mathrm{R}}{\mathrm{N}} = \frac{\nu - a}{r\varepsilon - a} = n = \frac{r^2}{\mathrm{P}^2} = \frac{rr'}{\mathrm{PP}'} = \frac{rr'}{pa}, \tag{61}$$

N denoting the length of the portion of the normal which is comprised between the meridian and the axis, and R denoting the length of the radius of curvature of the meridian. The projection of this radius on the focal vector is evidently the focal half chord of curvature, of which half chord the length may be here denoted by C; we see then that if we again project this half chord on the normal, the result is the normal itself, that is the portion N, because this double process of projection multiplies R twice successively by $n^{-\frac{1}{2}}$; and if, once more, the normal be projected on the focal vector, the third projection so obtained is equal in length to the semiparameter p, because, by (15) and (16),

$$\iota(r\varepsilon - a) + (r\varepsilon - a)\,\iota = 2p; \tag{62}$$

hence

$$\sqrt{(\mathrm{RN})} = \mathrm{C} = np = \frac{rr'}{a} = \frac{2rr'}{r + r'}, \tag{63}$$

that is, for any conic section, the geometrical mean between the radius of curvature and the normal is equal to the harmonic mean between the two focal distances; of which distances the second, namely r', is to be regarded as negative for the hyperbola, and infinite for the parabola, and the harmonic mean determined accordingly. We have also, for every conic section (if $r'a^{-1}$ be suitably interpreted),

$$\sqrt{n} = \frac{\mathrm{R}}{\mathrm{C}} = \frac{\mathrm{C}}{\mathrm{N}} = \frac{\mathrm{N}}{p} = \sqrt{\left(\frac{rr'}{pa}\right)}, \tag{64}$$

so that the semiparameter, the normal, the focal half chord of curvature, and the radius of curvature, are in continued geometrical progression: and the analysis may be verified, by calculating directly, on the same principles, the length of the normal, as follows:

$$\mathrm{N} = \sqrt{\{-(r\epsilon - a)^2\}} = \sqrt{\{r^2(e^2+1)+2r(p-r)\}} = \sqrt{\left(\frac{prr'}{a}\right)}. \tag{65}$$

The general relation of a conic section to a *directrix* is an immediate geometrical consequence of the equation (15), which has been here (in part) discussed, and may be regarded as its simplest interpretation. Some of the foregoing symbolical results respecting such a section admit of dynamical interpretations also; and, in particular, the expression $a\iota - a\epsilon$, which has been seen to represent, both in length and in direction, the perpendicular let fall from the second focus on the tangent, may suggest, by its composition, what is, however, a more immediate consequence of the equation (12), that in the undisturbed motion of a planet or comet about the sun, *the whole varying tangential velocity may be decomposed into two partial velocities, of which both are constant in magnitude, while one of them is constant in direction also.* The component velocity, which is constant in magnitude, but not in direction, is always in the plane of the orbit, and is perpendicular to the heliocentric radius vector of the body; the other component, which is constant in both magnitude and direction is parallel to the velocity at perihelion; and the magnitude of this fixed component is to the magnitude of the revolving one in the ratio of the excentricity e to unity. The author supposes that this theorem respecting a decomposition of the velocity in an excentric orbit is known, though he does not remember having met with it; but conceived that it might properly be mentioned here, as being a very easy and immediate consequence of the present analysis: respecting the general principles of which analysis, the reader is requested to consult the Abstract, already referred to, of the communication of November, 1844, printed at the commencement of the present volume of the Proceedings of the Academy. There is no difficulty in deducing, on the same principles, from formulæ of the present paper, the known differential equation,

$$\left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)^2 = \mathrm{M}\left(\frac{2}{r} - \frac{1}{a} - \frac{p}{r^2}\right), \qquad (66)$$

which connects the radial component of velocity with the heliocentric distance and the time, and may be integrated by the usual processes.

No. V.

December 8, 1845. (*See* page 150).

The following is the communication made by Sir William R. Hamilton on some additional applications of his theory of algebraic quaternions.

It had been shown to the Academy, at one of their meetings* in the last summer, that the differential equations of motion of a system of bodies attracting each other according to Newton's law, might be expressed by the formula:

$$\frac{d^2 a}{dt^2} = \Sigma \frac{m + \Delta m}{- \Delta a \sqrt{(- \Delta a^2)}} \tag{1}$$

in which a is the *vector* of any one such body, or of any elementary portion of a body, regarded as a material point, and referred to an arbitrary origin; m the constant called its mass; $a + \Delta a$, and $m + \Delta m$, the vector and the mass of another point or body of the system; Σ the sign of summation, relatively to all such other bodies, or attracting elements of the system; and d the characteristic of differentiation, performed with respect to t the time.

If we confine ourselves for a moment to the consideration of *two* bodies, m and m', and suppose r to be the positive number denoting the variable distance between them, so that r is the *length* of the vector $a' - a$, and, therefore, by the principles of this calculus,

$$r = \sqrt{\{-(a' - a)^2\}}; \tag{2}$$

we shall have, by the formula (1), the two equations,

$$\frac{d^2 a}{dt^2} = \frac{m' r^{-1}}{a - a'}, \quad \frac{d^2 a'}{dt^2} = \frac{m r^{-1}}{a' - a};$$

* See Appendix No. III., page xxxvii.

which may also be thus written,

$$m\frac{\mathrm{d}^2a}{\mathrm{d}t^2} = mm'r^{-3}(a' - a),$$

$$m'\frac{\mathrm{d}^2a'}{\mathrm{d}t^2} = mm'r^{-3}(a - a'),$$

and which give

$$0 = m\left(\delta a\frac{\mathrm{d}^2a}{\mathrm{d}t^2} + \frac{\mathrm{d}^2a}{\mathrm{d}t^2}\delta a\right) + m'\left(\delta a'\frac{\mathrm{d}^2a'}{\mathrm{d}t^2} + \frac{\mathrm{d}^2a'}{\mathrm{d}t^2}\delta a'\right) + 2\delta\frac{mm'}{r},$$

δa, $\delta a'$ being any arbitrary infinitesimal variations of the vectors a, a', and δr being the corresponding variation of r; because

$$\begin{aligned}&\delta a(a' - a) + (a' - a)\,\delta a + \delta a'(a - a') + (a - a')\,\delta a'\\ &\quad = -(\delta a' - \delta a)(a' - a) - (a' - a)(\delta a' - \delta a)\\ &\quad = -\delta\,.\,(a' - a)^2 = \delta\,.\,r^2 = 2r\delta r = -2r^3\delta\,.\,r^{-1}.\end{aligned}$$

And by extending this reasoning to any system of bodies, we deduce from the equation (1) this other formula, by which it may be replaced:

$$\tfrac{1}{2}\Sigma\,.\,m\left(\delta a\frac{\mathrm{d}^2a}{\mathrm{d}t^2} + \frac{\mathrm{d}^2a}{\mathrm{d}t^2}\delta a\right) + \delta\Sigma\frac{mm'}{r} = 0. \qquad (3)$$

Although it is believed that this result (3), if regarded merely *as a symbolic form*, is new, as well as the method by which it has been here obtained; yet if we transform it by the introduction of rectangular coordinates, x, y, z, making for this purpose

$$a = ix + jy + kz,\ a' = ix' + jy' + kz', \ldots \qquad (4)$$

and eliminating the squares and products of the three imaginary units, i, j, k, by the nine fundamental relations which were communicated to the Academy in 1843, namely,

$$\left.\begin{aligned}&i^2 = j^2 = k^2 = -1;\\ &ij = k,\ jk = i,\ ki = j;\\ &ji = -k,\ kj = -i,\ ik = -j;\end{aligned}\right\} \qquad (5)$$

we are conducted, from the equation (3), to a well-known equation of Lagrange, which may be written thus:

$$\Sigma \,.\, m\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\,\delta x + \frac{\mathrm{d}^2y}{\mathrm{d}t^2}\,\delta y + \frac{\mathrm{d}^2z}{\mathrm{d}t^2}\,\delta z\right) = \delta\Sigma \,.\, \frac{mm'}{r}\,;$$

where r, by (2), (4), (5), is equal to the known expression,

$$r = \sqrt{\{(x' - x)^2 + (y' - y)^2 + (z' - z)^2\}}.$$

If the law of the attraction were supposed different from that of the inverse square, a different function of r, instead of r^{-1}, should be multiplied by the product of two masses.

But further, it is not difficult so to operate on the formula (3), as to deduce from it another equation which shall be equivalent to the forms that were proposed by the present author, in his papers "On a General Method in Dynamics" (published in the Philosophical Transactions*), as being, at least theoretically, forms for the *integrals* of the differential equations of motion of any system of attracting bodies. For if we observe, that by the principles of the calculus of variations, combined with those of the method of vectors, we have the identity,

$$\delta\alpha\,\mathrm{d}^2\alpha + \mathrm{d}^2\alpha\,\delta\alpha = \mathrm{d}(\delta\alpha\,\mathrm{d}\alpha + \mathrm{d}\alpha\,\delta\alpha) - \delta(\mathrm{d}\alpha^2)\,;$$

and if we write

$$v = \sqrt{\left\{-\left(\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right)^2\right\}}, \qquad (6)$$

denoting by v the magnitude or degree (but not the direction) of the velocity of the body of which the vector is α; we may transform the equation (3) into the following:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Sigma.\frac{m}{2}\left(\delta\alpha\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \frac{\mathrm{d}\alpha}{\mathrm{d}t}\,\delta\alpha\right) + \delta\left(\Sigma\frac{mv^2}{2} + \Sigma\frac{mm'}{r}\right) = 0\,; \qquad (7)$$

which, when operated upon by the characteristic $\int_0^t \mathrm{d}t$, that is, when integrated once with respect to the time from 0 to t, becomes

$$\Sigma.\frac{m}{2}\left(\delta\alpha\frac{\mathrm{d}\alpha}{\mathrm{d}t} - \delta\alpha_0\frac{\mathrm{d}\alpha_0}{\mathrm{d}t} + \frac{\mathrm{d}\alpha}{\mathrm{d}t}\,\delta\alpha - \frac{\mathrm{d}\alpha_0}{\mathrm{d}t}\delta\alpha_0\right) + \delta\mathrm{F} = 0, \qquad (8)$$

* 1834, Part II. 1835, Part I.

if we make for abridgment

$$\text{F} = \int_0^t dt \left(\Sigma \frac{mv^2}{2} + \Sigma \frac{mm'}{r} \right), \qquad (9)$$

and denote by δa_0 and $\frac{da_0}{dt}$ the values which the variation of a, and the differential coefficient of that vector taken with respect to t, are supposed to have at the origin of the time. The definite integral denoted here by the letter F is the same which was denoted by the letter S in the Essays already referred to, and which was called, in one of those Essays, the *Principal Function* of the motion of a system of bodies; and if we now regard it as a function of the time t, and of all the final and initial vectors, $a, a', \ldots a_0, a'_0, \ldots$ of the various bodies of the system, and suppose (as we may) that its variation, taken with respect to all those vectors, is determined by an equation of the form,

$$0 = 2\delta\text{F} + \Sigma(\sigma\delta a - \sigma_0\delta a_0 + \delta a \cdot \sigma - \delta a_0 \cdot \sigma_0), \qquad (10)$$

in which σ, σ_0 are vectors, we are conducted, by comparison of the coefficients of the arbitrary variations of vectors, in the equations (8) and (10), to the two following systems of formulæ:

$$m\frac{da}{dt} = \sigma, \quad m'\frac{da'}{dt} = \sigma', \ldots; \qquad (11)$$

$$m\frac{da_0}{dt} = \sigma_0, \quad m'\frac{da'_0}{dt} = \sigma'_0, \ldots; \qquad (12)$$

of which the former may be regarded as intermediate, and the latter as final integrals of the differential equations of motion. The determination of the (vector) coefficient σ, from the variation of the (scalar) function F, is an operation of the same kind as the known operation of taking a partial differential coefficient, and may, in these new calculations, be called by the same name; but in order to be fully understood, it requires some new considerations, of which the account must be postponed to another occasion.

Consider a system of *three* attracting masses, m, m', m'', with their corresponding vectors, a, a', a''; and make for abridgment $a' - a = \beta$, and $a'' - a = \gamma$; we shall have, by (1), for the differential equations of motion of these three masses, referred to an arbitrary origin of vectors, the following:

$$\left.\begin{aligned}
\frac{d^2 a}{dt^2} &= \frac{m'}{-\beta\sqrt{(-\beta^2)}} + \frac{m''}{-\gamma\sqrt{(-\gamma^2)}}; \\
\frac{d^2(a+\beta)}{dt^2} &= \frac{m}{\beta\sqrt{(-\beta^2)}} + \frac{m''}{(\beta-\gamma)\sqrt{\{-(\beta-\gamma)^2\}}}; \\
\frac{d^2(a+\gamma)}{dt^2} &= \frac{m}{\gamma\sqrt{(-\gamma^2)}} + \frac{m'}{(\gamma-\beta)\sqrt{\{-(\gamma-\beta)^2\}}};
\end{aligned}\right\} \quad (13)$$

which give, for the internal or relative motions of m' and m'' about m, the equations:

$$\left.\begin{aligned}
\frac{d^2\beta}{dt^2} &= \frac{m+m'}{\beta\sqrt{(-\beta^2)}} + m''\left\{\frac{(\beta-\gamma)^{-1}}{\sqrt{\{-(\beta-\gamma)^2\}}} + \frac{\gamma^{-1}}{\sqrt{(-\gamma^2)}}\right\}; \\
\frac{d^2\gamma}{dt^2} &= \frac{m+m''}{\gamma\sqrt{(-\gamma^2)}} + m'\left\{\frac{(\gamma-\beta)^{-1}}{\sqrt{\{-(\gamma-\beta)^2\}}} + \frac{\beta^{-1}}{\sqrt{(-\beta^2)}}\right\}.
\end{aligned}\right\} \quad (14)$$

If we suppress the terms multiplied by m'' in the first of these equations (14), or the terms multiplied by m' in the second of those equations, we get the differential equation of motion of a binary system, under a form, from which it was shown to the Academy last summer, that the laws of Kepler can be deduced. If we take account of the terms thus suppressed, we have, at least in theory, the means of obtaining the perturbations.

Let m be the earth, m' the moon, m'' the sun; then β and γ will be the geocentric vectors of the moon and sun; and the laws of the disturbed motion of our satellite will be contained in the two equations (14), but especially in the first of these equations. By the principles of the present calculus we have the developments,

$$(\gamma-\beta)^{-1} = \gamma^{-1} + \gamma^{-1}\beta\gamma^{-1} + \gamma^{-1}\beta\gamma^{-1}\beta\gamma^{-1} + \ldots, \quad (15)$$

and

$$\frac{\sqrt{(-\gamma^2)}}{\sqrt{\{-(\gamma-\beta)^2\}}} = \left\{1 - \frac{\beta\gamma+\gamma\beta}{\gamma^2} + \frac{\beta^2}{\gamma^2}\right\}^{-\frac{1}{2}} = 1 + \frac{\beta\gamma+\gamma\beta}{2\gamma^2} + \ldots; \quad (16)$$

if then we reject the terms of the same order as $m''\beta^2\gamma^{-4}$, that is, terms depending on the inverse fourth power of the distance of the sun from the earth, the disturbing part of the expression for the second differential coefficient, taken with respect to the time, of the moon's geocentric vector, will reduce itself in this notation to the following:

$$\frac{-m''}{\sqrt{(-\gamma^2)}}\left(\frac{(\gamma-\beta)^{-1}\sqrt{(-\gamma^2)}}{\sqrt{\{-(\gamma-\beta)^2\}}}-\gamma^{-1}\right)=\tfrac{1}{2}m''(-\gamma^2)^{-\frac{3}{2}}(\beta+3\gamma^{-1}\beta\gamma). \quad (17)$$

This symbolic result admits of a simple geometrical interpretation. The symbol $\gamma^{-1}\beta\gamma$ denotes a vector in the plane of the two vectors β and γ, which has the same length as β, and is inclined at the same angle to γ, but at the other side of that line; so that γ bisects the angle between β and $\gamma^{-1}\beta\gamma$. If then we conceive a fictitious moon among the stars, so situated that either the sun, or a point opposite to the sun, as seen from the earth, bisects the arc of a great circle on the celestial sphere, between the fictitious and the actual moon (the bodies being here treated as points); and if we decompose the sun's disturbing force on the moon into two others, directed respectively towards the extremities of that celestial arc which is in this manner bisected: one component force, resulting from this decomposition, will be constantly ablatitious, tending directly to increase the distance of the moon from the earth, and bearing to the attractive force in the moon's undisturbed relative orbit, a ratio compounded of the direct ratio of half the mass of the sun to the sum of the masses of the earth and moon, and of the inverse ratio of the cubes of the distances of the sun and moon from the earth; and *the other component force, directed towards the fictitious moon*, will be *exactly triple of the ablatitious force* thus determined; provided that we still neglect all terms depending on the inverse fourth power of the sun's distance, as we have done in deducing the equation (17), of which the theorem here enunciated is an interpretation. A similar result, of course, holds good, for every satellite disturbed by the central body of a system. The theorem admits of being proved by considerations more elementary, but was suggested to the author by the analysis above described; which may be extended, by continuing the developments (15), (16), to the case of one planet disturbed by another, and to a more accurate theory of a satellite.

Without entering into any farther account at present of the attempts which he has made to apply the processes and notation of his calculus of quaternions, or method of vectors, to questions of physical astronomy, the author wished to state that he had found those processes, and that notation, adapt themselves with remarkable facility to questions and results respecting Poinsot's Theory of Mechanical Couples. A single *force*, of the ordinary kind, is naturally represented by a *vector*, because it is constructed or represented, in mathematical reasoning, by a straight line having direction; but also a *couple*, of the kind considered by Poinsot, is found, in Sir William Hamilton's analysis, to admit of being regarded as *the vector part of the product of two vectors*, namely, of those which represent respectively one of the two forces of the couple, and the straight line drawn to any point of its line of direction from any point of the line of direction of the other force. Composition of couples corresponds to addition of such vector parts; and the laws of equilibrium of several forces, applied to various points of a solid body, are thus included in the two equations,

$$\Sigma\beta = 0; \quad \Sigma(\alpha\beta - \beta\alpha) = 0; \tag{18}$$

the vector of the point of application being α, and the vector representing the force applied at that point being β. The condition of the existence of a single resultant is expressed by the formula,

$$\Sigma\beta \, . \, \Sigma(\alpha\beta - \beta\alpha) + \Sigma(\alpha\beta - \beta\alpha) \, . \, \Sigma\beta = 0. \tag{19}$$

Instead of the two equations of equilibrium (18), we may employ the single formula

$$\Sigma \, . \, \alpha\beta = -c, \tag{20}$$

c here denoting a scalar (or real) quantity, which is independent of the origin of vectors, and seems to have some title to be called the total *tension* of the system.

In mentioning finally some applications of his algebraic method to central surfaces of the second order, the author could not but feel that he spoke in the presence of persons, of whom several were much better acquainted with the general geometrical properties of those surfaces than he could pretend to be. But, while deeply conscious that he had much to learn in this department from his brethren of the Dublin School, as well as from mathematicians elsewhere, he ventured to hope that the novelty and simplicity of the symbolic forms which he was about to submit to their notice might induce some of them to regard the future development of the principles of his method as a task not unworthy of their co-operation. He finds, then, that if α and β denote two arbitrary but constant vectors, and if ρ be a variable vector, the equation of an ellipsoid with three arbitrary, and, in general, unequal axes, referred to the centre as the origin of vectors, may be put under the following form

$$(\alpha\rho + \rho\alpha)^2 - (\beta\rho - \rho\beta)^2 = 1. \qquad (21)$$

One of its circumscribing cylinders of revolution is denoted by the equation

$$- (\beta\rho - \rho\beta)^2 = 1; \qquad (22)$$

the plane of the ellipse of contact by

$$\alpha\rho + \rho\alpha = 0; \qquad (23)$$

and the system of the two tangent planes parallel hereto, by

$$(\alpha\rho + \rho\alpha)^2 = 1. \qquad (24)$$

A hyperboloid of one sheet, touching the same cylinder in the same ellipse, is denoted by the equation

$$(\alpha\rho + \rho\alpha)^2 + (\beta\rho - \rho\beta)^2 = -1; \qquad (25)$$

its asymptotic cone by

$$(\alpha\rho + \rho\alpha)^2 + (\beta\rho - \rho\beta)^2 = 0; \qquad (26)$$

and a hyperboloid of two sheets, with the same asymptotic cone (26), and with the tangent planes (24), is represented by the formula

$$(a\rho + \rho a)^2 + (\beta\rho - \rho\beta)^2 = 1. \qquad (27)$$

By changing ρ to $\rho - \gamma$, in which γ is a third arbitrary but constant vector, we introduce an arbitrary origin of vectors, or an arbitrary position of the centre of the surface as referred to such an origin; and the general problem of determining that individual surface of the second order (supposed to have a centre, until the calculation shall show in any particular question that it has none), which shall pass through *nine given points*, may thus be regarded as equivalent to the problem of finding *three constant vectors*, a, β, γ, which shall, for nine given values of the variable vector ρ, satisfy one equation of the form

$$\{a(\rho-\gamma)+(\rho-\gamma)a\}^2 \pm \{\beta(\rho-\gamma)-(\rho-\gamma)\beta\}^2 = \pm 1; \quad (28)$$

with suitable selections of the two ambiguous signs, depending on, and in their turn determining, the particular nature of the surface. It is not difficult to transform the equation (28), or those which it includes, so as to put in evidence some of the chief properties of surfaces of the second order, with respect to their circular sections.

The recent expressions may be abridged, if we agree to employ the letters s and v as characteristics of the operations of taking separately the scalar and vector parts of any quaternion to which they are prefixed; for then we shall have

$$a\rho + \rho a = 2\mathrm{s} \,.\, a\rho, \qquad \beta\rho - \rho\beta = 2\mathrm{v} \,.\, \beta\rho; \qquad (29)$$

so that, by making for abridgment $2a = a'$, $2\beta = \beta'$, the equation (21) of the ellipsoid (for example) will take the shorter form,

$$(\mathrm{s}.\, a'\rho)^2 - (\mathrm{v}.\, \beta'\rho)^2 = 1. \qquad (30)$$

Another modification of the notation, which, from its geometrical character, will often be found useful, or at least illustrative, may be obtained by agreeing to denote by the geometrical symbol BA the vector $\beta - a$, which is the difference of two other vectors a and β drawn to the two points A and B, from any common origin; so that BA is the vector *to* B *from* A. Denoting also by the symbol CBA the quaternion CB $\times$ BA, which is the product of the two vectors CB and BA; by DCBA the continued product DC $\times$ CB $\times$ BA, and so on: the foregoing equations of central surfaces may be transformed, and a great number of geometrical processes and results expressed under concise and not inelegant forms. For example, the symbols

$$\frac{\mathrm{V.ABC}}{\mathrm{AC}}, \quad (31), \qquad \text{and } \frac{\mathrm{S.ABCD}}{\mathrm{V.ABC}}, \qquad (32)$$

will denote, in length and in direction, the perpendiculars let fall, respectively, from the summit B on the base AC of a triangle, and from the summit D on the base ABC of a tetrahedron: the sextuple area of this tetrahedron ABCD being expressed in the same notation by the symbol S.ABCD.

The developments (15) and (16), with a great number of others, may be included in a formula which corresponds to Taylor's theorem, namely, the following:

$$f(a + \mathrm{d}a) = \left(1 + \frac{\mathrm{d}}{1} + \frac{\mathrm{d}^2}{1.2} + \ldots\right) f a; \qquad (33)$$

the only new circumstance being, that in interpreting or transforming the separate terms, for example, the term $\frac{1}{2}\mathrm{d}^2 fa$, of the resulting development of the function $f(a + \mathrm{d}a)$, if a and its differential $\mathrm{d}a$ denote vectors, we must in general employ *new rules of differentiation*, having indeed a very close affinity to the known rules, but modified by the non-commutative character of the operation of multiplication in this calculus of vectors or of quaternions. It is thus that, instead of writing $\mathrm{d}.a^2 = 2a\mathrm{d}a$, $\delta.a^2 = 2a\delta a$, we have been obliged to write

$$\mathrm{d}.a^2 = a.\mathrm{d}a + \mathrm{d}a.a; \ (34) \qquad \delta.a^2 = a.\delta a + \delta a.a. \ (35).$$

No. VI.

METEOROLOGICAL JOURNAL,

COMMENCING

1st JANUARY, 1845, and ending 31st DECEMBER, 1845,

BY

GEORGE YEATES.

The instruments employed, and the general circumstances of the mode of observing, have been described in the preliminary observations to the Tables of the year 1843, in the 2nd volume of the Proceedings of the Academy, Appendix V.

JANUARY.						FEBRUARY.						MARCH.					
	Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.
	Max.	Min.					Max.	Min.					Max.	Min.			
1	47°	33°	30.140	. . .	W.	1	42°	24°	29.340	.006	W. by N.	1	49°	42°	29.800	.090	S. E.
2	47	35	30.070	. . .	W.	2	35	29	30.040	.120	W. by N.	2	46	36	29.950	.100	N. W.
3	48	38	29.770	.040	S. W.	3	48	36	30.000	. . .	N. W.	3	46	39	29.750	.085	N. E.
4	50	34	30.140	.040	S. W.	4	48	35	30.240	. . .	W. by N.	4	49	33	29.870	. . .	S. E.
5	52	38	29.900	.139	S. W.	5	44	39	30.100	. . .	W.	5	42	32	29.990	. . .	N. E.
6	55	49	29.940	. . .	W. by S.	6	46	33	30.100	. . .	W. N. W.	6	36	29	30.280	. . .	N. E.
7	54	43	30.120	.525	W. by S.	7	40	32	29.950	. . .	S. E.	7	43	35	30.290	.006	N. E.
8	49	39	30.060	. . .	S. E.	8	39	32	29.970	. . .	S. E.	8	43	36	30.220	. . .	N. E.
9	52	48	29.870	. . .	E. by S.	9	40	34	29.700	.100	W.	9	40	28	30.330	.025	N. E.
10	54	45	29.490	. . .	E. S. E.	10	49	40	29.530	. . .	W.	10	51	40	30.240	.100	N. W.
11	54	40	29.370	.099	S. W.	11	46	30	29.860	.143	W. by N.	11	51	35	30.100	. . .	N. by E.
12	49	35	29.730	. . .	S. E.	12	37	30	30.180	. . .	W. by N.	12	40	25	29.870	. . .	N. N. E.
13	48	39	29.250	.220	E.	13	52	43	29.780	.020	W.	13	40	29	29.750	. . .	E. N. E.
14	52	41	29.620	.390	W. N. W.	14	54	34	29.800	.002	W.	14	32	19	29.660	. . .	W. N. W.
15	49	42	29.480	.080	E.	15	50	35	30.080	. . .	W. by N.	15	44	20	29.750	. . .	N. E.
16	49	33	29.970	.100	W. S. W.	16	47	40	29.340	. . .	S. E.	16	37	29	29.750	.110	N. E.
17	51	33	29.760	.005	S. W.	17	48	40	29.790	. . .	S. E.	17	36	21	29.780	. . .	N. by W.
18	53	35	29.300	.315	S. W.	18	48	40	29.900	. . .	S. by E.	18	37	27	29.730	.015	W.
19	48	34	29.160	.030	S. W.	19	48	39	30.030	. . .	S. by E.	19	43	31	29.770	.020	N. W.
20	43	30	29.500	1.000	N. W.	20	45	35	30.060	. . .	S. W.	20	41	29	30.180	.240	S. W.
21	43	27	30.340	.005	E.	21	44	33	29.700	. . .	S. E.	21	43	31	30.120	. . .	W. by S.
22	53	30	30.070	. . .	W.	22	45	37	29.270	.160	N. E.	22	53	46	30.050	. . .	W. by N.
23	54	42	29.600	. . .	S. W.	23	50	37	29.400	.020	W. by N.	23	59	55	30.050	.025	W.
24	52	39	29.700	.230	W. S. W.	24	46	38	29.720	.005	W. by N.	24	52	35	30.150	.180	W.
25	53	36	29.680	. . .	W. by S.	25	48	36	29.880	.006	E.	25	50	43	29.800	. . .	W.
26	54	40	29.670	.100	W.	26	50	37	29.690	.180	N. W.	26	51	38	29.800	. . .	W.
27	53	34	29.300	.140	W.	27	50	33	29.840	. . .	E. by S.	27	55	47	29.660	.250	N. W.
28	45	30	28.900	.028	N. W.	28	48	41	29.750	. . .	E. by S.	28	48	45	29.570	.100	W. by N.
29	42	27	29.250	.004	N. W.							29	45	37	30.160	.020	W. by N.
30	42	23	29.200	.005	N. W.							30	55	37	29.720	.010	S.
31	40	21	29.470	. . .	N. E.							31	56	37	30.220	.016	W.

	APRIL.				
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	50°	42°	30.060	.015	S. E.
2	56	43	29.920	. . .	N. E.
3	55	40	29.890	. . .	N.
4	57	33	29.890	. . .	N.
5	52	42	30.060	. . .	N. E.
6	50	32	29.970	. . .	N. E.
7	48	28	29.880	. . .	W.
8	59	39	29.450	. . .	N. W.
9	51	35	29.040	.110	W. by S.
10	49	38	29.200	. . .	W. by N.
11	48	35	29.640	.007	N.
12	49	34	29.670	.037	W. by N.
13	49	41	29.340	.145	W. by S.
14	50	42	29.660	.050	W. by S.
15	50	40	30.230	. . .	E.
16	53	31	30.460	. . .	N. E.
17	55	38	30.460	. . .	N. E.
18	59	47	30.260	.002	N. E.
19	54	44	30.140	. . .	N. E.
20	53	35	30.170	. . .	E.
21	59	35	30.120	. . .	E.
22	59	36	29.900	. . .	N. E.
23	55	43	29.800	. . .	N. E.
24	55	43	29.770	.008	N. E.
25	65	40	29.530	. . .	S. W.
26	53	47	29.140	.150	S. W.
27	50	49	29.140	.245	S. W.
28	61	47	29.570	.075	S. W.
29	61	45	29.800	.065	S. W.
30	61	42	29.800	. . .	S. W.

	MAY.				
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	61°	57°	29.600	. . .	S. W.
2	52	45	29.600	.060	S. W.
3	54	44	29.048	.110	S. W.
4	50	44	29.900	.045	N. W.
5	48	37	30.000	.010	W. by S.
6	49	41	29.660	.220	W.
7	50	40	29.880	.140	N. E.
8	49	38	29.500	.175	E.
9	50	39	29.450	.220	N. E.
10	49	32	29.560	.200	N. W.
11	60	34	29.800	. . .	S. W.
12	60	40	29.870	.070	W. by N.
13	60	38	30.300	.025	N. by E.
14	59	41	30.880	. . .	S. W.
15	60	52	30.450	. . .	W. by N.
16	60	49	30.440	. . .	W. by N.
17	60	49	30.200	. . .	W. by N.
18	59	44	30.160	. . .	W. by N.
19	60	47	29.920	. . .	W. by N.
20	60	39	29.990	.020	E.
21	61	38	29.800	. . .	E. by N.
22	60	42	30.040	. . .	E. by S.
23	60	40	29.920	. . .	E. by N.
24	60	47	29.940	.003	N. E.
25	61	47	29.850	. . .	E. by N.
26	60	45	29.740	. . .	E. by N.
27	58	43	29.800	.240	N. E.
28	58	45	30.000	. . .	N. E.
29	58	45	29.370	.040	E. by N.
30	58	45	30.020	.002	E. by N.
31	65	39	30.090	. . .	N. E.

	JUNE.				
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	60°	47°	29.990	.118	S. W.
2	67	47	29.840	. . .	S. W.
3	70	50	29.330	. . .	S. by W.
4	57	41	29.370	.450	W. by S.
5	64	49	29.220	.040	S. W.
6	61	51	29.260	.030	W. by S.
7	60	46	29.680	.025	S.
8	60	43	29.880	1.300	W.
9	59	41	30.260	.005	W. by S.
10	65	55	30.350	. . .	W.
11	69	53	30.260	. . .	E.
12	73	50	30.200	. . .	S. W.
13	80	50	30.300	. . .	W.
14	77	49	30.230	. . .	S. E.
15	74	52	30.000	.018	E. by S.
16	72	51	29.770	.196	S. W.
17	69	49	29.880	. . .	E.
18	68	41	29.840	. . .	W. by S.
19	68	50	30.620	. . .	W. by S.
20	65	55	30.200	. . .	N. E.
21	69	50	30.200	. . .	S. W.
22	67	46	30.160	. . .	W.
23	66	52	30.220	. . .	S. W.
24	72	51	29.730	.070	S. W.
25	64	47	29.860	.040	N. W.
26	65	45	29.890	. . .	S. W.
27	64	46	29.380	.350	S. E.
28	64	44	29.490	.460	N. W.
29	63	36	29.820	. . .	S. W.
30	64	46	29.700	.035	S. W.

JULY.						AUGUST.						SEPTEMBER.					
	Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.
	Max.	Min.					Max.	Min.					Max.	Min.			
1	64°	53°	29.250	.110	W.	1	65°	44°	29.400	.026	W. by N.	1	70°	50°	30.330	. . .	W.
2	65	51	29.860	.030	S.	2	63	41	29.490	.110	W. by N.	2	68	54	30.260	. . .	N. E.
3	64	50	29.580	1.030	N. E.	3	65	48	29.470	.045	S. W.	3	70	50	30.250	.025	N. E.
4	61	55	29.920	.120	W. by N.	4	65	43	29.640	.130	S. E.	4	65	41	30.240	. . .	E. by N.
5	65	42	30.160	.040	S. W.	5	66	45	29.750	.130	N.	5	70	43	30.200	. . .	N. E.
6	68	49	29.960	. . .	N. E.	6	66	44	29.700	.260	S. W.	6	66	52	30.200	. . .	E. by S.
7	62	49	29.850	.190	S. W.	7	66	46	29.760	.100	W.	7	67	52	30.275	. . .	E.
8	69	50	29.750	.010	S. W.	8	65	45	29.900	.168	W.	8	68	58	30.100	. . .	E.
9	69	48	29.870	.220	S. E.	9	65	46	29.580	.870	N. W.	9	70	41	30.000	. . .	W. by S.
10	61	49	29.680	.246	N. E.	10	64	52	29.690	. . .	N. W.	10	76	55	30.099	. . .	W. by S.
11	65	44	29.780	.130	N. N. W.	11	66	50	29.790	.008	N. W.	11	70	52	30.070	. . .	E.
12	64	44	29.980	.010	N. W.	12	63	52	30.030	.011	S. W.	12	69	48	29.982	. . .	E.
13	63	48	29.900	.189	N. N. W.	13	64	49	30.200	. . .	S. W.	13	69	44	29.800	. . .	E.
14	65	49	30.140	.025	N. N. W.	14	64	52	30.900	. . .	N. W.	14	66	40	29.500	.160	W. by N.
15	64	44	30.230	. . .	S. W.	15	62	53	29.950	.008	N. E.	15	68	39	29.422	. . .	W. by S.
16	66	44	30.050	. . .	S. W.	16	64	45	30.100	. . .	N.	16	65	35	30.000	.020	S. E.
17	66	56	29.960	.175	S. W.	17	61	46	29.840	. . .	N. W.	17	70	42	29.070	.242	S. W.
18	68	52	30.150	. . .	S. E.	18	62	52	29.600	.080	W. by S.	18	73	40	28.970	.018	W. by N.
19	77	46	30.100	. . .	S. E.	19	63	51	29.660	.100	W.	19	68	35	29.530	.018	W. by N.
20	68	49	30.130	. . .	S. E.	20	64	48	29.660	.086	N. W.	20	62	29	29 680	.065	S. W.
21	66	50	30.080	. . .	N. E.	21	62	49	29.958	.008	N. W.	21	64	33	29.430	.022	N. E.
22	71	51	30.060	. . .	N E.	22	65	46	30.122	. . .	S. W.	22	60	36	29.800	. . .	W. by N.
23	67	50	30.070	. . .	N. E.	23	66	51	29.868	.030	S. W.	23	55	30	30.332	. . .	N. E.
24	65	50	30.020	. . .	N. E.	24	70	51	29.980	.012	S. W.	24	58	30	30.200	. . .	S.
25	65	49	30.020	. . .	N. W.	25	70	56	29.680	.145	S. W.	25	58	35	29.582	.050	S. W.
26	66	47	29.900	. . .	N. W.	26	66	50	29.980	.182	N. W.	26	60	44	29.700	. . .	N. W.
27	65	50	29.650	.070	S. W.	27	69	50	30.230	. . .	S. W.	27	61	47	29.792	.260	W.
28	65	40	29.640	.005	N. W.	28	67	56	30.360	.012	N. W.	28	65	46	29.790	.015	N. W.
29	66	42	29.760	.220	W.	29	66	57	30.382	. . .	N. W.	29	58	42	29.780	.010	S. W.
30	65	42	29.430	.160	S. by E.	30	67	54	30.440	.002	N. by W.	30	56	47	29.570	. . .	W.
31	62	46	29.100	.190	S. W.	31	66	54	30.445	. . .	W.						

OCTOBER.						NOVEMBER.						DECEMBER.					
	Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.
	Max.	Min.					Max.	Min.					Max.	Min.			
1	62°	43°	29.702	.043	S. W.	1	58°	31°	30.150	.006	W. by N.	1	42°	33°	29.600	.130	S. W.
2	58	47	29.660	.200	S. W.	2	52	44	30.140	.006	S. E.	2	45	37	29.580	.025	W.
3	53	48	29.280	.800	W.	3	53	40	30.190	.002	S. E.	3	44	32	29.240	.008	W. by S.
4	61	49	29.289	.620	E. by N.	4	53	34	29.800	. . .	S. W.	4	39	32	29.600	.008	W.
5	55	35	29.900	.020	S. E.	5	54	44	29.520	. . .	S.	5	49	38	29.340	.260	W.
6	54	44	29.720	. . .	E. by N.	6	57	51	29.300	.090	S. E.	6	46	36	29.160	.008	W.
7	55	49	29.500	.110	N. W.	7	57	43	29.200	.300	S. E.	7	43	32	29.780	.015	W.
8	54	38	29.176	. . .	S. W.	8	61	45	29.350	.500	W.	8	48	28	29.890	. . .	W.
9	54	39	29.190	.300	E. by S.	9	50	43	29.400	.338	W. by S.	9	50	42	30.150	. . .	W.
10	55	42	29.230	.008	S.	10	48	41	29.280	.155	W. by N.	10	47	35	30.380	.002	W.
11	55	40	29.230	.135	S. W.	11	51	39	29.230	.035	W. by S.	11	51	41	30.030	.006	W.
12	55	38	29.982	.020	S. W.	12	50	38	29.500	.002	N. W.	12	47	32	30.400	. . .	W.
13	61	48	29.900	.160	S.	13	49	34	29.850	.008	S. E.	13	40	24	30.420	. . .	W.
14	66	55	30.130	.055	S.	14	43	30	29.900	.008	S. W.	14	47	27	30.100	.045	W.
15	65	54	30.000	. . .	W.	15	51	39	29.570	.012	W. by S.	15	47	30	29.800	.029	W.
16	56	44	30.100	. . .	W.	16	54	44	29.030	.565	W.	16	46	41	29.760	. . .	W.
17	56	48	29.900	. . .	W.	17	51	40	29.500	.060	W.	17	48	42	29.860	.266	S. E.
18	67	50	30.060	.008	W.	18	52	37	29.060	.130	W.	18	48	34	29.580	.460	W.
19	61	53	30.220	. . .	W.	19	56	46	28.680	.315	W.	19	48	33	28.930	.100	W. by N.
20	64	53	30.000	.045	W.	20	42	32	28.950	.043	S. W.	20	44	33	28.880	.182	N. W.
21	55	39	30.400	. . .	W. by N.	21	47	35	29.382	. . .	S. W.	21	43	33	29.840	.300	W. by N.
22	54	46	30.450	. . .	W.	22	42	30	29.580	. . .	W. by N.	22	49	30	29.350	.065	W. by N.
23	54	46	30.450	. . .	W.	23	40	30	29.940	.070	W.	23	53	37	29.620	.015	W. by N.
24	54	46	30.170	. . .	W.	24	41	33	30.080	. . .	N. W.	24	47	29	30.190	. . .	W.
25	57	36	30.390	. . .	W.	25	51	35	29.860	.064	W.	25	48	32	30.180	.010	W. by N.
26	49	32	30.232	. . .	W. by N.	26	55	43	29.580	.030	W.	26	50	37	29.700	. . .	S. W.
27	52	45	30.000	. . .	S. W.	27	55	51	29.700	.065	W. by S.	27	51	34	29.960	.240	W.
28	56	49	29.900	.008	S. W.	28	55	47	29.270	. . .	E. by S.	28	54	40	29.450	1.090	W. by N.
29	56	49	29.700	.085	S. E.	29	52	41	29.600	.006	W.	29	45	32	29.700	.060	W. by S.
30	58	49	29.850	.030	W. by S.	30	46	35	29.780	. . .	W.	30	58	42	29.370	.050	W.
31	54	32	30.760	.030	W.							31	53	37	29.790	. . .	S.

	MEAN TEMPERATURE OF LAST 3 YEARS.			RAIN LAST 3 YEARS.		
	1843.	1844.	1845.	1843.	1844.	1845.
January	40.9	41.8	42.6	2.589	1.031	3.495
February	37.8	38.6	40.8	1.635	2.308	.762
March	40.8	43.0	39.9	1.643	2.042	1.412
April	49.8	51.4	46.9	2.645	.545	.909
May	53.5	55.3	49.8	4.644	.316	1.680
June	58.9	59.0	57.1	2.277	1.539	3.131
July	62.0	60.0	56.8	1.773	2.148	3.170
August	63.2	57.9	57.2	1.541	3.473	2.523
September	59.6	57.0	54.0	.637	1.845	.705
October	48.0	49.9	50.0	3.302	2.530	2.677
November	43.7	48.7	45.0	2.425	3.742	2.810
December	47.6	41.1	40.0	0.138	2.192	1.539
Mean of last three years . .	50.6	50.1	48.3	25.249	23.711	26.628

No. VII.

The following is an abstract of the paper read by Professor Harrison on the 26th of January, 1846. (*See* page 184).

It is generally known that Baron Cuvier, in the "Regne Animale," mentions two distinct species of Cassowary, one, the Galeated, or the Struthio Casouarius, found in several of the islands of the Indian Archipelago; the other, the Casouarius (or Dromaius) Novæ Hollandiæ.

The first, or galeated cassowary, has the bill compressed laterally, the head surmounted with a bony prominence, covered with a corneous substance; the skin of the head and top of the neck naked, tinted with a sky-blue and flame colour, with pendant caruncles, like those of the turkey; the wings have stiff feathers, without barbs, and serve as weapons in fighting; the claw of the internal toe is much the strongest. Next to the ostrich, this is the largest bird in nature, from which, however, it differs in internal organization, the intestines being short, and the cæca and cloaca small, and no intermediate stomach between the crop and gizzard; it lives on fruit and eggs, but not on grain. Its eggs are of a green colour. The second, or New Holland cassowary, has the bill depressed, no helmet on the head, naked round the ear only; the plumage is brown, thicker, and more bearded; no caruncles or alar spines, and the claws are nearly equal. On its internal organization the Baron makes no remark, neither does he allude to the peculiar condition of the trachea, which forms a striking discriminating character between it and the galeated bird. Dr. Knox, the distinguished anatomist and lecturer in Edinburgh, in an extremely interesting paper, read before the Wernerian Natural History Society, and published in the Edinburgh Philosophical Journal, in April, 1823, has entered more fully into a comparison of the internal organization of these two species, and has given the first account I have met with, of the remarkable tracheal sac in the emu. His anatomical description, however, of this curious appendix, is brief and imperfect, which may be accounted for by the specimen he examined being greatly mutilated before it fell into his possession. His account of it is as follows:—"At the fifty-second ring, counting from the glottis, there is found a large muscular bag, about the size of a man's head, into which the windpipe opens by a large orifice, occasioned by a deficiency of part of the circumference, in about thirteen tracheal rings; or rather the rings, instead of closing round to form the tube of the trachea, expand outwards, and are attached to the sides of the bag; it has no communication with any of the air cells"—p. 36. "This muscular bag is as large as the human head, is closely attached to the sides of the trachea and expanded rings, is situated in the neck, immediately above the bone called the merrythought; it was seen by me in the female, though it is probable the male also possesses it. It is quite peculiar to this bird, no such appendage having been ever seen attached to the trachea of any of the feathered creation, nor do I know of anything analogous to it in any other animal, excepting in the chameleon, to the upper portion of whose trachea there is appended a comparatively large membranous bag."—p. 138. "It has not the most distant resemblance to the tracheal appendages found in other birds. In thus differing so singularly and mysteriously from the analogous structure of birds of the old and new continents, it fully confirms the opinions of some naturalists, that the living productions of Australia will, when properly examined, be found to present peculiarities altogether wonderful, and, perhaps, yet, for a long period, quite inexplicable."—p. 139.

Meckel, in the tenth volume of his Comparative Anatomy, alluded to the discovery of this air sac by Fremery, in the year 1819, but states that he has noticed it in a vague and imperfect manner. I have not had an opportunity of consulting this work of Fremery. Meckel does not describe this air sac with that clearness and precision so characteristic of this writer. His account rather consists of quotations from Knox, and of the varying statements of other writers, than of any observations of his own. Carus, in his System of Comparative Anatomy, makes no mention of it. It does not appear to have been known to Cuvier, as there is no notice of it either in his Anatomie Comparée, or in his Regne Animale, neither is it alluded to in the new edition of that invaluable work, now in course of publication. In the elaborate and excellent article "Aves," in the Encyclopædia of Anatomy and Physiology, edited by Dr. Todd, of London, and which article is from the pen of Mr. Owen, who is justly regarded as the first comparative anatomist and zoologist of the present day, this peculiar appendage is alluded to; but, strange to say, the brief and imperfect account which is there given of it is wholly incorrect, a circumstance I can only attribute to some accidental inadvertence. Mr. Owen says, "that the trachea of the emu is remarkable for a sudden dilatation; but, in this instance, the cartilaginous rings do not preserve their integrity at the dilated part, but are wanting posteriorly, where the tube is completed by membranes only." I have made some researches into the writings of other authors also, but have not met with any accurate description of this apparatus.

As I have lately had an opportunity of examining this appendix in the living bird, as well as in one recently dead, I shall state such facts as I have observed in the former condition, as well as the appearances I have remarked in the latter. I may first observe, that this exists both in the male and female; for some time since I dissected a male emu, and found the opening in the trachea, but the soft parts were too much injured by decomposition to admit of an accurate examination of the sac itself. The specimen from which the following account is taken is an adult, or probably, an old female.

The cervical air-bag (fig. 1) occupies a broad and deep depression on the forepart of the neck, immediately above the sternum and furculum bone, which latter in this bird is small and imperfect. The sac is not observable either in the living or the dead bird, unless distended, when it slowly bulges forwards and laterally, and fills the depression above-mentioned; it is not, however, even then so prominent as to cause any remarkable deformity. On examining this region during life, the sac being undistended, I found the skin very moveable; it gave to the hand the sensation of great warmth, when contrasted with the surrounding parts. The entire of the trachea was very moveable to either side; but on fixing it steadily, and carrying the fingers along its forepart down to the sternum, the tube was felt a little above the latter to be flattened or depressed, but no slit or opening could be distinguished; the sac was perfectly flaccid and compressed, and the communication with the trachea was closed. This examination did not appear to cause any peculiar uneasiness in the part, or to excite any general irritation in the respiratory system. This bird, however, is very restless and timid, and very impatient under any restraint; it is also possessed of great muscular power, three or four men being required to secure it, and the attempt to do so is by no means free from danger, either to itself or to those around, as it struggles with great violence, and strikes with its powerful claws in every direction. I succeeded twice, however, to my satisfaction, in feeling this region; and I think I am warranted in concluding that the tracheal opening is usually closed, and that it may be opened at the pleasure of the animal. The plumage in this situation is thin and scanty; the feathers are chiefly of the double species, that is, two barbs proceeding from one quill; the barrel of the latter is small, and filled with pith, and the feathers are fine and hair-like. This, indeed, is the general character of the plumage of the struthiones.

A few days after the death of one of these birds (a female), I carefully examined the sac, larynx, and trachea, and the following is an accurate account of the several appearances. The integuments covering the air-bag differ in no respect from the general investment of the neck; beneath the skin is a strong and red muscular lamina, expanded over its sides and forepart; the fibres are chiefly longitudinal, but several strong fasciculi run in different directions; the subjacent areolar tissue contains numerous nerves and vessels, especially veins. On opening the bag, the tracheal orifice (fig. 2) is seen distinct and well-defined, of the form of a long parallelogram; the mucous membrane of the tube is continued around its free margin, is reflected over, and adheres to the anterior and lateral parts of the tube considerably beyond the edges of the opening, and then expands in all directions to line this capacious reservoir, which is sufficiently large to contain at least a quart or three pints of fluid. This membrane is soft and vascular; numerous capillaries and large veins, with long tortuous nerves, are seen distinctly through it. The tracheal opening (fig. 4) is situated about three inches above the furculum, and in the middle line of the tube; it is two inches and a half long, and scarcely half an inch wide, but it can be easily extended to three inches in the vertical and to three-quarters of an inch in the transverse direction; it is produced by a deficiency in the anterior part of six rings (in the two specimens I have examined the number was the same); the cartilage above and that below the opening is broad and well defined; the extremities of the six lateral cartilages are sharp, thin, and very moveable; there is no dilatation, at least to any appreciable degree, in this part of the tube, and the rings are not all extended or continued into the parietes of the sac, but, on the contrary, are bent towards each other of opposite sides, and can, by a little lateral pressure, be brought into contact. On the interior of the back part of the trachea, exactly opposite the opening in the median line, a remarkable prominence, or vertical keel-like projection is observed standing forward into the tube, and presenting a ridge of rounded bifid points or tubercles; when the sides of the opening are approximated, the anterior extremities of the six rings come into contact with and are supported by this posterior ridge, so that the trachea is there divided into two channels, one at either side of this middle line; and thus, when the opening into the sac is closed, the respiratory passage is maintained free and uninterrupted, while at the same time its anterior wall is well supported against any collapse into the cavity, and is also enabled to resist the weight or pressure of the external atmosphere, under the suction influence of the inspiratory efforts: this posterior keel-like projection extends for some distance on the back part of the trachea, both above and below the opening. When the trachea is relaxed in the longitudinal direction, the rings are all approximated, and those bounding the opening are a little overlapped by that above and that below it, and the apposition of the several segments is still further secured by the pressure of the superincumbent soft parts. This cervical air-bag bears no analogy to the air-cells disseminated through different parts of the bodies of birds; such are formed of cellular tissue, but this is an extension of the mucous surface, and has no communication with the air-cells, excepting through the trachea and lungs.

The TONGUE (fig. 1) is small, flat, thin, and triangular; its surface presents but few papillæ, but is studded with innumerable small points, orifices of mucous follicles; its margins are neatly fringed with five or six loose, denticulated folds on either side, some of which are a quarter of an inch in length. From the base of the tongue proceeds backwards a thick semilunar fold of membrane, of the same colour and texture; from its centre is a short, conical projection, analogous in its position, in front of the glottis, to an epiglottis, and capable of acting as such to a slight extent; it is devoid of cartilage, but when the tongue is retracted, this process can cover the anterior half of the glottis. The Hyoid bone, or rather cartilage, supports the tongue by a broad basis, from the centre of which a short, strong style passes forwards in the median line; a similar process descends in front of the thyroid cartilage, and is attached to it, and to the forepart of the trachea, by elastic ligament; the cornua are pliant and elastic, and curved backwards in a tortuous form; they support the pharynx and fauces, and admit of considerable expansion. The glottis, or superior larynx is well developed, and bears some resemblance to the rima in mammalia; its aspect is obliquely upwards and backwards, and is placed, as usual in birds, on the posterior plane of the trachea, though not to the same extent; but it is devoid of all spines, tubercles, and papillæ, nor has it the chink-like form so common in that class. The thyroid cartilage bounds the larynx in front, the cricoid behind: on the upper edge of the latter are two thin broad plates, passing forwards, from the inner side of the centre of each there is a prominent and firm fibro-mucous body, projecting inwards; these bodies bear some analogy to arytenoid cartilages; they nearly touch, and can easily be made to do so, whereby the opening is divided into two parts; the anterior, small and triangular, can be covered by the epiglottis when the tongue is retracted; the posterior is round or oval, and can only be closed by the action of the surrounding muscles pressing its sides into contact during the act of deglutition. These two portions bear a close resemblance in form to the human rima glottidis, when subdivided into two by the approximation of the long, anterior, basilar processes of the arytenoid cartilages (fig. 1)

The trachea is of considerable length, the rings are all cartilaginous, compressible, and elastic; they are 104 in number in the female, (I did not count them in the male specimen); sixty-four are above the tracheal opening, six correspond to the open portion of the tube, and there are thirty-four between this and the division into the right and left bronchi; in each of the latter are six semicircular cartilages, and three rudimental fragments; beyond these, the air-tubes abruptly become membranous and muscular, and no cartilages are continued into the lungs. The form of the tracheal tube is somewhat transversely elliptical, but is indented posteriorly through its whole extent; that is, each ring is curved posteriorly, so as to be convex towards the canal, and concave towards the spine; this general indentation is increased by the slightest pressure; the œsophagus is closely connected to it, and when distended, during the deglutition of any large substance, would appear to derive some accommodation from this structure. This posterior indentation is much increased in depth opposite the tracheal opening, and thus accounts for the corresponding vertical prominence internally already described; in this particular situation, the cartilages are also somewhat differently modified, as we shall notice presently. A yellowish elastic structure extends the whole length of this posterior depression on the trachea (fig. 5); this increases in strength inferiorly, and terminates at the division into the two bronchi; beneath this elastic ligament, opposite the tracheal opening, and a little above and below it, short, but strong, transverse muscular bands pass across the groove, and are inserted into the cartilages at either side (fig. 5); these fibres, by approximating this attachment, will tend to preserve, and even protrude the keel-like projection within, and thus enable it the better to support the approximated ends of the lateral cartilages in front; the longitudinal elastic ligamentary tissue admits of the extension of the trachea, and can restore it to its state of rest, on the subsidence of the extending force.

The tracheal cartilages present some difference in size and form in different portions of the tube; their diameter gradually, but slightly, increases from above downwards, and a very trifling swell is observable in the region of the opening. The upper and middle rings present flattened surfaces, and their edges, above and below, are connected by elastic membrane; those below the opening, though forming larger circles, are small, round, and weak, and have an oblique direction downwards and forwards; this portion of the tube is very elastic; the four or five last cartilages are deficient posteriorly, in a narrow, angular interstice, which is closed by lining membrane, weak muscular fibres, and elastic tissue; from the last ring no internal ridge or bar proceeds, but from the angle between the bronchi the mucous membrane rises mesially in a prominent falx-like fold, which is increased or diminished in depth and tension in proportion as the trachea and bronchial tubes are shortened and contracted, or elongated and distended; the bronchial cartilages are little more than semicircles, their extremities being somewhat thick and expanded; the two superior present, externally and posteriorly, small projecting processes, into which some fibres from the long cervical muscles are inserted; the posterior wall of each bronchus is musculo-membranous; although there is no inferior larynx, yet each bronchial opening,—which is of an oval form, the long axis from before backwards, or, during life, from above downwards,—is capable of great alteration in size, figure, and tension of its lateral boundaries; these changes can be effected by the varying degrees of inspiration and expiration, by extension and flexion of the neck, and also by the action of the tracheal muscles.

The six cartilages which are concerned in the tracheal opening, deserve particular notice; they are peculiarly elastic, and appear composed, each, of two symmetrical portions, one on either side; each lateral portion is crescentic (fig. 6); the anterior cornu, flat, thin, and very moveable, is enveloped in mucous membrane, and engaged in the edge of the tracheal opening; the posterior cornu meets the corresponding cornu from the other side, in the posterior indentation, and both project forwards from the posterior wall of the trachea, and present the appearance internally already noticed. These posterior cornua, from the right and left crescentic cartilages, are, in some, connected together by a narrow and thin cartilaginous structure; in others, by a narrow line of dense cellular tissue; so that we may regard this portion of the trachea as composed not of six cartilaginous rings, open in front, but of twelve crescentic cartilages, six on each side; their anterior cornua either bounding the sides of the opening when this is open, or in contact, when it is closed; their posterior cornua bent forwards, so as to form the keel-like prominence internally, which will come into contact with and support the anterior cornua when these are approximated and the orifice closed, and the tracheal canal thereby divided into two lateral tubes.

From an examination of the several parts concerned in this curious apparatus, and from observing the animal during life, I am led to infer that it possesses the voluntary power of not only expanding this air-bag, but also of retaining it in that state for an indefinite time, without any continued muscular exertion, and that it can either rapidly or slowly contract, or empty it, and perfectly close its communication with the trachea. It has been remarked that on some days it is not dilated; on others it is frequently expanded, and as frequently contracted, in a few moments, or retained in a distended state for a considerable time; and on some occasions it remains in that condition when the bird is at rest, or apparently asleep. When about to fill it, he raises and slightly extends the neck, and darts it a little forward; little or no muscular exertion is apparent, and the bag swells out, most probably by an expiratory effort, the glottis being previously closed, and the muscular wall of the sac being relaxed, so as to admit of more easy dilatation; perhaps, also, the posterior transverse muscular fibres of the trachea may, by increased contraction, tend to divaricate the anterior cornua of the cartilages, whilst the extension of the neck and elongation of the trachea have separated the cartilages above and below the opening; once the bag has been filled by this expiratory effort, the glottis being closed, it may be retained in that state, even should this opening become relaxed, provided the walls of the sac do not contract, and respiration may continue without the reservoir being affected, further than that the included air may be more or less changed by the admixture of fresh air from each inspiratory current: the closure of the glottis, and an expiratory effort, then appear to be the simple agencies whereby the distension of the sac is effected.

The sac can be emptied by the contraction of its muscular covering, the distending force having ceased, and the air may be expelled by expiration, or it may be drawn into the lungs by inspiration; the elasticity of the cartilages, and the compression of the surrounding parts, will then approximate the edges of the opening, which will be supported by the internal vertical projection on the back part of the tube, and thus the orifice will become so perfectly closed, that inspiration can have no effect in drawing within it the superincumbent soft parts.

When, from surveying this curious and elaborate structure, we turn our attention to its use, and endeavour to explain the design of this anomalous arrangement, we are at once met by the fact, that although this bird is in all respects so similar to the ostrich, and to the Indian cassowary, yet in it alone is this bag developed. Were a similar structure found in all the struthiones, we should have had little hesitation in connecting it—though, no doubt, erroneously—with some of the peculiar habits and endowments of this class generally; but such not being the case, we naturally ask, is there any peculiarity of climate, or any other circumstance in the locality in which this species is found, that will explain the necessity of it, or account for its presence, or is there any peculiar power or faculty possessed by this bird of which those allied to it are deprived? or are we to regard it as one of those examples of creative omnipotence which often displays itself in the variety of its works, without any other obvious result but the mere manifestation of that power?—a remark which appears strongly verified by many of the peculiarities of the living productions of Australia.

Dr. Knox considers that the emu has been furnished with this peculiar provision to preserve it amidst those dangers from sudden floods to which New Holland is particularly exposed. "The sandy plains of this country," he says, "are, during a great part of the year, inundated, and become then boundless marshes; and the plains generally are exposed to sudden inundations. The rivers, moreover, running westward from the great chain of mountains, terminate in vast muddy inland marshes; the emu, forced to seek his food amidst these fens, may, when obliged to have recourse to swimming (which must often be the case), fill the muscular bag of the trachea with air, and thus convert it into a swimming bladder. It may also assist the bird in escaping from his pursuers; but on this I do not mean to insist, as this organ is wanting in the galeated cassowary and in the ostrich, both equally remarkable for speed of foot." Dr. Knox further remarks, that when the bag has been distended by an expiratory effort, and the glottis retained in a closed state, the air may be alternately circulated between the lungs, air-cells, and tracheal bag, without the bird being necessitated to allow it to escape, in order again to perform the act of inspiration, and thus give it an additional advantage in running. This explanation appears extremely probable, and, no doubt, if the inspiratory efforts are thereby rendered less frequent, this creature may be enabled to sustain its running flight (which, in speed, is said to surpass the race-horse or the grey-hound) for a longer time, and with less fatigue, than the quadruped or man, who are incapable of keeping up long-continued rapid progression, not so much from debility in the muscular system generally, as from a failure in the inspiratory muscles, which, under such exciting and exhausting circumstances, are called on to exert additional force, in order to maintain the due quality of the blood, as well as to regulate the current of the circulation, and which exertions, when too long continued, are speedily followed by that overwhelming and well-known, though almost indescribable, sensation denominated fatigue. It appears to me, also, that this organization may still further minister to the respiratory function, by extending the surface of the mucous membrane on which the chemico-vital changes in the blood are effected. The lining membrane of this reservoir presents not only a very extensive surface, but it is also as highly organized as that lining the trachea and bronchial tubes, with which it is continuous; numerous capillaries and tortuous nerves branch throughout its texture, and several large veins course irregularly along its wall; it is, therefore, highly probable that the same changes which are effected in the blood through the parietes of the minute pulmonary capillaries, and through the thin coats of the large veins which traverse the air-cells in the bodies of birds generally, may all take place on the lining membrane of this cervical air-bag; and that this additional respiratory agency will be supplied at that very time when the function of respiration is required in the highest degree to maintain the muscular exertion and the nervous energy which the animal evinces during its rapid excursions. This adjunct to the respiratory apparatus may even be the more necessary to this animal as a compensation for the imperfectly-developed, or almost rudimental wings, which are not only of little or no avail in locomotion, but which, from the absence of those large air-cells and blood-vessels which exist in the wings of other birds, can here in no way contribute to the respiratory function.

That this peculiar organization is connected with the vocal powers of this bird, I conceive there can be very little question; it is with surprise I have read the remark of Meckel, that this bird has no voice. Those who have frequently visited these birds in the Zoological Gardens, must have noticed the different sounds they emit; in fact, they have two distinct voices, just as they possess distinct organs; the most ordinary is a harsh, disagreeable, hissing voice, not unlike that of the common goose; this is frequently heard, as the bird follows visitors round the enclosure in expectation of food; this voice I attribute to the structure, or organization of the superior glottis. The other, and more peculiar sound, is only occasionally emitted, probably because, while in a state of captivity, the ordinary excitements are not so frequently present; this resembles a low, hollow sound, not unlike that caused by gently striking a large drum, or moving an empty barrel; sometimes it is sharp, short, and sudden; at other times it is long and muffled, like the rolling of thunder, or of a smoothly-running distant carriage; sometimes it is soft, continued, and rather melodious; but at others it is disagreeably interrupted by harsh and rough grunting sounds. The animal only occasionally emits this voice; on visiting the Gardens, in hope of hearing it, I have been frequently disappointed; on other occasions, the birds have repeated it several times. The care-taker informs me that in his morning visits to open the aviary and feed the birds, they frequently make this extraordinary noise, and which he compares to the sound of thunder. The ear detects this sound as proceeding from the upper part of the sternum, that is, from the position of the sac, and, while making it, the animal extends and alters the curve of the neck, and fills this tracheal bag; there can be no doubt, therefore, that this voice is connected, in part, with the inferior glottis, or the two narrow bronchial openings, and, in part, if not essentially, with this peculiar appendix; there is nothing in the superior larynx, or in the inferior, alone, that can account for it, whereas the great capacity of this reservoir, its free communication with the trachea, immediately above the narrow bronchial aperture, its sudden distension, and as sudden contraction, or the alternate partial action and relaxation of the distending and compressing agencies, together with the free and elastic vibrating borders of the tracheal opening, the resisting wall behind, and the long and softly resonant tube, leading upwards, may, I think, satisfactorily account for these peculiar vocal phenomena. What may have been the design of imparting to this being this peculiar endowment, it would be as vain to speculate upon as to attempt to account for the infinite variety of voice that prevails throughout the animal world; that it is voluntary I have no doubt, and may be exercised either for sexual attraction or social union, or as indicative of nervous emotion, the result of anger, terror, or alarm.

This remarkable air-bag is not only peculiar to this animal, but there is nothing exactly analogous to it in any other member of the class Aves. In the trachea of several water birds there have been long observed peculiar swellings, or dilatations of the tube, in the structure of which the entire of a certain number of rings are engaged, but there is no distinct sac or bag. The laryngeal or tracheal sac in the chameleon, bears, as Dr. Knox has remarked, some remote analogy to it; the laryngeal sacs in certain of the quadrumana, and even the ventricles in the human larynx, and the sacculi laryngis, which lead from each upwards and forwards, may be regarded as rudimental conditions of the same structure.

Explanation of the Plate.

Fig. 1.—Superior glottis, tongue fimbriated.

Fig. 2.—Front view of the tracheal sac; a few feathers remain.

Fig. 3.—Sac laid open, and drawn over to the right side: *a*, opening in the trachea: *b*, sharp, thin extremities of the six rings: *c*, prominence on the back of the trachea: *d*, section of the cutaneous covering; *e*, of the muscular; *f*, the lining mucous membrane, extending over the front and outside of the rings, and then continued round the edges of the opening into the trachea.

Fig. 4.—Section of the trachea: *a*, *b*, *c*, as in last figure.

Fig. 5.—Posterior view of the sac: *a*, indentation on the trachea corresponding to the internal ridge in figs. 3 and 4; *b*, longitudinal elastic ligament, dissected off, and drawn to one side; *c*, transverse muscular fasciculi, attached to the convexities of the cartilages on either side, raised on a small twig.

Fig. 6.—Horizontal view of three tracheal cartilages: *a*, from the upper portion; *c*, from the lower; *b*, corresponds to the opening, shews two semicircular cartilages, their posterior cornua projecting forwards into the tube, and ending in round and slightly bifid points; *d d*, mucous membrane of the trachea and sac.

No. VIII.

ACCOUNT

OF THE

ROYAL IRISH ACADEMY,

FROM 1ST APRIL, 1845, TO 31ST MARCH, 1846.

THE CHARGE.

				£	s.	d.	£	s.	d.
To balance in favour of the Public on 31st March, 1845,				. . . . .			119	11	0
Parliamentary Grant for 1845 (paid 1st August, 1845),				300	0	0			
Quarterly Warrants from Treasury,				146	17	8			
Total from Treasury,							446	17	8
INTEREST ON STOCK:									
Half year's on £1643 19 6, 3¼ per Cents.				26	14	3			
Ditto, ,, 1117 1 10, 3 ,,				16	15	1			
Ditto, ,, 1643 19 6, 3¼ ,,				26	14	3			
Ditto, ,, 867 1 10, 3 ,,				13	0	2			
				83	3	9			
Less Commission for receiving Dividends,	£0	5	4						
Stamp for Power of Attorney for receiving Dividends,	1	0	0						
				1	5	4			
Total Interest on Stock,							81	18	5
AMOUNT OF STOCK SOLD:									
To £250 3 per Cent. Consols, sold at 98½,				246	5	0			
33 days' Interest,					13	6			
				246	18	6			
Power of Attorney for sale of Stock,	1	0	0						
Commission for sale of ditto,	0	6	3						
				1	6	3			
Total Amount of Stock sold,							245	12	3
RENT OF STABLE, due 1st November, 1845,				. . . . .			21	0	0
							914	19	4

	£	s.	d.	£	s.	d.
Brought forward, . . .				914	19	4
Publications sold:						
Transactions and Proceedings sold here, .	39	16	8			
Ditto, sold in London by Messrs. Boone,	50	18	2			
Ditto, sold in Dublin by Messrs. Hodges and Smith, ...	9	7	6	100	2	4
Total Amount of Books sold, .						
Outstanding Subscriptions for the Purchase of Messrs. Hodges and Smith's MSS., collected during the Year:						
T. F. Kelly, Esq., ...	1	1	0			
D. H. Kelly, Esq., ...	1	1	0			
J. T. Banks, M. D., ...	1	1	0			
John E. V. Vernon, Esq., ...	5	0	0			
Durham Dunlop, Esq., ...	1	1	0			
M. O. R. Dease, Esq., ...	1	0	0			
Sir P. Crampton, Bart., ...	2	0	0			
Sir P. N. Nugent, Bart., ...	2	0	0			
Rev. Dean Lyons, ...	1	0	0			
Halliday Bruce, Esq., ...	1	0	0			
Total Amount of Manuscript Fund,				16	4	0
Life Compositions:						
Daniel Connolly, LL. D., ...	21	0	0			
John Ball, Esq., ...	18	18	0			
Robert Forster, Esq., ...	21	0	0			
Walter Sweetman, Esq., ...	21	0	0			
James C. Sherrard, Esq., ...	21	0	0			
E. King Tennison, Esq., ...	21	0	0			
Total Life Compositions, . .				123	18	0
Entrance Fees:						
W. Hogan, Esq., ...	5	5	0			
J. A. Galbraith, Esq., ...	5	5	0			
J. F. Waller, Esq., ...	5	5	0			
James Jameson, Esq., ...	5	5	0			
N. P. O'Gorman, Esq.	5	5	0			
Lord Farnham, ...	5	5	0			
Lord Wallscourt, ...	5	5	0			
W. Le Fanu, Esq., ...	5	5	0			
William Henn, Esq., ...	5	5	0			
Digby Pilot Starkey, Esq., ...	5	5	0			
Charles Bournes, Esq., ...	5	5	0			
W. C. Dobbs, Esq., ...	5	5	0			
P. J. Blake, Esq., ...	5	5	0			
	68	5	0	1155	3	8

	£	s.	d.	£	s.	d.
Brought forward, . . .	68	5	0	1155	3	8
Wyndham Goold, Esq., ...	5	5	0			
C. C. King, Esq., ...	5	5	0			
Daniel Connolly, LL. D.	5	5	0			
J. S. Close, Esq., ...	5	5	0			
David Moore, Esq., ...	5	5	0			
James Claridge, Esq., ...	5	5	0			
Right Hon. D. R. Pigot, ...	5	5	0			
J. Talbot, Esq., ...	5	5	0			
Adolphus Cooke, Esq., ...	5	5	0			
James S. Eiffe, Esq., ...	5	5	0			
William Lloyd, Esq., ...	5	5	0			
Benjamin Wilme, Esq., ...	5	5	0			
T. Jolliffe Tuffnell, Esq., ...	5	5	0			
Charles W. Williams, Esq., ...	5	5	0			
E. K. Tenison, Esq., ...	5	5	0			
Mathew Baker, Esq., ...	5	5	0			
Rev. Classon Porter, ...	5	5	0			
Conyngham Ellis, Esq., ...	5	5	0			
Very Rev. Henry Cotton, Dean of Lismore,	5	5	0			
Rickard Deasy, Esq., ...	5	5	0			
Arthur R. Nugent, Esq., ...	5	5	0			
Rev. J. Connell, ...	5	5	0			
Robert Francks, Esq., ...	5	5	0			
Pierce Morton, Esq., ...	5	5	0			
Richard Cane, Esq., ...	5	5	0			
J. T. Evans, Esq.	5	5	0			
Robert C. Williams, Esq., ...	5	5	0			
H. W. Massy, Esq.	5	5	0			
R. R. Madden, Esq., ...	5	5	0			
Earl of Enniskillen, ...	5	5	0			
Thomas Butler, Esq., ...	5	5	0			
C. W. Levinge, Esq., ...	5	5	0			
Stephen O'Meagher, Esq., ...	5	5	0			
Total Entrance Fees, . . .				241	10	0
ANNUAL SUBSCRIPTIONS AND ARREARS:						
W. Hogan, Esq., ... 1845,	2	2	0			
Charles Hanlon, Esq., ... ,,	2	2	0			
Rev. R. Butler, ... ,,	2	2	0			
Charles Vignoles, Esq., ... ,,	2	2	0			
John Mollan, Esq., ... ,,	2	2	0			
Henry Close, Esq., ... ,,	2	2	0			
F. Churchill, M. D., ... ,,	2	2	0			
Rev. Francis Crawford, ... ,,	2	2	0			
Rev. Dr. West, ... ,,	2	2	0			
	18	18	0	1396	13	8

		£	s.	d.	£	s.	d.
Brought forward,		18	18	0	1396	13	8
John Hamilton, Esq.,	1845,	2	2	0			
William Murray, Esq.,	,,	2	2	0			
W. S. O'Brien, Esq., M.P.,	,,	2	2	0			
Edward Hutton, Esq.,	,,	2	2	0			
C. T. Webber, Esq.,	,,	2	2	0			
J. S. Cooper, Esq.,	,,	2	2	0			
Alexander Ferrier, Esq.,	,,	2	2	0			
William Longfield, Esq.,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
Jacob Owen, Esq.,	,,	2	2	0			
W. B. Wallace, Esq.,	,,	2	2	0			
Henry Watson, Esq.,	1842,	2	2	0			
Ditto,	1843,	2	2	0			
Ditto,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
M. O. R. Dease, Esq.,	,,	2	2	0			
J. G. Abeltshauser, Esq.,	,,	2	2	0			
William Mac Dougal, Esq.,	,,	2	2	0			
Dr. O'Grady,	,,	2	2	0			
Hon. J. King,	,,	2	2	0			
Durham Dunlop, Esq.,	,,	2	2	0			
William Edington, Esq.,	,,	2	2	0			
T. F. Kelly, Esq.,	,,	2	2	0			
William Hill, Esq.,	,,	2	2	0			
Sir William Betham,	,,	2	2	0			
Thomas Cather, Esq.,	,,	2	2	0			
J. Anster, LL.D.,	,,	2	2	0			
W. R. Wilde, Esq.,	,,	2	2	0			
J. Huband Smith, Esq.,	,,	2	2	0			
A. E. Gayer, LL.D.,	,,	2	2	0			
Dr. Osborne,	,,	2	2	0			
W. Barker, M.D.,	,,	2	2	0			
G. D. Latouche, Esq.,	,,	2	2	0			
Robert Tighe, Esq.,	,,	2	2	0			
Dr. Montgomery,	,,	2	2	0			
W. T. Mulvany,	,,	2	2	0			
Rev. James Wills,	,,	2	2	0			
Goddard Richard, Esq.,	,,	2	2	0			
W. E. Hudson, Esq.,	,,	2	2	0			
J. H. Jellet, Esq.,	,,	2	2	0			
W. T. Lloyd, Esq.,	,,	2	2	0			
A. Smith, M.D.	,,	2	2	0			
W. T. Mac Cullagh, Esq.,	,,	2	2	0			
R. C. Walker, Esq.,	,,	2	2	0			
James Apjohn, M.D.,	,,	2	2	0			
		113	8	0	1396	13	8

		£	s.	d.	£	s.	d.
Brought forward,		113	8	0	1396	13	8
George Wilkinson, Esq.,	1845,	2	2	0			
Sir Thomas Staples, Bart.,	,,	2	2	0			
Edward Bewley, M. D.	,,	2	2	0			
F. M. Jennings, Esq.,	,,	2	2	0			
Lord Farnham,	,,	2	2	0			
Most Rev. Archbishop of Dublin,	,,	2	2	0			
Lord Wallscourt,	,,	2	2	0			
F. W. Burton, Esq.,	,,	2	2	0			
Robert Law, Esq.,	,,	2	2	0			
Robert Ball, Esq.,	,,	2	2	0			
William Drennan, Esq.,	,,	2	2	0			
Gerald Fitzgibbon, Esq.,	,,	2	2	0			
J. Magee, Esq.,	,,	2	2	0			
R. L. Ogilby, Esq.,	,,	2	2	0			
Thomas Grubb, Esq.,	,,	2	2	0			
C. W. Hamilton, Esq.,	,,	2	2	0			
Dr. Toleken,	,,	2	2	0			
William Monsell, Esq.,	1844,	2	2	0			
Rev. W. Lee,	1845,	2	2	0			
John Ball, Esq.,	,,	2	2	0			
E. E. H. Orpen, Esq.,	,,	2	2	0			
Sir John Kingston James, Bart.,	1845,	2	2	0			
Thomas Oldham, Esq.,	,,	2	2	0			
C. G. Otway, Esq.,	1843,	2	2	0			
Ditto,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
H. F. C. Logan, D. D.,	,,	2	2	0			
Hon. F. Ponsonby,	,,	2	2	0			
Rev. James Reid,	,,	2	2	0			
William Blacker, Esq.,	,,	2	2	0			
Philip Reade, Esq.,	,,	2	2	0			
Dean of Ossory,	,,	2	2	0			
Sir Lucius O'Brien, Bart.,	,,	2	2	0			
A. B. Kane, Esq.,	,,	2	2	0			
Right Hon. Chief Baron,	,,	2	2	0			
Hon. Justice Crampton,	,,	2	2	0			
Edmund Davy, Esq.,	,,	2	2	0			
Sir P. Crampton, Bart.,	,,	2	2	0			
J. D'Alton, Esq.,	1844,	2	2	0			
Dr. Beauchamp,	1845,	2	2	0			
E. Cane, Esq.,	,,	2	2	0			
W. Grimshaw, M. D.,	,,	2	2	0			
John Finlay, LL. D.,	,,	2	2	0			
R. A. Wallace, Esq.,	,,	2	2	0			
A. C. Stirling, Esq.,	,,	2	2	0			
		207	18	0	1396	13	8

		£	s.	d.	£	s.	d.
Brought forward,		207	18	0	1396	13	8
T. E. Beatty, Esq.	1845,	2	2	0			
H. H. Joy, Esq.,	,,	2	2	0			
M. M'Master, Esq.,	,,	2	2	0			
C. Bolton, Esq.,	,,	2	2	0			
Sir Robert Kane, M.D.,	,,	2	2	0			
G. J. Allman, M. D.,	,,	2	2	0			
F. W. Conway, Esq.,	,,	2	2	0			
M. Longfield, Esq.,	,,	2	2	0			
Sir Edward Borough, Bart.,	,,	2	2	0			
W. Andrews, Esq.,	,,	2	2	0			
Acheson Lyle, Esq.,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
R. Cully, Esq.,	,,	2	2	0			
Sir Richard Morrison,	1843,	2	2	0			
Ditto,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
James Patten, M. D.,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
John Davidson, Jun., Esq ,	,,	2	2	0			
Samson Carter, Esq.,	,,	2	2	0			
Henry L. Lyndsay, Esq.,	,,	2	2	0			
William Roberts, Esq.,	,,	2	2	0			
G. A. Fraser, Esq.,	,,	2	2	0			
Rev. R. Chato,	,,	2	2	0			
G. Carr, Esq.,	,,	2	2	0			
Dean of Kildare,	,,	2	2	0			
J. S. Eiffe, Esq.,	,,	2	2	0			
W. Gregory, Esq.,	,,	2	2	0			
Ditto,	1846,	2	2	0			
E. J. Cooper, Esq ,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
M. Mac Master, Esq.,	1846,	2	2	0			
A. Jacob, M.D.,	1845,	2	2	0			
O. Sproule, Esq.,	,,	2	2	0			
J. Hart, M.D.,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
Thomas Stack, Esq.,	,,	2	2	0			
G. A. Kennedy, Esq.,	,,	2	2	0			
Abraham Abell, Esq.,	1844,	2	2	0			
Ditto,	1845,	2	2	0			
B. J. Chapman, Esq.,	,,	2	2	0			
Dr. Reid,	,,	2	2	0			
Wyndham Goold, Esq.,	,,	2	2	0			
Charles Vignoles, Esq.,	,,	2	2	0			
R. Graves, M.D.,	,,	2	2	0			
		302	8	0	1396	13	8

		£	s.	d.	£	s.	d.
Brought forward, . . .		302	8	0	1396	13	8
F. Churchill, M. D., ...	1846,	2	2	0			
D. Moore, Esq., ...	1845,	2	2	0			
R. A. Wallace, Esq., ...	,,	2	2	0			
W. B. Wallace, Esq., ...	,,	2	2	0			
P. D. Hardy, Esq., ...	,,	2	2	0			
G. A. Hamilton, Esq., M. P., ...	,,	2	2	0			
R. W. Smith, Esq., ...	,,	2	2	0			
Sir Matthew Barrington, Bart., . . .	,,	2	2	0			
Very Rev. Dean Disney, ...	1843,	2	2	0			
Ditto, ...	1844,	2	2	0			
Ditto, ...	1845,	2	2	0			
James Pim, Jun., Esq., ...	,,	2	2	0			
William Stokes, M. D., ...	,,	2	2	0			
George Cash, Esq., ...	1845,	2	2	0			
E. J. Clarke, Esq., ...	,,	2	2	0			
J. Burrowes, Esq., ...	1846,	2	2	0			
Captain H. James, ...	,,	2	2	0			
Sir M. Chapman, Bart., ...	,,	2	2	0			
Sir Robert Kane, M. D., ...	,,	2	2	0			
C. T. Webber, Esq., ...	,,	2	2	0			
Rev. R. Butler, ...	,,	2	2	0			
James O'Grady, M. D., ...	1844,	2	2	0			
Ditto, ...	1845,	2	2	0			
William Hill, Esq., ...	1846,	2	2	0			
J. Nelson, Esq., ...	1845,	2	2	0			
Total Amount of Annual Subscriptions,					354	18	0
THE TOTAL CHARGE, . . .		. . . .	.	. £	1751	11	8

THE DISCHARGE.

ANTIQUITIES PURCHASED.	£	s.	d.	£	s.	d.
Browne, small bronze sword, 3rd Jan. 1846,	2	0	0			
Campbell, D., for a crystal ball, 2nd Apr. 1845, ...	3	0	0			
Glennon, Richard, porpyhry hammer, 21st January, 1846, ...	0	10	0			
Kearney, Patrick, for gold bracelet, . . .	50	11	0			
Kerrigan, William, for silver pin and bracelet, 19th March, 1845, ...	6	0	0			
M'Cullagh, James, plated antique ornament, 16th December, 1844, ...	2	0	0			
Ditto, brass kettle and spear head, 13th Feb., 1846, ...	3	10	0			
Underwood, James, old seal, 6th May, 1844,	0	10	0			
Wallace, Stephen, bronze antique, 21st Jan., 1845, ...	0	15	0			
Wynne, H., antique silver ring, 18th Nov., 1845, ...	0	3	0			
Total amount of Antiquities, . . .				68	19	0
BOOKS, PRINTING, AND STATIONERY.						
Barthes and Co., duty and carriage on books, 19th January, 1846, ...	2	16	0			
Bennett, Francis, cleaning copperplates, 17th February, 1845, ...	1	4	0			
Camden Society, subscription for the year 1843–44, ...	2	0	0			
Curry, Eugene, deciphering wooden tablets, 1st July, 1845, ...	2	16	0			
D'Alton, John, for Annals of Boyle and History of Ireland, 24th May, 1845, ...	1	0	0			
Gill, M. H., printing, 23rd December, 1844,	69	5	3			
Hanlon, G. A., wood engravings, 27th Jan. 1845, ...	2	5	0			
Hodges and Smith, Transactions, vol. xx. Petrie's Round Towers, 6th August, 1845,	622	11	0			
Jones, J. F., Books, &c., 4th August, 1845, .	4	4	2			
Ditto, for Memoirs of Dr. O'Connor, 11th March, 1846, ...	3	3	0			
	711	4	5			
				68	19	0

	£	s.	d.	£	s.	d.
Brought forward, . . .	711	4	5	68	19	0
M'Dowell, William, cleaning copperplates to January 26th, 1846, ...	17	15	10			
Marshall, A., Post Office Directory, 13th December, 1845, ...	0	4	0			
Mullen, George, bookbinding, 13th October, to 7th February, 1845, ...	34	19	8			
Perry, J. H., and Co., ink, ...		17	0			
O'Shaughnessy, J. J., tinted prints, 7th Aug. 1845, ...	3	7	0			
Plunkett, James, Drawings, 21st June, 1845, to 10th January, 1846, ...	0	18	0			
Ponsonby, E., ink, &c., 29th October, 1845,	0	7	0			
Sharpe, Charles, Transactions, 31st January, 1846, ...	1	3	2			
Tallon, John, Jun., Stationery, 19th May, 1845, ...	1	13	8			
Underwood, J. H., Burton's History of Kilmainham, 12th December, 1845, . . .	0	5	0			
Wiseheart, red ink and paper, 3rd May, 1845,	0	1	1			
Webb, Thomas, cartridge paper, 26th May, 1845, ...	0	1	9			
Total Books, Printing, and Stationery,				772	17	7
Irish Manuscripts purchased.						
Baggot, B. W., Irish MS., 6th Dec. 1845, .	6	0	0			
Total of Irish MS., ...				6	0	0
Coals, Candles, &c.						
Allen, William, oil, &c., 20th March, 1846,	0	12	1			
Alliance Gas Co., gas to 31st Dec. 1845, . .	13	9	9			
Ditto, for coke and carriage, Oct. 13th, 1845, ...	1	9	6			
Edmundson, J., and Co., oil, 13th June, 1846,	1	17	6			
Hoey, C., 2 tons coal, and carriage, 31st Jan. 1846, ...	1	13	0			
Kiernan, James, load bogwood, 22nd Nov. 1845, ...	1	0	0			
Piele, Thomas, 1 ton coal, and carriage, 3rd July, 1845, ...	0	16	0			
Wilson, T. P., 15 tons coal, and carriage, .	18	0	0			
Total Amount Coals, &c., ...				38	17	10
				886	14	5

	£	s.	d.	£	s.	d.
Brought forward, . . .	. . . . .			886	14	5
CONTINGENCIES.						
Carriage of parcels from Athlone, 3rd May, 1845, ...	0	1	0			
Ditto, from Liverpool, 16th May, 1845, ...	0	4	10			
W. Hamilton, carriage of parcels, 29th April, 1845, ...	0	2	0			
Steam Co., freight of a case, 30th May, 1845,	0	12	0			
Freight of books from London, 8th August, 1845, ...	0	5	6			
Fannin and Co., freight of a parcel, 4th Dec. 1845, ...	0	5	0			
Freight of parcels from London, 12th March, 1845, ...	1	2	5			
Clifford and Co., gum, April 4th, 1845, . .	0	2	6			
Ditto, varnish, ditto, . . .	0	8	1			
Clibborn, E., carriage of antiquities, . . .	0	10	6			
Boyle and Co., collecting Boone's draft in London, ...	0	1	3			
Maguire for twine, 9th May, ...	0	4	0			
Rooke, C., for 98 *3d.* stamps, ...	1	4	6			
Postage and postage stamps, ...	2	17	9			
Maguire for twine, ...	0	1	9			
Clibborn, E., sundries for cleaning house, due 16th January, 1846, ...	10	0	0			
Total Contingencies, ...				18	3	1
REPAIRS OF HOUSE, &c.						
Barrington, F., repairing stove, 3rd February, 1846, ...	2	17	8			
Brown, John, cleaning windows, 28th June, 1846, ...	1	10	0			
Ditto, painting ditto, 21st July, 1845, .	3	0	0			
Ditto, glazing ditto, 4th December, 1845,	0	13	9			
Ditto, varnishing the hall-door, 12th Dec., 1845, ...	0	12	0			
Ditto, painting lantern on roof of library, 24th December, 1845, ...	2	9	6			
Ditto, sundry painting, 18th Feb., 1845,	7	14	0			
Ditto, cleaning windows, 21st Jan., 1846,	1	10	0			
Murphy, J., sweeping chimneys, 4th March, 1845, ...	1	5	6			
	21	12	5			
				904	17	6

	£	s.	d.	£	s.	d.
Brought forward, . . .	21	12	5	904	17	6
Murphy, J., sweeping chimneys, 18th Oct. 1845, ...	1	7	6			
Malone, John, cleaning ashpit, 27th Jan. 1846,	0	4	0			
Rounds, Brothers, painting museum, &c., .	13	15	6			
Walsh, J., for Academy's proportion of cost of pump in yard, ...	0	10	0			
Total Repairs of House, &c., . . .				37	9	5
Furniture Repairs, &c.						
Boylan, P., satin paper, May 22nd, 1845, .	0	4	0			
Casey, P., repairing bells, &c., Novem. 15th, 1845, ...	2	5	6			
Duffy, John, glass case, ... £14 0s. 0d.						
Cash advanced for materials, as in last audit, ... £6 0s. 0d.	8	0	0			
Edmundson and Co., fitting up gas, 14th Aug. 1845, ...	85	8	3			
Ditto, candlesticks, &c., . . .	3	5	8			
Malone, P., shaking carpets, 22nd Oct. 1845,	0	3	6			
Perry, James and Co., coal scuttle, 14th June, 1845, ...	3	12	0			
Pim, Brothers, and Co., velvet, &c., 8th Nov. 1845, ...	1	18	11			
Porter, D., earthenware, 12th April, 1845, .	0	2	0			
Ditto, ditto, 13th April, 1845, .	0	7	9			
Sibthorpe and Son, glazing cases, &c., 4th August, 1845, ...	11	4	9			
Sparks, R. W., table cover, &c., 14th April, 1845, ...	1	18	7			
Total Furniture Repairs, &c., . . .				118	10	11
Rent, Taxes, and Insurance.						
Symes, A., half year's rent of house, ending 1st August, 1845, ...	52	4	6			
Ditto, ditto, 1st Feb. 1846,	52	4	6			
Taxes, ministers' money, one year, 15th Oct. 1845, ...	2	15	5			
Ditto, pipe water, one year, ending 24th June, 1845, ...	1	10	0			
Ditto, for watering streets, one year, . . .	0	9	2			
	109	3	7	1060	17	10

	£	s.	d.	£	s.	d.
Brought forward, . . .	109	3	7	1060	17	10
Boyle, Low, Pim, and Co., proportion of poor rate on stable for 1843, 1844, 1845, . . .	0	13	1			
Globe office, one year's insurance, ending 26th December, 1845, ...	5	13	6			
National office, one year's insurance, ending 26th December, 1845, ...	5	13	6			
Ditto, ditto, additional insurance, ditto,	4	2	6			
Total Amount of Rent, Taxes, and Insurance,				125	6	2
Salaries, Servants' Wages, &c.						
Clibborn, Edward, half year's salary, due 16th June, 1845, ...	60	0	0			
Ditto, ditto, due 16th December, 1845, ...	75	0	0			
Ditto, quarter year's ditto, due 16th March, 1846, ...	37	10	0			
Ball, Robert, Esq., Treasurer, one year, due 16th March, 1846, ...	21	0	0			
Kane, Sir Robert, Secretary to Council, due 16th March, 1846, ...	21	0	0			
Mac Cullagh, James, LL.D., Secretary to the Academy, due 16th March, 1846, . . .	21	0	0			
Drummond, Rev. W. H., D. D., Librarian, due 16th March, 1846, ...	21	0	0			
Curry, A., attending meetings, &c., 23rd June, 1845, ...	1	2	6			
Ditto, from 14th July, 1844, to 16th March, 1845, ...	1	17	6			
Halahan, G., medicine for porter, 22nd Dec. 1845, ...	0	5	8			
Hamilton, William, porter, wages, 16th March to 16th June, 13 weeks, ...	8	11	2			
Ditto, 16th June to 27th December, 28 weeks, ...	18	8	8			
Ditto, allowance to himself and wife at Christmas, ...	2	2	0			
Ditto, wages, from 27th Dec. to 16th March, 1846, 11 weeks, . .	7	4	10			
Kenny, Thomas, attending as hall porter, during the porter's sickness, two weeks, .	1	5	0			
Kane, B., and daughter, attending meetings,	2	11	6			
Magill, John, messenger, 20 weeks, $1\frac{1}{2}$ days, @ 15*s.*, ...	15	3	9			
	315	2	7			
				1186	4	0

	£	s.	d.	£	s.	d.
Brought forward, . . .	315	2	7	1186	4	0
Sloane, H. C., attending meetings, 17th April to 26th May, 1845, ...	1	2	6			
Plunkett, J., cleaning and arranging museum,	2	12	0			
Smith, John, delivering notes, &c., 19th April to 4th August, 1845, ...	0	13	6			
Todhunter, Isaac, making up accounts, &c., 22nd March, 1845, to 16th March, 1846, 51 weeks @ 20*s*., ...	51	0	0			
Total Salaries, Servants' Wages, &c.,				370	10	7
TOTAL DISCHARGE, ...	. . . . . £			1556	14	7
Balance in favour of the Public, . .	. . . .			194	17	1
				1751	11	8

STATE OF THE BALANCE.

	£	s.	d.
In Bank of Ireland, ...	105	19	9
In Treasurer's hands, as per Account, ...	88	17	4
	£194	17	1

The Treasurer reports, that there is to the credit of the Academy in the Bank of Ireland, £867 1*s*. 10*d*. in Three per Cent. Consols, and £1643 19*s*. 6*d*. in Three and a quarter per Cent. Government Stock, the latter known as the Cunningham Fund.

(Signed,) ROBERT BALL,
Treasurer.

31*st March*, 1846.

No. IX.

METEOROLOGICAL JOURNAL,

COMMENCING

1st JANUARY, 1846, AND ENDING 31st DECEMBER, 1846,

BY

GEORGE YEATES.

The instruments employed, and the general circumstances of the mode of observing, have been described in the preliminary observations to the Tables of the year 1843, in the 2nd volume of the Proceedings of the Academy, Appendix V.

	JANUARY.				
	Thermom. Max.	Thermom. Min.	Barom.	Rain.	Wind.
1	51°	36°	29.900	.200	W.
2	41	30	30.280	. . .	W.
3	49	31	30.160	.060	W.
4	54	35	29.880	.180	S. W.
5	41	33	30.100	.110	W.
6	54	39	29.890	. . .	W.
7	51	45	30.080	. . .	W.
8	52	47	30.390	. . .	S. W.
9	53	29	30.490	.006	S. W.
10	47	32	30.230	. . .	S. W.
11	52	33	30.150	.008	S.
12	47	42	29.790	.006	S. by E.
13	45	40	29.380	.030	S. E.
14	48	44	29.260	.370	S. E.
15	43	39	29.600	. . .	W.
16	41	36	29.600	. . .	S.
17	48	44	29.520	.080	S. W.
18	49	30	29.550	.018	S. by E.
19	47	36	28.780	.240	E.
20	51	39	29.090	.500	W.
21	47	38	29.140	.015	E.
22	51	43	28.700	.300	W.
23	51	41	29.000	.030	S. E.
24	57	41	29.400	.040	W.
25	53	45	28.990	.230	S. W.
26	55	47	28.890	.020	S. W.
27	52	45	29.300	.030	S. W.
28	51	41	29.150	.160	S. W.
29	50	40	29.290	.200	W. by N.
30	51	42	29.670	.002	S. W.
31	56	49	29.760	.008	S. W.

	FEBRUARY.				
	Thermom. Max.	Thermom. Min.	Barom.	Rain.	Wind.
1	55°	40°	29.880	.100	W. by S.
2	48	35	29.780	.380	W.
3	52	39	29.600	. . .	S. W.
4	54	36	29.900	.050	W.
5	46	39	29.800	.018	W.
6	44	36	29.870	. . .	W.
7	52	42	29.790	. . .	W.
8	48	35	30.120	. . .	N. W.
9	43	29	30.340	. . .	N. W.
10	41	32	30.400	. . .	N. E.
11	41	28	30.240	. . .	W.
12	44	36	30.200	. . .	W.
13	47	39	30.200	. . .	W. by N.
14	47	39	30.030	. . .	W.
15	47	37	30.300	. . .	W.
16	46	38	30.320	. . .	W. by N.
17	47	36	30.200	. . .	W. by N.
18	47	37	30.020	. . .	N. W.
19	46	40	29.970	. . .	W.
20	47	39	29.990	. . .	E.
21	53	43	29.860	. . .	S. W.
22	58	47	29.470	.060	S. W.
23	56	49	29.570	.008	S. W.
24	59	48	29.130	.200	S. E.
25	56	47	29.420	.045	W. by S.
26	54	42	29.480	.012	S. E.
27	57	46	29.420	.040	S.
28	57	47	29.540	.054	S. W.

	MARCH.				
	Thermom. Max.	Thermom. Min.	Barom.	Rain.	Wind.
1	57°	44°	29.700	. . .	S. E.
2	56	44	29.640	.040	S. S. E.
3	52	44	29.360	.460	S. W.
4	51	44	28.990	.220	W.
5	51	39	29.970	. . .	W. by S.
6	50	36	29.520	.050	W. by S.
7	49	36	29.680	.090	W. by N.
8	51	35	30.020	. . .	N. W.
9	48	39	30.220	.058	N. W.
10	53	34	30.320	. . .	N. W.
11	53	45	30.370	.145	N. E.
12	51	36	30.530	. . .	W. by S.
13	54	45	30.290	.020	W. by S.
14	52	46	29.980	. . .	W. by N.
15	54	43	29.890	.405	W.
16	54	41	29.250	.200	S. W.
17	47	33	29.470	.100	N. W.
18	43	30	29.620	. . .	N. W.
19	39	25	25.570	.020	S.
20	37	26	29.680	. . .	N. E.
21	47	26	29.090	.105	S. W.
22	47	35	28.930	.245	W. by S.
23	47	36	28.930	.008	W. by S.
24	50	37	29.190	.010	W. by S.
25	52	37	29.190	.100	S. W.
26	48	35	29.590	.180	N. by W.
27	50	36	29.780	.100	W.
28	51	35	29.570	.002	N. W.
29	47	30	30.020	.020	N. E.
30	48	29	29.990	.002	S. E.
31	49	40	29.570	.080	E. by S.

APRIL.						MAY.						JUNE.					
	Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.		Thermom.		Barom.	Rain.	Wind.
	Max.	Min.					Max.	Min.					Max.	Min.			
1	51°	41°	29.200	.460	W.	1	55°	44°	30.260	.260	W.	1	69°	48°	30.150	. . .	E.
2	55	39	29.030	.200	E. by S.	2	64	52	30.230	. . .	W.	2	79	52	30.208	. . .	E.
3	52	38	29.150	.460	S. W.	3	62	51	30.120	. . .	E.	3	84	56	30.264	. . .	E.
4	48	37	29.420	.200	E. by S.	4	62	50	29.640	.025	S. by E.	4	83	54	30.160	. . .	E.
5	47	36	29.000	.500	E. by S.	5	62	47	29.528	.014	S. E.	5	84	59	30.140	. . .	S. E.
6	41	35	29.240	1.360	N. E.	6	64	45	29.480	.300	S. W.	6	85	56	30.122	. . .	S. E.
7	47	31	29.370	. . .	E. by N.	7	64	41	29.650	.100	W. by S.	7	84	53	30.033	. . .	S. E.
8	49	33	29.380	. . .	W.	8	58	42	29.930	. . .	W. by S.	8	82	55	29.800	. . .	W.
9	51	34	29.700	.004	W.	9	62	44	29.800	. . .	W. by N.	9	77	60	29.732	. . .	S. E.
10	53	32	29.780	.006	W.	10	63	43	29.650	.060	W. by S.	10	76	59	29.831	.040	S. W.
11	59	42	29.240	.080	S. E.	11	60	44	29.864	.035	W. by S.	11	76	54	30.141	. . .	W.
12	52	46	29.000	.120	S. E.	12	62	47	29.722	.130	S.	12	76	54	30.295	. . .	W.
13	60	47	29.260	.160	S. W.	13	57	46	29.858	.140	E.	13	78	56	30.230	. . .	W.
14	60	45	29.660	.270	S. E.	14	58	45	30.074	. . .	N. E.	14	77	57	30.232	. . .	E.
15	59	42	29.760	.008	W.	15	64	38	29.950	. . .	S.	15	84	57	30.351	. . .	E.
16	57	47	29.990	. . .	E. by N.	16	68	40	29.600	. . .	N. E.	16	75	56	30.423	. . .	S. E.
17	60	42	29.980	.210	N. W.	17	64	48	29.120	. . .	E.	17	78	49	30.370	. . .	S. E.
18	55	38	30.150	.060	W. by S.	18	51	29	28.900	.004	N. E.	18	80	54	30.282	. . .	S. E.
19	56	37	30.260	.020	N. W.	19	66	38	29.300	. . .	S. W.	19	83	51	30.100	. . .	W.
20	53	33	30.200	. . .	N. E.	20	66	43	29.070	.065	S. W.	20	85	60	30.200	. . .	N. E.
21	55	40	29.900	.420	E.	21	63	46	29.760	.100	N. E.	21	73	59	30.151	.022	S. by E.
22	47	42	29.850	.500	E.	22	66	50	30.092	.099	S. W.	22	76	58	29.921	. . .	S. W.
23	50	42	29.980	.002	E.	23	57	51	30.230	.220	N. E.	23	74	54	29.720	.160	W. by S.
24	53	41	29.880	.020	E. by N.	24	65	54	30.240	. . .	N. W.	24	69	49	29.520	.100	W.
25	53	35	29.960	. . .	S.	25	66	54	30.286	. . .	W. by N.	25	76	40	29.500	.550	W.
26	56	40	29.940	. . .	N. E.	26	66	50	30.250	. . .	N. W.	26	69	51	29.460	.060	S. W.
27	48	32	30.000	.018	W. by N.	27	65	49	30.180	. . .	S. E.	27	68	51	29.500	.002	W. by S.
28	51	44	29.960	.004	N. E.	28	66	44	30.230	. . .	S. E.	28	71	51	29.650	.240	S. E.
29	51	31	30.230	. . .	W. by N.	29	66	41	30.390	. . .	S. W.	29	67	51	29.490	.090	S. W.
30	57	42	30.280	. . .	W.	30	68	52	30.340	. . .	E.	30	67	52	29.720	.210	S. W.
						31	74	48	30.264	. . .	E.						

JULY.	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	68°	53°	29.800	.025	S. W.
2	68	57	29.960	.080	S. W.
3	73	59	30.060	.004	S. W.
4	75	54	30.070	. . .	E. by S.
5	81	54	29.550	.010	N. E.
6	72	52	29.390	.120	W.
7	59	50	29.830	. . .	N. W.
8	62	53	29.650	.140	S. E.
9	71	55	29.646	.060	S. E.
10	68	57	29.953	.004	S. E.
11	68	52	30.176	. . .	S. W.
12	68	53	30.143	. . .	S. E.
13	73	55	30.042	. . .	S. E.
14	79	58	29.110	. . .	W. by N.
15	72	55	29.862	. . .	S. W.
16	75	55	29.620	.100	S. W.
17	72	50	29.430	.220	W.
18	65	51	29.083	.200	S. W.
19	68	47	29.500	.080	W.
20	68	49	29.782	.080	W.
21	70	55	29.700	.060	W.
22	69	53	29.860	.045	S. W.
23	66	59	29.550	.010	S. W.
24	72	52	29.650	.065	S. W.
25	70	50	29.853	. . .	W.
26	72	49	30.100	.040	W.
27	72	60	30.000	.120	S. W.
28	78	63	30.070	. . .	S. W.
29	78	55	30.100	.160	S. E.
30	63	52	30.082	.640	N. E.
31	63	57	30.100	.620	E.

AUGUST.	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	73°	56°	29.857	.040	N. E.
2	72	66	29.850	.020	N. E.
3	76	62	29.850	.260	S. E.
4	77	56	29.900	. . .	W.
5	74	60	29.930	. . .	N. E.
6	74	52	30.020	. . .	N. E.
7	74	59	29.788	. . .	E.
8	68	61	29.650	.210	S. E.
9	71	56	29.840	.260	N. W.
10	71	52	29.937	. . .	S. W.
11	68	54	29.950	.040	W.
12	70	51	30.112	. . .	W.
13	72	51	29.600	.460	N. W.
14	64	44	29.900	. . .	S. E.
15	69	55	29.600	1.050	W.
16	68	54	29.800	.045	S. W.
17	70	51	29.758	.640	S. W.
18	63	51	29.308	.140	N. E.
19	62	50	29.728	.210	N. W.
20	60	55	29.726	.010	N. W.
21	64	59	29.880	. . .	S. W.
22	64	54	30.100	. . .	N. W.
23	70	55	30.229	.140	W. by N.
24	68	47	30.252	. . .	E. by N.
25	66	50	30.314	. . .	E.
26	66	43	30.230	. . .	E.
27	67	42	30.140	. . .	W.
28	69	55	30.146	.440	S. E.
29	66	49	30.100	. . .	W.
30	69	51	30.120	. . .	S. W.
31	71	53	30.190	.140	S. W.

SEPTEMBER.	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	66°	42°	30.392	. . .	W. by N.
2	65	49	30.372	. . .	N. W.
3	71	48	30.352	. . .	S. W.
4	70	54	30.352	. . .	W.
5	72	58	30.182	. . .	S. E.
6	71	48	29.970	. . .	W.
7	71	51	29.928	. . .	E.
8	70	56	29.720	.100	N. W.
9	69	52	30.032	. . .	W. by N.
10	65	44	30.432	.006	W. by N.
11	65	44	30.460	. . .	W. by N.
12	67	54	30.500	. . .	E.
13	70	55	30.500	. . .	E.
14	66	47	30.343	.004	E.
15	70	47	30.311	. . .	E.
16	69	49	30.250	. . .	E.
17	70	47	30.074	.006	E.
18	68	52	29.920	. . .	E.
19	68	54	29.763	. . .	E.
20	69	56	29.500	.030	E.
21	67	57	29.690	. . .	E.
22	64	57	29.700	. . .	E. by S.
23	62	54	29.310	.420	E. S. E.
24	65	54	29.400	.940	E. S. E.
25	64	54	29.800	.008	N. W.
26	65	57	29.400	.020	S. W.
27	67	47	29.630	.180	N. W.
28	62	45	29.431	.820	N. W.
29	51	45	29.431	.070	N. W.
30	54	42	29.760	.020	W.

OCTOBER.					
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	61°	51°	29.910	. . .	W.
2	64	40	29.760	. . .	W.
3	63	41	29.741	. . .	W.
4	59	44	29.472	. . .	S. by E.
5	60	52	29.350	.400	N. E.
6	60	47	29.200	.120	W. by N.
7	59	48	29.100	. . .	S. W.
8	57	48	29.373	.170	S. W.
9	58	47	29.250	.440	S. W.
10	64	54	29.250	.040	S. W.
11	60	49	29.474	.100	E. by N.
12	60	46	29.600	.008	W. by S.
13	57	40	30.182	.040	N. by E.
14	53	45	29.100	.220	W.
15	53	44	28.900	.300	N. by W.
16	55	43	29.212	.022	N. by W.
17	54	41	29.422	.140	W.
18	56	46	29.630	.040	W. N. W.
19	58	46	29.620	.960	S. W.
20	56	45	29.500	.080	W N. W.
21	51	42	28.700	.120	W.
22	54	41	29.300	.360	N. W.
23	50	37	29.820	.030	N. W.
24	49	41	29.500	.040	W. by S.
25	54	37	30.250	.006	N. E.
26	51	33	29.872	.126	W. by S.
27	50	41	30.330	.015	W. by N.
28	52	41	30.150	. . .	S. W.
29	53	44	30.182	.120	N. W.
30	54	40	30.282	.040	N. W.
31	53	42	30.124	. . .	S. W.

NOVEMBER.					
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	55°	50°	29.892	.032	S. W.
2	56	51	29.784	.030	S. E.
3	57	34	29.900	.070	S. E.
4	54	46	29.860	.200	S. E.
5	58	43	30.086	. . .	S. by E.
6	56	53	30.050	.080	S. by E.
7	57	50	30.231	. . .	W. by N.
8	56	51	30.422	. . .	S. E.
9	51	30	30.462	. . .	S. E.
10	50	35	30.520	. . .	S. by E.
11	48	34	30.500	. . .	N. W.
12	50	38	30.500	. . .	E. by S.
13	49	47	30.494	. . .	E. by S.
14	51	44	30.390	. . .	. S. E.
15	48	46	30.172	. . .	S. E.
16	48	45	29.950	. . .	E. S. E.
17	54	47	29.648	.220	S. W.
18	55	47	29.650	.060	S. E.
19	56	44	29.570	.120	S. W.
20	53	46	28.600	.240	S. S. E.
21	49	42	29.410	.070	W. by S.
22	51	36	29.360	.070	W. S. W.
23	49	39	29.626	.080	W.
24	58	49	29.516	.018	W.
25	55	45	29.464	.008	W.
26	48	43	29.250	.050	W. by N.
27	46	40	29.500	.250	E. by N.
28	47	29	29.600	. . .	W.
29	41	25	29.932	.050	N.
30	36	27	30.084	. . .	N. E.

DECEMBER.					
	Thermom.		Barom.	Rain.	Wind.
	Max.	Min.			
1	36°	24°	29.760	.100	S. W.
2	39	29	29.496	. . .	N. W.
3	34	25	29.714	. . .	W.
4	36	27	30.022	. . .	N. W.
5	41	31	29.912	.006	W.
6	43	36	29.996	. . .	N. W.
7	41	32	30.220	. . .	N. W.
8	44	40	30.400	. . .	N. W.
9	47	43	30.332	. . .	N. W.
10	45	42	29.923	.010	N. N. W.
11	44	26	29.900	. . .	N. N. W.
12	31	24	29.900	.015	N. N. W.
13	33	22	29.960	. . .	N. W.
14	30	24	29.700	. . .	W. by N.
15	31	24	29.642	. . .	N.
16	34	25	29.910	. . .	W. by N.
17	37	32	29.826	.140	W. by N.
18	39	25	30.112	. . .	W. by S.
19	47	36	29.826	.008	W. by S.
20	51	41	29.650	. . .	W. S. W.
21	49	43	29.078	.010	W. by N.
22	45	33	29.172	.010	W. by N.
23	41	34	29.000	.060	E. by N.
24	37	30	29.260	.140	E. by S.
25	37	25	29.900	.140	N. by W.
26	32	23	30.350	. . .	N. N. W.
27	36	16	30.350	. . .	N. N. W.
28	43	34	30.390	. . .	W. S. W.
29	46	40	30.400	.020	W. S. W.
30	48	42	30.440	.050	S. W.
31	48	44	30.500	.004	S. W.

	RAIN.	MEAN TEMP.		RAIN.	MEAN TEMP.
January	2.843	44.4	July	2.883	61.8
February . . .	967	45.5	August	4.105	61.0
March	2.660	43.5	September . . .	2.624	57.9
April	5.082	45.8	October	3.937	53.1
May	1.552	55.0	November . . .	1.648	56.3
June	1.474	65.5	December . . .	.713	35.6
				30.488	

No. X.

ABSTRACT

OF

THE ACCOUNTS OF THE ACADEMY,

FOR

THE YEAR ENDING 31ST OF MARCH, 1847.

Abstract of the Accounts of the Royal Irish Academy,

FOR THE YEAR ENDING 31st MARCH, 1847.

THE CHARGE.

	£	s.	d.	£	s.	d.
To balance in favour of the Academy, April 1, 1846				194	17	1
Parliamentary Grant	300	0	0			
Treasury warrant for Rent and Taxes	146	17	8			
				446	17	8
Transactions sold				26	1	0
Interest on Stock				79	8	8
One year's rent of Stable				20	9	1
Annual Subscriptions and Arrears	357	0	0			
Entrance Fees	136	10	0			
Life Compositions	99	15	0			
				593	5	0
Donations for purchase of "Domnach Airgid"				212	10	0
Total				£1573	8	6

THE DISCHARGE.

	£	s.	d.	£	s.	d.
Antiquities purchased				22	10	8
Books purchased	165	18	8			
Printing Transactions and Papers, &c.	340	5	11			
Binding	78	3	6			
Stationery	7	6	10			
				591	14	11
Coals, Gas, &c.				51	18	9
Furniture and repairs, &c.				13	4	4
Repairs of House				50	3	10
Rent, Taxes, and Insurance				124	13	1
Salaries, Wages, &c.				366	7	2
Contingencies				50	3	7
Paid on account of the "Domnach Airgid"				200	0	0
Total Discharge				1470	16	4
Balance in favour of the Academy				102	12	2
Total				£1573	8	6

	£	s.	d.	
In $3\frac{1}{4}$ per Cent. Stock	£1643	19	6	Conyngham Fund.
In 3 per Cent. consols	867	1	10	

State of the Balance.

	£	s.	d.
In the Bank of Ireland	£61	17	1
In the Treasurer's hands	40	15	1
	£102	12	2

ROBERT BALL, *Treasurer*.

No. XI.

ACCOUNT

OF THE

ROYAL IRISH ACADEMY,

FROM 1ST APRIL, 1846, TO 31ST MARCH, 1847.

THE CHARGE.

	£	s.	d.	£	s.	d.
To balance in favour of the Public on 31st March, 1846, ...	. . . . .			194	17	1
Parliamentary Grant for 1846 (paid 18th August, 1846), ...	300	0	0			
Quarterly Warrants from Treasury, . . .	146	17	8			
Total from Treasury, . . .				446	17	8
INTEREST ON STOCK:						
Half year's on £1643 19 6, $3\frac{1}{4}$ per Cents.	26	14	3			
Ditto, ,, 867 1 10, 3 ,, .	13	0	1			
Ditto, ,, 1643 19 6, $3\frac{1}{4}$,, .	26	14	3			
Ditto, ,, 867 1 10, 3 ,, .	13	0	1			
Total Interest on Stock, . .				79	8	8
TRANSACTIONS AND PROCEEDINGS SOLD:						
Transactions, Proceedings, and Taylor's Memoirs, sold here, ...	12	18	0			
Transactions and Proceedings sold in London, by Messrs. Boone, £14 12*s.* less Commission, £1 9*s.*, ...	13	3	0			
Total amount of Books sold, .				26	1	0
RENT OF STABLE, to 1st November, 1846, £21, less Poor-rate, 10*s.* 11*d.*	. . . . .			20	9	1
LIFE COMPOSITIONS:						
William Lloyd, M. D., ...	21	0	0			
Charles Haliday, Esq., ...	21	0	0			
Rev. William Reeves, ...	21	0	0			
Forward, ...	63	0	0	767	13	6

	£	s.	d.	£	s.	d.
Brought forward,	63	0	0	767	13	6
Rev. Henry Tibbs (non-resident),	15	15	0			
J. O. Curran, M. B.,	21	0	0			
Total Life Compositions,				99	15	0
ENTRANCE FEES:						
J. C. Deane, Esq.,	5	5	0			
John Alcorn, Esq.,	5	5	0			
P. Bevan, M. D.,	5	5	0			
M. H. Stapleton, M. B.,	5	5	0			
J. B. Kennedy, Esq.,	5	5	0			
J. O. Curran, M. B.,	5	5	0			
Abraham W. Baker, Esq.,	5	5	0			
George Lefroy, Esq.,	5	5	0			
John Aldridge, M. D.,	5	5	0			
T. P. Boyd, Esq.,	5	5	0			
Rev. R. M'Ghee,	5	5	0			
J. T. Beasly, Esq.,	5	5	0			
John Tyrrell, Esq.,	5	5	0			
Charles Haliday, Esq.,	5	5	0			
Charles P. M'Donnell, Esq.,	5	5	0			
F. J. Sidney, Esq.,	5	5	0			
W. N. Hancock, Esq.,	5	5	0			
William Brooke, Esq.,	5	5	0			
Rev. W. Reeves,	5	5	0			
D. J. Corrigan, M. D.,	5	5	0			
J. K. Ingram, Esq.,	5	5	0			
Leonard Dobbin, Esq.,	5	5	0			
Right Hon. Justice Perrin,	5	5	0			
Rev. H. Tibbs,	5	5	0			
Hon. and Rev. W. Wingfield,	5	5	0			
M. P. Darcy, Esq.,	5	5	0			
Total Entrance Fees,				136	10	0
ANNUAL SUBSCRIPTIONS:						
Charles Hanlon, Esq., ... 1846,	2	2	0			
W. M. O'Grady, M. D., ,,	2	2	0			
R. Adams, Esq., ,,	2	2	0			
A. Cane, Esq., ,,	2	2	0			
C. W. Hamilton, Esq., ,,	2	2	0			
Rev. Dr. West, ,,	2	2	0			
William Hogan, A. M., ,,	2	2	0			
Algernon Preston, Esq., ,,	2	2	0			
John Hamilton, Esq., ,,	2	2	0			
J. S. Cooper, Esq., ,,	2	2	0			
F. W. Barton, Esq., ,,	2	2	0			
Forward,	23	2	0	1003	18	6

		£	s.	d.	£	s.	d.
Brought forward,		23	2	0	1003	18	6
G. Fitzgibbon, Esq.,	1846,	2	2	0			
Rev. J. G. Abeltshauser,	,,	2	2	0			
William Edington, Esq.,	,,	2	2	0			
F. W. Conway, Esq.,	,,	2	2	0			
Most Rev. Archbishop of Dublin,	,,	2	2	0			
Sir Lucius O'Brien, Bart.,	,,	2	2	0			
Thomas Cather, Esq.,	,,	2	2	0			
Alexander Ferrier, Jun., Esq.,	,,	2	2	0			
William M'Dougall, Esq.,	,,	2	2	0			
Aquilla Smith, M. D.,	,,	2	2	0			
John Mollan, M. D.,	,,	2	2	0			
Rev. J. Wills,	,,	2	2	0			
J. Huband Smith, Esq.,	,,	2	2	0			
Sir John K. James, Bart.,	,,	2	2	0			
C. C. King, Esq.,	,,	2	2	0			
Thomas Beatty, M. D.,	,,	2	2	0			
Henry Clare, Esq.,	,,	2	2	0			
R. Tighe, Esq.,	,,	2	2	0			
Hon. F. Ponsonby,	,,	2	2	0			
William Murray, Esq.,	,,	2	2	0			
William Longfield, Esq.,	,,	2	2	0			
G. D. La Touche, Esq.,	,,	2	2	0			
James M'Donnell, Esq.,	,,	2	2	0			
Charles Bournes, Esq.,	,,	2	2	0			
E. J. Cooper, Esq.,	,,	2	2	0			
Alexander Taylor, Esq.,	,,	2	2	0			
J. F. Waller, Esq.,	,,	2	2	0			
G. J. Allman, M. D.,	,,	2	2	0			
Philip Reade, Esq.,	,,	2	2	0			
Edward Hutton, Esq.,	,,	2	2	0			
G. Yates, Esq.,	,,	2	2	0			
Rev. H. F. Logan, D. D.,	,,	2	2	0			
W. S. O'Brien, Esq., M.P.,	,,	2	2	0			
Edward Bewley, M. D.,	,,	2	2	0			
Rev. W. Lee,	,,	2	2	0			
Right Hon. Chief Baron,	,,	2	2	0			
J. Davidson, Jun., Esq.,	,,	2	2	0			
Rev. C. Porter,	,,	2	2	0			
Sir M. Barrington, Bart.,	,,	2	2	0			
J. Magee, Esq.,	,,	2	2	0			
M. O. R. Deane, Esq.,	,,	2	2	0			
N. P. O'Gorman, Esq.,	,,	2	2	0			
H. C. Beauchamp, M. D.,	,,	2	2	0			
James Pim, Jun., Esq.,	,,	2	2	0			
William Stokes, M. D.,	,,	2	2	0			
Forward,		117	12	0	1003	18	6

		£	s.	d.	£	s.	d.
Brought forward,		117	12	0	1003	18	6
James Talbot, Esq.,	1846,	2	2	0			
William Blacker, Esq.,	,,	2	2	0			
George Carr, Esq.,	,,	2	2	0			
D. P. Starkey, Esq.,	,,	2	2	0			
G. A. Frazer, Esq.,	,,	2	2	0			
R. W. Townsend, Esq.,	,,	2	2	0			
Rev. F. Crawford,	,,	2	2	0			
Hon. James King,	,,	2	2	0			
R. Cully, Esq.,	,,	2	2	0			
Rev. J. Galbraith,	,,	2	2	0			
H. G. Hughes, Esq.,	,,	2	2	0			
W. E. Hudson, Esq.,	,,	2	2	0			
Francis L'Estrange, Esq.,	,,	2	2	0			
Rev. George Longfield,	,,	2	2	0			
William Roberts, Esq.,	,,	2	2	0			
W. T. Kent, Esq.,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
M. Longfield, LL. D.,	,,	2	2	0			
J. Osborne, M. D.,	,,	2	2	0			
J. Hart, M. D.,	,,	2	2	0			
M. R. Sausse, Esq.,	,,	2	2	0			
Samson Carter, Esq.,	,,	2	2	0			
T. F. Kelly, Esq.,	,,	2	2	0			
Hon. Judge Crampton,	,,	2	2	0			
W. T. Mulvany, Esq.,	,,	2	2	0			
T. N. Redington, Esq.,	,,	2	2	0			
Lord Farnham,	,,	2	2	0			
Benjamin Wilme, Esq.,	,,	2	2	0			
W. C. Dobbs, Esq.,	,,	2	2	0			
James Claridge, Esq.,	,,	2	2	0			
James Jameson, Esq.,	,,	2	2	0			
J. S. Eiffe, Esq.,	,,	2	2	0			
James Apjohn, M. D.,	,,	2	2	0			
Robert Mallett, Esq..	,,	2	2	0			
Capt. Sterling,	,,	2	2	0			
Rev. Samuel Haughton,	,,	2	2	0			
William Lefanu, Esq.,	,,	2	2	0			
J. Neilson, Esq.,	,,	2	2	0			
W. Monsell, Esq.,	1845,	2	2	0			
John Phillips, Esq.,	1846,	2	2	0			
F. M. Jennings, Esq.,	,,	2	2	0			
T. Oldham, Esq.,	,,	2	2	0			
Lord Wallscourt,	,,	2	2	0			
Durham Dunlop, Esq.,	,,	2	2	0			
John Finlay, Esq.,	,,	2	2	0			
Forward,		212	2	0	1003	18	6

		£	s.	d.	£	s.	d.
Brought forward,		212	2	0	1003	18	6
Rev. N. J. Halpin,	1846,	2	2	0			
William Henn, Esq.,	,,	2	2	0			
William Andrews, Esq.,	,,	2	2	0			
James S. Close, Esq.,	,,	2	2	0			
Rev. J. H. Jellett,	,,	2	2	0			
Sir R. Morrison,	,,	2	2	0			
Charles Doyne, Esq.,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
Sir Edward Borough, Bart.,	,,	2	2	0			
Hon. and Very Rev. H. Pakenham, Dean of St. Patrick's,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
W. T. Lloyd, Esq.,	,,	2	2	0			
P. J. Blake, Esq.,	,,	2	2	0			
Robert Law, Esq.,	,,	2	2	0			
John M'Mullen, Esq.,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
C. Bolton, Esq.,	,,	2	2	0			
J. O'Driscoll, Esq.,	,,	2	2	0			
Abraham Palmer, Esq.,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
Sir Thomas Staples, Bart.,	,,	2	2	0			
R. C. Walker, Esq.,	,,	2	2	0			
Sir William Betham,	,,	2	2	0			
H. H. Joy, Esq.,	,,	2	2	0			
R. Sharpe, Esq.,	,,	2	2	0			
Arthur Jacob, M. D.,	,,	2	2	0			
W. Grimshaw, Esq.,	,,	2	2	0			
O. Sproule, Esq.,	,,	2	2	0			
J. M. Neligan, M. D.,	,,	2	2	0			
John Anster, LL. D.,	,,	2	2	0			
E. S. Clarke, M. D.,	,,	2	2	0			
Rev. Edward Marks, D. D.,	,,	2	2	0			
W. R. Wilde, Esq.,	,,	2	2	0			
E. Davy, Esq.,	,,	2	2	0			
R. W. Smith, Esq.,	,,	2	2	0			
G. A. Kennedy, M. D.,	,,	2	2	0			
Rev. James Reid,	,,	2	2	0			
Robert Reid, M. D.,	,,	2	2	0			
John Toleken, M. D.,	,,	2	2	0			
Francis Barker, M. D.,	1845,	2	2	0			
Ditto,	1846,	2	2	0			
J. T. Evans, M. D.,	,,	2	2	0			
Thomas Grubb, Esq.,	,,	2	2	0			
Edward Getty, Esq.,	,,	2	2	0			
Forward,		304	10	0	1003	18	6

	£	s.	d.	£	s.	d.
Brought forward, . . .	304	10	0	1003	18	6
Samuel Ferguson, Esq., ... 1845,	2	2	0			
Ditto, ... 1846,	2	2	0			
William Barker, M.B., ... „	2	2	0			
M. P. D'Arcy, Esq., ... 1847,	2	2	0			
James Patten, M. D., ... 1846,	2	2	0			
R. R. Madden, M. D., ... 1847,	2	2	0			
R. J. Graves, M. D., ... 1846,	2	2	0			
George Yates, Esq., ... 1847,	2	2	0			
Very Rev. James Gregory, Dean of Kildare, ... 1846,	2	2	0			
George Cash, Esq., ... „	2	2	0			
John Burrowes, Esq., ... 1847,	2	2	0			
Charles Vignoles, Esq., ... „	2	2	0			
Charles Hanlon, Esq., ... „	2	2	0			
Aquilla Smith, M. D., ... „	2	2	0			
Very Rev. Henry Cotton, Dean of Cashel, ... „	2	2	0			
Wyndham Goold, Esq., ... „	2	2	0			
George Wilkinson, Esq., ... 1846,	2	2	0			
Ditto, ... 1847,	2	2	0			
Rev. Thomas Stack, ... 1846,	2	2	0			
Ditto, ... 1847,	2	2	0			
Henry L. Lindsey, Esq., ... 1846,	2	2	0			
P. D. Hardy, Esq., ... „	2	2	0			
Ditto, ... 1847,	2	2	0			
James Pim, Jun., Esq., ... „	2	2	0			
William Smith O'Brien, Esq., M. P., „	2	2	0			
Total amount of Annual Subscriptions,				357	0	0
List of Subscriptions for the Purchase of the "Domnach Airgid" Irish Manuscript:						
C. P. Croker, M. D., ...	1	0	0			
J. Mac Cullagh, LL. D., ...	25	0	0			
Thomas Jennings, Esq., ...	1	0	0			
F. M. Jennings, Esq., ...	1	0	0			
G. W. Hemans, Esq., ...	1	0	0			
Rev. H. Lloyd, D. D., ...	25	0	0			
John Burrowes, Esq., ...	1	0	0			
William Hill, Esq., ...	0	10	0			
Arthur R. Nugent, Esq., ...	1	0	0			
Hon. Judge Crampton, ...	1	0	0			
Rev. Chas. Strong, ...	1	0	0			
Hon. G. S. Gough, ...	1	0	0			
Forward, ...	59	10	0	1360	18	6

	£	s.	d.	£	s.	d.
Brought forward,	59	10	0	1360	18	6
Thomas Fortescue, Esq.,	1	10	0			
Lord Wallscourt,	1	1	0			
Alexander Ferrier, Esq., Jun.	1	1	0			
J. R. Corballis, Esq.,	1	1	0			
Alexander Parker, Esq.,	1	0	0			
Aquilla Smith, M. D.,	1	0	0			
His Grace the Duke of Leinster,	5	0	0			
Marquis of Kildare,	5	0	0			
Right Hon. H. Labouchere, M.P.,	10	10	0			
Robert Forster, Esq.,	1	0	0			
Miss Edgeworth,	1	0	0			
C. K. Tenison, Esq.,	1	0	0			
Very Rev. Henry Cotton, Dean of Cashel,	1	0	0			
Rev. James Wilson, D. D.,	1	0	0			
Rev. S. Butcher,	1	0	0			
Bindon Blood, Esq.,	1	0	0			
T. N. Redington, Esq.,	2	2	0			
Rev. W. H. Drummond, D. D.,	1	0	0			
R. Franks, Esq.,	1	0	0			
Rev. T. R. Robinson, D. D.,	1	0	0			
A. B. Hope, Esq., M. P.,	3	0	0			
Earl of Leitrim,	10	0	0			
James Talbot, Esq.,	1	0	0			
William Lloyd, Esq.,	1	0	0			
William Stokes, M. D.,	2	2	0			
J. Davidson, Jun., Esq.,	1	0	0			
G. J. Allman, M.D.,	1	0	0			
Captain T. A. Larcom, R. E.,	5	0	0			
Bishop of Chichester,	1	0	0			
Bishop of St. David's,	5	0	0			
Robert Tighe, Esq.,	1	0	0			
W. Sweetman, Esq.,	5	0	0			
Charles Bournes, Esq.,	0	10	0			
Henry Courtney, Esq.,	1	0	0			
John Wynne, Esq.,	5	0	0			
J. O'Ferrall, M. D.,	1	1	0			
N. P. O'Gorman, Esq.,	1	0	0			
W. R. Wilde, Esq.,	1	1	0			
Wyndham Goold, Esq.,	1	0	0			
M. M. O'Grady, M. D.,	1	0	0			
Charles Vignoles, Esq.,	1	0	0			
M. P. D'Arcy, Esq.,	1	0	0			
Jacob Owen, Esq.,	1	0	0			
Charles Hanlon, Esq.,	1	0	0			
Col. Harry D. Jones, R. E.,	2	0	0			
Forward,	152	9	0	1360	18	6

	£	s.	d.	£	s.	d.
Brought forward, ...	152	9	0	1360	18	6
G. A. Grierson, Esq., ...	2	0	0			
E. J. Cooper, Esq., ...	10	0	0			
Haliday Bruce, Esq., ...	1	0	0			
Conyngham Ellis, Esq., ...	1	0	0			
A. B. Cane, Esq., ...	1	0	0			
Sir W. R. Hamilton, LL.D.	1	0	0			
Rev. Charles Mayne, ...	1	0	0			
W. E. Hudson, Esq., ...	5	0	0			
Thomas Cather, Esq., ...	1	0	0			
William Lefanu, Esq., ...	1	1	0			
A. Whyte Baker, Esq., ...	1	0	0			
Rev. Charles Graves, A.M., ...	10	0	0			
J. R. D., by E. Clibborn, Esq., ...	5	0	0			
Rev. J. H. Todd, D.D., ...	10	0	0			
Sir John M'Neill, ...	5	0	0			
Sir Thomas Esmonde, Bart., M.P., ...	5	0	0			
Total Amount of Subscriptions, .				212	10	0
THE TOTAL CHARGE, ...			£	1573	8	6

THE DISCHARGE.

Antiquities purchased, Etc.	£	s.	d.	£	s.	d.
Brown, Robert, repairing copper antiques, 26th May, 1846, ...	0	4	0			
Chambers, H., copper coins, 20th June, 1846,	0	1	0			
Donegan, John, gold ornament, 11th May, 1847, ...	2	0	0			
Ditto, gold brooch, 29th January, 1847, . .	2	0	0			
Enniskillen, Earl of, crozier, ...	1	5	6			
Gore, Francis, old glass bottle, 1st May, 1846,	0	2	6			
Griffith, Wm., antique gold ornament, 29th January, 1847, ...	2	15	0			
Kennan, Wm., for repairing antiquities, 26th June, 1846, ...	0	4	2			
Keegan, Charles, antique glass bottle, 1st February, 1847, ...	0	1	0			
Leary, Nicholas, coin and pike-head, 2nd November, 1846, ...	0	5	0			
M'Donagh, B., Cork, for repairing antiques, 27th June, 1846, ...	0	2	6			
Rice, Paul, antique bronze sword, 19th Feb., 1847, ...	1	0	0			
Scanlan, John, ancient spear-head, 20th June, 1846, ...	0	10	0			
Strickland, Thos., gold bracelet, 8th Feb., 1847,	9	5	0			
Underwood, James, two iron swords, 6th April, 1846, ...	1	0	0			
Wright, Antony, encaustic tiles, Aug. 9, 1846,	1	15	0			
Total amount of Antiquities, . . .				22	10	8
Books, Printing, and Stationery, Etc.						
Bellew, G., Stationery, 3rd December, 1846,	0	2	2			
Brown, J., initial wafers, 29th July, 1846, .	0	1	0			
Camden Society, subscription to 1st May, 1845, ...	1	0	0			
Du Noyer, G. V., drawings, 19th August, 1846, ... £5 0 0						
Ditto, do. 8th December, 1846, ... 1 11 6						
Ditto, do. 9th February, 1847, ... 4 10 0	11	1	6			
Forward, ...	12	4	8	22	10	8

	£	s.	d.	£	s.	d.	£	s.	d.
Brought forward, . . .				12	4	8	22	10	8
Edwards, Geo., manuscript copy of forfeited estates, ...	£25	0	0						
Ditto, for collating the above and index, ...	3	10	0						
Ditto, Ledwich's Antiquities, ...	1	1	0						
				29	11	0			
Ferrier and Co., ink, 15th June, 1846, ...	£0	4	6						
Ditto, do., 4th July, 1846, . .	0	1	9						
				0	6	3			
Geological Society, for wood-cuts, 10th July, 1846, ...				0	7	6			
Gill, M. H., petty printing, to 20th March, 1846, ...	£37	14	7						
Ditto, on account of printing Transactions and Proceedings, 23rd December, 1846, .	200	0	0						
				237	14	7			
Hanlon, G., wood-cuts, 15th Dec., 1846, . .				9	19	6			
Hodges and Smith, books, to 31st December, 1845, ...	£43	6	6						
Ditto, do., 31st December, 1846, ...	35	5	2						
				78	11	8			
Jones, blank account books, 26th August, 1846, ...				0	2	3			
M'Dowell, George, copperplate engraving, 11th August, 1846, .	£15	19	6						
Ditto, do., 22nd February, 1847,	8	8	0						
				24	7	6			
Marshall, A., Directory, 4th January, 1847,				0	10	6			
Millard, Thomas, wood-cuts, 7th August, 1846, ...				3	0	0			
Mullen, George, bookbinding, to 22nd May, 1845, ...	£18	6	9						
Ditto, do., to 31st December, 1846,	32	4	9						
Ditto, do., to 4th March, 1847, .	27	12	0						
				78	3	6			
O'Shaughnessy, J. J., copperplate printing, 24th August, 1846, .	£24	5	0						
Ditto, for tracing-paper, 22nd May, 1846, ...	0	1	4						
Ditto, pencils, 27th June, 1846,	0	2	6						
				24	8	10			
Forward, ...				499	7	9	22	10	8

				£	s.	d.	£	s.	d.
Brought forward, . . .				499	7	9	22	10	8
Oriental Translation Fund for 1845, ...	£10	10	0						
Ditto, for 1846,	10	10	0						
Ditto, for 1847,	10	10	0						
				31	10	0			
O'Neill and Duggan, wafers, 30th July, 1846, ...	£0	3	0						
Ditto, blank books, 19th February, 1847, . . .	1	3	0						
				1	6	0			
O'Reilly, John, books, 21st December, 1846, ...	£0	16	0						
Ditto, ditto, 27th February, 1847, ...	0	15	0						
				1	11	0			
Plunket, J., drawings, 25th April, 1846, ...	£0	7	6						
Ditto, do., 17th May, 1846,	1	0	0						
Ditto, do., 1st Aug., do. .	3	10	0						
Ditto, do., 13th Nov., do. .	3	1	0						
Ditto, do., 17th Oct., do. .	4	8	0						
Ditto, do., 13th Feb., 1847,	17	0	0						
				29	6	6			
Porter, Jos., books, 26th February, 1847, .				5	0	0			
Rathborne, J. and H., wafers, &c., 8th December, 1846, ...				0	2	6			
Ray Society, subscription for 1845, ...				1	1	0			
Robertson, John, book, 6th August, 1846 .				0	2	0			
Sharpe, for books, 1st October, 1846, . . .				1	17	6			
Taylor, R. and J. E., Scientific Memoirs, . .				16	4	0			
Talon, J., envelopes, &c., 21st Dec., 1846, . .				4	4	6			
Wiseheart, J., red ink, &c., 14th January, 1847, ...				0	2	2			
Total Books, Printing, &c., ...							591	14	11

Coals, Gas, &c.

	£	s.	d.	£	s.	d.
Alliance Gas Company, gas to 31st December, 1846, ...	17	19	9			
Edmundson, J. and Co., for oil, 10th January, 1846, ...	2	3	0			
Hoey, Charles, for 31 tons coals, and carriage, to 5th March, 1847, ...	31	16	0			
Total amount Coals, &c., ...				51	18	9
Forward, ...	. . . . .			666	4	4

	£	s.	d.	£	s.	d.
Brought forward, . . .				666	4	4
Repairs of House.						
Brown, John, glass, 24th July, 1846, . . . £0 4 6						
Ditto, cleaning windows, to 2nd June, 1846, . . . 3 0 0	3	4	6			
Casey, Paul, repairing grates, &c., 17th October, 1846, . . .	4	8	2			
Malone, P., cleaning carpets, 30th October, 1846, . . .	0	3	6			
Murphy, J., sweeping chimneys, 12th March, 1847, . . .	1	14	6			
Roberton, J., sweeping chimneys, 31st March, 1846, . . .	0	15	0			
Sibthorpe, H., and Son, glazing, 11th November, 1846, . . .	0	5	8			
Surman, George, carpenters' work, 12th Feb., 1846, . . .	39	12	6			
Total Repairs of House, . . .				50	3	10
Furniture and Repairs.						
Brown, John, painting, 27th January, 1846, .	1	10	0			
Edmundson, J., and Co., gas-fittings, ditto, .	3	16	10			
Nannetti, G., ornamental pedestal, 31st October, 1845, . . .	0	15	0			
Surman, George, repairs of sundries, 15th April, 1844, . . .	7	2	6			
Total Furniture and Repairs, . . .				13	4	4
Rent, Taxes, and Insurance.						
National Insurance Company, insurance, to 25th December, 1847, . . .	9	16	0			
Globe Insurance Company, do., to 25th December, 1847, . . .	5	13	6			
Rent of house, one year, to 1st Feb., 1847,	104	9	0			
Ministers' Money, to 29th September, 1846,	2	15	5			
Pipe Water Tax, to 24th June, 1846, . . .	1	19	2			
Total Rent, Taxes, and Insurance, .				124	13	1
Forward, . . .				854	5	7

					£	s.	d.	£	s.	d.
Brought forward,								854	5	7
SALARIES, WAGES, &c.										
Ball, Robert, Treasurer, one year, to 16th March, 1847,					21	0	0			
Clibborn, E., one year, to 16th March, 1847,					150	0	0			
Curry, A., sundry attendances, to 26th March, 1847,					5	5	0			
Drummond, Rev. W. H., D. D., Librarian, 16th March, 1847,					21	0	0			
Graves, Rev. Charles, A. M., Secretary of Council, 18th March, 1847,					21	0	0			
Hamilton, William, Hall Porter, to 13th March, 1847,	£34	4	8							
Ditto, Christmas allowance, do.	2	2	0							
					36	6	8			
Lockhart, J., suit of livery for Hall Porter, 26th March, 1846,					4	4	0			
Magill, John, Messenger, to 12th Sept., 1846,					18	15	0			
O'Brien, Thomas, do., to 27th March, 1847,					17	5	0			
Todd, Rev. J. H., D. D., Secretary of Academy, 16th March, 1847,					21	0	0			
Singleton, John, hat for Hall Porter, 26th March, 1847,					0	15	0			
Todhunter, Isaac, Accountant, to 27th March, 1847,					48	0	0			
Woodhouse, John, for livery buttons, 26th March, 1847,					1	16	6			
Total Salaries, Wages, &c.,								366	7	2
CONTINGENCIES.										
Bannin, William, rings for keys, 6th Feb., 1847,					0	5	8			
Barthes and Lowell, duty and charges on books, 9th July, 1846,					0	13	0			
Boone, T. and W., duty and charges on books,					2	8	6			
Box, W. R., gutta percha, 9th July, 1847,					0	8	6			
Boyle, Low, Pim, and Co., commission for receiving dividends on Stock,					0	5	2			
Clibborn, Edward, allowance for cleaning house, to 16th July, 1846,	£5	0	0							
Ditto, do., to 16th January, 1847,	5	0	0							
Ditto, do., to 16th January, 1846, omitted in last account,	5	0	0							
					15	0	0			
Forward,					19	0	10	1220	12	9

	£	s.	d.	£	s.	d.
Brought forward, . . .	19	0	10	1220	12	9
Clifford and Co., gum, &c., 9th July, 1846,	0	8	10			
Curry, E., carriage of parcel from Sligo, . .	0	2	0			
Daniel, P., brass nails, &c., 27th Oct., 1847, .	0	5	9			
Donnelly, M., cleaning ash pit, ...	0	3	0			
Gerty, E., coach-hire of deputation, 2nd September, 1846, ...	3	18	0			
Hodges and Sons, for nails, 6th March, 1847,	0	1	3			
Johnson, T., gum, &c., 31st January, 1846,	0	2	7			
Kennan, William, carriage of keg of bog butter, 31st July, 1846, ...	0	2	6			
Leg, William, repairing cask, 5th August, 1846, ...	0	2	4			
Maguire, twine, to 31st January, 1846, . .	0	16	3			
Pamplin, for duty and charges on books, 12th May, 1846, ...	1	2	6			
Rigby, W. and J., wad-punches, 2nd January, 1847, ...	0	5	0			
Sharp, R., repairing clock, &c., 31st January, 1847, ...	2	8	6			
Salmon and Co., one stone plaster of Paris, 17th October, 1846, ...	0	1	0			
Stamp receipt for Treasury warrant, . . .	0	5	0			
Yates, George, for tin cases, &c., 27th June, 1846, ...	1	13	3			
Freight and carriage of sundry packages from Liverpool and London, &c., ...	5	6	7			
Postages, and Post Office orders, ...	13	18	5			
Total Amount of Contingencies, . .				50	3	7
IRISH MANUSCRIPT.						
Paid on account of the purchase of the "Domnach Airgid,"	. . . . .			200	0	0
Total Discharge, ...	. . . . .		£	1470	16	4
Balance in favour of the Public, . .	. . . . .			102	12	2
Total amount of Charge, ...	. . . . .		£	1573	8	6

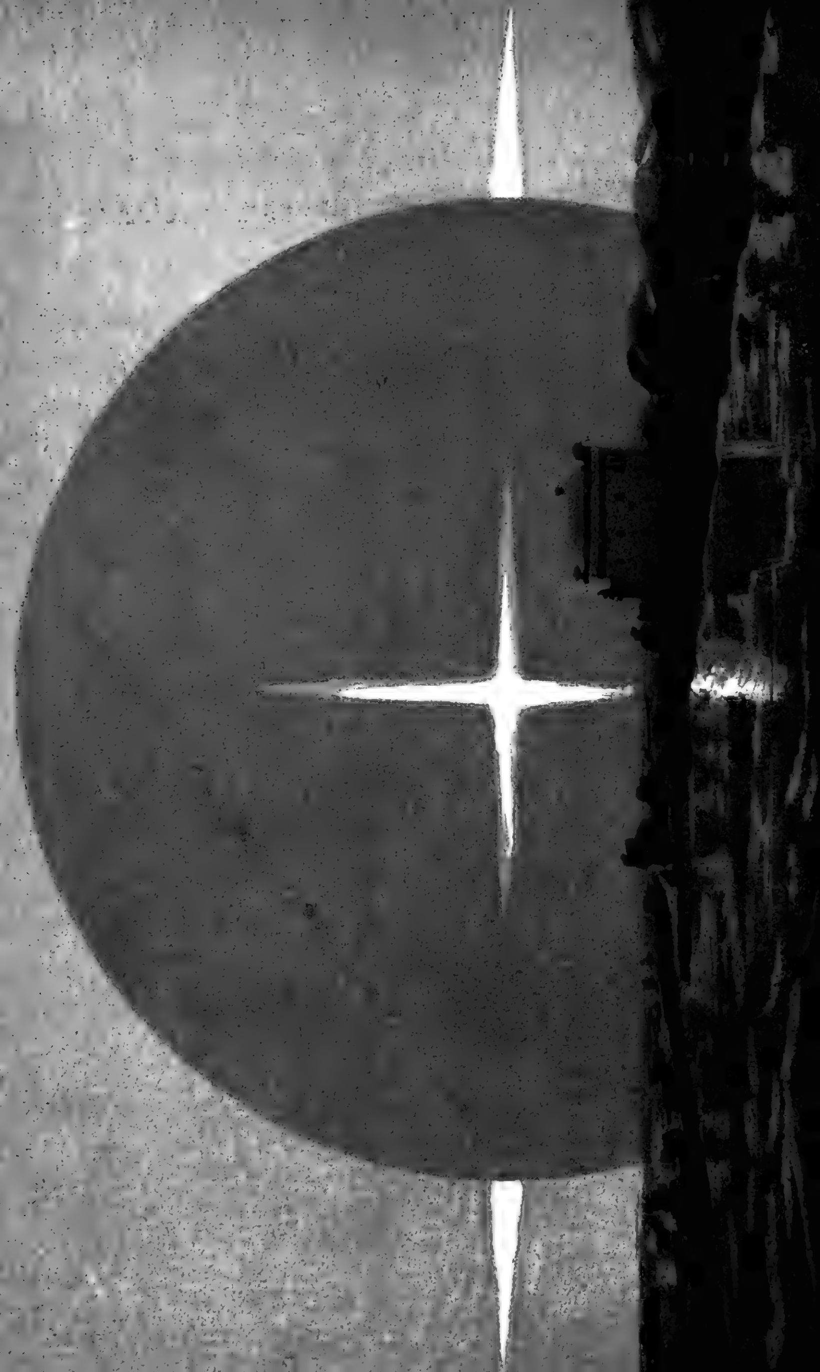
STATE OF THE BALANCE.

	£	s.	d.
In Bank, ...	61	17	1
In Treasurer's hands, as per Account, ...	40	15	1
	£102	12	2

The Treasurer reports that there is to the credit of the Academy in the Bank of Ireland, £867 1*s.* 10*d.*, in Three per Cent. Consols, and £1643 19*s.* 6*d.*, in Three and a quarter per Cent. Government Stock, the latter known as the Conyngham Fund.

(Signed,) ROBERT BALL,
Treasurer.

31*st March*, 1847.



15

Su nana

Nos infrascripti fidem facimus et per praesentes attestamur has annexas duas copias transumptas fuisse per fr͡em Michaëlem Clery ex duobus libris manuscriptis, quibus a linguae Hybernicae peritioribus hucusque fides adhibebatur, uno nimirum vetustissimo codice pertinente ad Clam y Ululenner, et altero, qui est vera copia celeberrimi et vetustissimi codicis nunc Dublinii reservati; easdemque copias de verbo ad verbum sine styli, ordinis, aut substantiae rerum mutatione aut corruptione cum eisdem nos protothypis per omnia concordare. In cuius rei fidem et testimonium his subscripsimus in deserto nostrae mansionis die 29 Aprilis 1636

Fr Bernardinus Clery
Guardianus Dunny

Fr Mauritius Ultanus

Fr Mauritius Ultanus

Augusti 1644

Addictissimi

Mountgarret

Thomas Archiepūs Cashellensis

Nicolaus Epūs Fernensis

D. Lo. Henry

Daniel O Bryen

Bellings

B. Preston

Rob: Lynch

Her: O Shaghnusse

Excellentissime Dñe

Pergratū nobis fuit pro Catholicâ fide demicantibus Belgicarū rerū summam, in Vᵉ Excellentissime Dñe translatam fuisse, et cum vestri animi propensio, et studia in gentem, et causam nostrum satis nota sint et perspecta et quo affectu Austriacæ Domui adhæserint Hiberni, non sit qui dubitet, Excellentiam ergo vestram quanto potest favore res nostras prosecuturā confidimus. Quo in statu sint aperiet Rᵈᵘˢ Pater Hugo de Burgo Ordinis Sᵗⁱ Francisci Religiosus, cui ut indubitata fides adhibeatur rogamus

Excellentissime Dñe
Vestræ
Addictissimi

Kilkeniæ 24ᵃ die Augusti 1644

Thomas Archiep. Cashelensis
Malachias Archiep. Tuamensis
Fr. Lo. Fernens.
Mountgarret
Daniel O'Brien
B. Prestoun
Rob. Lynch
Bellings
Der. O Shaghnusse

…DEMY

Contingencies.			Furni… Re…	…rors in Balances not explained.			Total Discharges.			Balances carried forward.		
£	s.	d.	£	£	s.	d.	£	s.	d.	£	s.	d.
46	8	11	129	. . . . .			1880	1	6½	790	14	8
22	18	5	53	. . . . .			1016	11	9½	823	19	2½
23	7	5	7	. . . . .			1030	8	4½	511	9	8
13	13	7½	. . .	2	5	0	853	17	1	566	12	4
39	11	2½	. . .	. . . . .			873	16	4	259	4	3
18	0	7½	. . .	. . . . .			770	3	3½	212	19	5½
34	13	5½	. . .	. . . . .			1173	5	7½	166	17	1
24	19	5½	. . .	. . . . .			1131	1	2½	164	17	10½
25	5	6½	22	. . . . .			884	14	8	230	3	1½

No. IV.

SYNOPSIS OF THE ACCOUNTS OF THE ROYAL IRISH ACADEMY,

FROM MARCH 17, 1816, TO MARCH 31, 1846;

PREPARED FROM THE RETURNS AS CERTIFIED BY THE GOVERNMENT AUDITORS.

Printed by Order of Council, 16th February, 1846.

From	To	Stock at End of Year			Total annual Amount of private Income	Private Income							Public Income		Proceeds of Stock sold		Balances of previous Audit	Total Charges
			3½ per Cents	3 per Cents		Interest on Stocks	Books sold	Rent of Stalls	Life Compositions	Entrance Fees	Annual Subscriptions	Donations to purchase MSS	Allowance from Treasury	Grants from Parliament				
		£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
[illegible]	5 January	These amounts are in the old, or Irish currency, the remainder are in British currency	1400 0 0	......	948 15 4½	80 10 0	21 19 4½	17 1 3	591 10 0	96 13 9	141 1 0	. .	249 19 6	769 0 0	....	..	703 1 4	2670 16 2½
5 January	5 January		1800 0 0	. .	533 3 4	71 17 11½	11 19 1½	31 2 6	329 17 6	28 8 9	56 17 6	. .	166 13 0	350 0 0	....	...	790 14 8	1640 11 0
5 January	5 January		2500 0 0	......	226 5 10	82 5 0	3 0 0	17 1 3	15 10 0	39 16 1	58 13 6	. . .	166 13 0	325 0 0		...	823 19 2½	1511 18 0½
5 January	5 January		3000 0 0	......	415 1 9	96 5 0	3 15 0	17 1 3	188 16 6	45 10 0	63 11 0	......	166 13 0	325 0 0	......	......	511 9 8	1418 4 5
5 January	5 January		3250 0 0	......	399 15 3	111 5 0	3 0 0	17 1 3	152 0 0	22 15 0	63 14 0	......	166 13 0	......	......	......	566 12 4	1133 0 7
5 January	5 January		3450 0 0	......	315 12 0	121 0 0	3 10 0	17 1 3	......	......	171 0 9†	......	83 6 6	325 0 0	......	......	259 4 3	983 2 9
5 January	5 January		3900 0 0	......	552 3 9	129 17 6	165 4 9	17 1 3	139 5 0	17 1 3	63 14 0	......	249 19 6	325 0 0	......	......	212 19 5½	1340 2 8½
5 January	5 January		4162 10 0	......	312 9 0	143 2 6	3 5 0	17 1 3	45 10 0	62 11 3	40 19 0	......	166 13 0	650 0 0	......	......	166 17 1	1295 19 1
5 January	5 January		4362 10 0	......	358 7 0	154 0 0	13 5 0	17 1 3	68 5 0	29 8 9	77 7 0	. . .	166 12 11	325 0 0	. .	......	164 17 10½	1014 17 9½
5 January	5 January	4 per Cents.	4100 0 0		242 1 0¾	162 9 6¾	0 7 6	.	42 0 0	31 10 0	35 14 0	. .	146 17 8	[illegible] 0 0	......	Irish. 230 3 1½	British. 212 9 0¾	901 7 9½
5 January	5 January	200 0 0	4100 0 0	......	342 1 0	151 17 6	12 14 6	31 10 0	21 0 0	68 5 0	56 14 0	......	146 17 8	300 0 0	......	......	262 4 10¾	1051 3 6¼
5 January	5 January	242 6 0	3984 12 0	......	456 6 10	149 3 0	20 10 10	15 15 0	84 0 0	105 0 0	81 18 0	......	146 17 8	300 0 0	......	......	342 13 2	1245 17 8
5 January	5 January	242 6 0	3984 12 0	......	470 9 3	149 3 0	10 15 3	15 15 0	115 10 0	52 10 0	126 16 0	......	182 16 8	299 10 10	2727 6 1	......	548 3 1	4228 5 11
5 January	5 January		1500 0 0	3150 0 0	466 13 5	144 15 0	33 3 6	21 0 0	106 1 0	63 0 0	98 14 2	......	146 13 7	299 3 9	......	......	381 9 3	1294 0 3
5 January	5 January		1500 0 0	3150 0 0	497 5 0	147 0 0	24 11 0	21 0 0	141 18 0	57 15 0	104 18 0	......	148 9 6	300 0 0	......	......	496 2 3	1441 16 9
5 January	31 March		1500 0 0	3150 0 0	519 8 6	220 10 0	19 6 6	21 0 0	64 1 0	75 15 0	115 16 0	......	192 6 3	300 0 0	......	......	137 2 4	1148 17 1
1 April	31 March	.	1500 0 0	3150 0 0	655 11 6	147 0 0	23 9 6	21 0 0	231 0 0	115 10 0	117 12 0	......	153 17 0	375 0 0	......	......	336 4 11	1520 13 5
1 April	31 March		1500 0 0	2000 0 0	602 2 9	132 0 0	28 1 9	21 0 0	204 15 0	105 0 0	111 6 0	......	153 17 0	300 0 0	1044 6 0	......	133 15 7	2233 15 10
1 April	31 March	. . Cunningham Fund	1500 0 0	2000 0 0	667 14 0	97 10 0	26 6 0	21 0 0	231 0 0	147 0 0	144 18 0	......	153 17 0	300 0 0	462 0 8	......	340 3 11	1924 4 7
1 April	31 March	. . .	1500 0 0	1500 0 0	614 5 7	97 10 0	11 14 7	21 0 0	222 12 0	120 15 0	140 14 0	......	153 17 0	300 0 0	......	......	160 14 0	1328 16 7
1 April	31 March	.	1500 0 0	1500 0 0	641 5 0	97 10 0	5 2 0	21 0 0	195 6 0	120 15 0	201 12 0	......	148 12 6	300 0 0	......	......	202 4 1	1292 1 7
1 April	31 March	. . .	1500 0 0	1500 0 0	734 0 5	97 10 0	26 9 5	21 0 0	186 18 0	162 15 0	239 8 0	......	146 17 8	300 0 0	......	......	227 8 3	1408 6 4
1 April	31 March	. .	1500 0 0	1344 10 2	706 1 0	95 15 4	20 3 8	21 0 0	207 18 0	84 0 0	277 4 0	......	146 17 8	300 0 0	142 4 0	Rebatement of Poor Rate.	130 11 0	1425 14 5
1 April	31 March		1500 0 0	1367 5 8	592 13 1	94 6 7	2 14 6	21 0 0	165 18 0	81 0 0	224 14 0	......	146 17 8	300 0 0	......	1 1 8	74 3 9	1114 16 2
1 April	31 March	. .	1551 4 1	1286 11 3	736 12 8	97 0 8	37 19 0	21 0 0	170 2 0	162 15 0	247 16 0	......	146 17 8	300 0 0	530 12 0	......	59 1 4	1173 3 8
1 April	31 March		1643 19 6	1117 1 10	624 6 8	88 7 4	8 17 4	21 0 0	99 15 0	110 5 0	296 2 0	......	146 17 8	300 0 0	......	......	108 9 4½	1239 13 8
1 April	31 March	.	1643 19 6	1117 1 10	462 8 7	95 6 4	14 6 3	21 0 0	15 15 0	89 5 0	226 16 0	......	146 17 8	300 0 0	334 2 3	......	210 13 9½	1414 2 3½
1 April	31 March*		1643 19 6	1117 1 10	680 14 0	76 8 0	......	21 0 0	241 10 0	210 0 0	331 16 0	658 5 6	146 17 8	900 0 0	......	......	210 5 6½	2716 2 8½
1 April	31 March*		1643 19 6	867 1 10	923 6 9	81 18 5	100 2 4	21 0 0	123 18 0	241 10 0	354 18 0	16 4 0	146 17 8	300 0 0	245 12 3	......	110 11 0	1751 11 8

From	To	Books purchased	Printing and Paper	Binding	Stationery	Coals, Candles, &c.	Contingencies	Furniture and Repairs	Repairs of House and Alterations	Rent, Taxes, and Insurance	Salaries, Wages, &c.	Government Stock purchased	Manuscripts purchased	Premiums	Antiquities purchased	Commission on collecting Subscriptions	Errors in Balances not explained	Total Discharges	Balance carried forward
		£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
[illegible]	5 January	52 10 8½	71 6 2	......	9 19 1½	48 1 6	16 8 11	129 17 5½	277 3 5½	592 17 [illegible]	111 10 0	330 8 3	......	50 0 0	......	59 18 8	. . .	1880 1 6½	790 14 8
5 January	5 January	12 4 6½	132 10 9	.....	......	32 15 3	22 18 5	53 10 5	9 15 5½	151 6 2½	108 5 0	568 6 0	. . .	. .	. . .	31 19 9	.	1016 11 9½	823 19 2½
5 January	5 January	4 1 6	97 11 5½	21 3 2½	......	21 19 10	23 7 5	7 5 0	2 11 6	213 6 6½	198 16 0	110 19 8	......	17 14 0		11 6 3	. .	1039 8 1½	511 9 8
5 January	5 January	29 9 10	129 17 7½	13 5 4	......	31 13 5	13 13 7½	......	13 1 5	189 8 10	188 7 0	224 4 11	......	......	......	20 15 1	+2 5 0	853 17 1	566 12 4
5 January	5 January	58 5 11	31 19 2½	20 12 0	......	20 9 11	39 11 2½	......	6 1 4	203 16 10	201 13 8	267 17 10	......	10 0 0	......	13 8 5	......	873 16 4	259 4 3
5 January	5 January	50 10 2	76 1 9	7 18 6	......	25 12 4	18 0 7½	......	6 0 0	190 6 4	211 0 0	173 19 7	......	......	......	8 14 0	......	770 3 3½	212 19 5½
5 January	5 January	27 1 5½	114 4 3½	59 4 11	......	35 8 0	34 13 5½	. . .	63 1 3	160 13 6	211 0 0	395 18 9	......	60 0 0	......	12 0 0	......	1173 5 7½	166 17 1
5 January	5 January	35 12 11	106 11 4	2 14 2	......	31 1 4	24 19 5½	......	8 16 3	179 13 0	211 0 0	418 3 9	......	75 0 0	......	7 9 0	......	1131 1 2½	164 17 10½
5 January	5 January	29 8 7	347 1 1½	8 2 6	. . . .	31 16 4	25 5 6½	22 14 10	27 13 3	169 18 6	211 0 0	. .	......	. . .	. . .	8 14 0	. . .	854 14 8	230 3 1½
5 January	5 January	35 12 9	98 16 9	1 9 0	. . . .	26 17 1¼	28 2 11½	. . . .	12 5 7½	155 1 1½	194 15 5	. . . .	......	77 13 0¾	......	5 9 2	......	639 2 11¼	262 4 10¾
5 January	5 January	38 11 11	1 10 0	2 10 0	......	26 16 3½	16 15 4	55 5 0	27 3 8	152 18 5½	194 15 5	181 18 4	......	......	......	7 5 11½	......	708 10 4½	342 13 2
5 January	5 January	50 14 1	42 16 6	24 6 0	......	29 15 3½	16 4 8	......	6 18 9	148 13 0½	194 15 5	......	......	170 0 0	......	13 10 10	......	697 14 7	548 3 1
5 January	5 January	71 9 11	305 6 11	11 5 0	......	29 0 9	80 11 10	......	60 11 8	151 0 0½	195 8 0½	2718 12 5	104 3 7	74 11 9	......	14 14 9	......	3846 16 8	381 9 3
5 January	5 January	116 10 4	......	10 8 6	2 2 2	27 2 10	27 1 3	......	18 7 0	141 11 3	202 19 6	139 7 10	68 16 7	......	......	13 7 9	......	797 18 0	496 2 3
5 January	5 January	88 10 7	533 13 8	51 7 8	5 17 2	33 3 0	14 2 10	......	39 19 0	156 2 9	204 0 0	......	142 13 3	20 0 0	......	15 4 6	......	1304 14 5	137 2 4
5 January	31 March	30 17 6	11 3 3	42 16 0	11 16 7	17 7 0	29 8 0	5 0 0	20 19 5	233 5 11	326 19 2	......	......	70 0 0	......	12 18 6	......	812 11 4	336 4 11
1 April	31 March	179 17 2	82 5 0	8 6 8	6 3 9	25 4 4	26 7 4	......	527 13 1	145 16 6	204 0 0	......	47 5 0	110 0 0	......	23 1 0	—1 5 0	1357 7 10	133 15 7
1 April	31 March	219 1 7	5 1 0	10 7 4	6 18 6	35 19 1	51 19 3	......	961 13 2	155 11 0	216 10 0	......	165 0 0	44 0 0	......	21 1 0	......	1593 1 11	340 3 11
1 April	31 March	656 1 6	255 4 1	235 5 8	6 10 0	32 3 1	20 16 8	38 16 4	90 7 1	158 16 2	243 1 2	......	......	......	......	26 2 10	......	1763 10 7	160 14 0
1 April	31 March	213 11 6	149 19 5	51 5 0	8 18 3	42 9 1	21 4 0	......	19 18 1	160 1 6	218 6 8	......	87 15 0	......	......	24 4 0	+1 0 0	1026 12 6	202 4 1
1 April	31 March	152 10 7	65 17 1	62 5 0	6 15 7	48 18 8	27 15 8	......	85 18 8	162 1 7	244 16 5	......	88 6 6	......	73 10 0	25 17 7	......	1064 13 4	227 8 3
1 April	31 March	169 4 1	227 14 6	60 14 0	23 4 3	55 10 0	30 8 10	......	137 7 8	159 18 5	235 4 1	......	40 0 0	39 10 0	......	29 9 0	......	1277 14 7	130 11 0
1 April	31 March	115 12 4	390 14 4	151 8 10	5 6 6	45 17 6	36 6 7	......	135 3 9	140 10 11	250 13 6	......	18 8 0	19 19 4	4 0 0	28 9 1	......	1351 10 8	74 3 9
1 April	31 March	51 19 0	285 1 8	49 0 6	8 6 9	47 6 1	17 11 11	......	56 8 2	149 19 3	250 7 9	115 19 2	......	......	......	23 14 7	......	1055 14 10	59 1 4
1 April	31 March	90 17 0	558 11 1	84 14 4	5 13 10	24 17 5	35 0 8	......	72 10 10	147 11 10	177 8 10½	263 16 2	......	......	113 11 6	......	......	1574 14 3½	198 9 4½
1 April	31 March	10 10 0	13 10 6	......	1 2 0	38 18 2	21 5 8	17 17 6	16 3 6	159 17 11	343 18 8	388 8 4	26 19 0	......	20 8 8	......	......	1058 19 11	210 13 9½
1 April	31 March	78 13 7	316 17 8	106 12 2	2 0 6	0 2 5	51 9 7	18 3 8	7 8 6	138 9 1	262 12 2	49 18 7	93 9 0	19 15 10	56 11 0	......	......	1243 16 9	210 5 6½
1 April	31 March*	81 0 3	9 8 6	52 14 6	11 19 9	61 5 7	45 1 7	63 1 5	198 3 11	134 0 4½	265 4 1	......	1358 11 0	......	393 0 9	......	......	2676 11 8½	119 11 0
1 April	31 March*	611 0 4‡	93 19 8	34 19 8	3 0 6	38 17 10	18 3 1	118 10 11	37 9 5	125 6 2	370 10 7	......	6 0 0	......	68 19 0	......	......	1556 17 2	194 14 6

* The accounts for these dates have not yet been certified by the Audit Commissioners.

† This Sum contains the Life Compositions and Entrance Fees, which are not specified in the audited Account for this Year.

‡ Of this sum, £622 11s. was paid Mr. Petrie, that being the balance due him on account of the Essay on the Round Towers.